2020 年淡水河口潮汐不對稱性調查

Investigation of Tidal Asymmetry at the Danshuei River Estuary in 2020

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Abstract

Tidal asymmetry refers to the periodic difference between failing and rising tides whose information can be used for sediment analysis, water circulation analysis, and water depth navigation along the river from upstream to estuary. Tidal propagation in the estuary itself is affected by topography and astronomical tidal constituents. Data of water level in 6 sites along Danshuei River in northern Taiwan was taken every hour in 1 year period of 2020 starting from January to December. In this study, to analyze the tidal asymmetry, the harmonic analysis and tidal skewness were used. From the result of the water level measurement, the tidal wave propagates in uniformly due to the shallow water constituent from upstream to downstream. The largest constituent along the river estuaries was the principal lunar semidiurnal constituent M2 and followed by principal solar semidiurnal constituents S2 which is astronomical constituent. The tidal form result ranged from 0.22 to 0.3 stated that mixed, mainly semi-diurnal nature was determined. The simulated water surface yields the best results with a low root mean square value and an energy ratio that approaches 1.0. The next analysis, conducting the six combinations of tidal constituent, is to reveal the behavior of tidal wave of the Danshuei River with harmonic analysis and tidal skewness method. According to harmonic method, the principal lunar and shallow water over tides constituents, M2 and M4, are involved to measure the behavior of tidal asymmetry. The relative phase degree of M2 and M4 constituents ranged from 15.34 to 178.962 indicated that flood dominance of tidal wave was observed. Also, it is validated by tidal skewness approach which results in positive value of skewness at all stations. Also, the M4/M2 amplitude ratio ranged between 0.027 to 0.141 shows that the river has significant tidal distortion. The M2 and M4 interaction was the main contributor of tidal asymmetry in this study followed by M2-S2-MS4 combination.

Keywords: Tidal asymmetry, Harmonic analysis, Tidal skewness, Danshuei River

1. Introduction

Tidal asymmetry is generated from the behavior of tidal ocean itself within arbitrary change leading to the shallow water areas. Tidal asymmetry refers to a periodic difference between the falling and rising tidal periods. Specifically, a shorter duration of the rising tide indicates a flood-dominant tidal asymmetry, and a shorter duration of falling tide indicates an ebb-dominant tidal asymmetry [8]. In estuarine and coastal systems, tidal asymmetry influences sediment movement and water circulation system. As a result, the tidal dynamic potentially generate morphological changes in the water system eventually [2].

Several factors contribute to this phenomenon, including the nonlinear effects of tidal interaction with topography and astronomical tide constituents; including bottom friction, river inflow, bathymetric changes, and the morphology of such systems [7]. Bottom frictions are essential in the dissipation of tidal energy and the evolution of wave energy from the major tidal elements to other frequencies. These frequency sets are referred to as shallow water tidal components [12]. River inflow has a significant impact on estuarine tidal dynamics. River flow, on the other hand, can raise the average water level, reducing tidal amplitudes and slowing the celerity of tidal propagation, so deforming the tidal waves [12]. The shape of tidal systems can influence their tidal dynamics. Estuaries with funneling shapes and decreasing cross-sections can dampen tidal wave energy by reflection against barriers [4].

Any combination of tidal constituents with angular frequencies $\omega_1, \omega_2, \omega_3$ (pair or triads constituents) hat satisfy either the frequency relationship $2\omega_1 = \omega_2$ or $\omega_1 + \omega_2 = \omega_3$ can contribute to tidal asymmetry [15]. This method is also called harmonic method, which requires the calculation of amplitude and phases of tidal constituents. The contribution of different combinations of constituents to the overall asymmetry in the rise and fall of water levels can be conveniently quantified adopting a skewness-based approach which was proposed by Nidzieko (2010) [13] and extended by Song et al. (2011) [5].

A three-dimensional hydrodynamic model was applied to the Danshuei River estuarine system in northern Taiwan to investigate the influence of flood-ebb, spring-neap tidal cycles, and salinity distribution on tidal mixing, residual circulation, stratification, and tidal asymmetry. The Danshuei River is formed by the confluence of Tahan Stream, Hsintien Stream, and Keelung River. The tide influences and exposes the downstream parts of all three streams to seawater incursion. They create Taiwan's biggest estuary system when combined [9]. It has been determined that on the Danshuei River, using data from 2008, the river has flood dominance in the estuary system, and the simulation results have also been validated by observations [14]. Correspondingly, the aim of the current study is to examine the tidal dynamics and tidal wave deformation based on continuous field measurement of water level at several location along estuary within a new 2020 dataset using tidal harmonic and tidal skewness method.

2. Material and Method

2.1. Data Collection

The Danshuei River watershed encompasses 2726 km2, with a maximum length of approximately 70 km from the headwaters to the river mouth; it drains from a mountainous area with a maximum elevation of 3500 km. The estuarine reach is more than 20 km in length, and the average river width and depth are about 600 m and 6.5 m, respectively (**Figure 1**) [16].

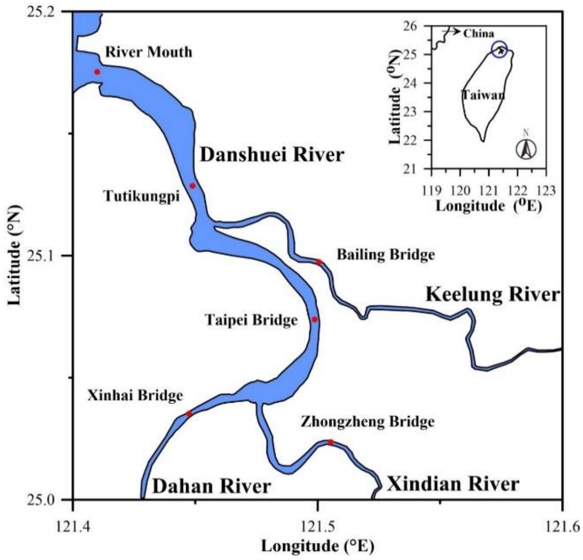


Figure 1. Location Map of the Study Area

The essential data required to carry out the study aim is water level observations. Furthermore, hourly water level data are taken at 6 sites: River Mouth, Tutikungpi, Bailing Bridge, Taipei Bridge, Xinhai Bridge, and Zhongzheng Bridge. These locations' coordinates and data record length are provided in **Table 1.**

Table 1. Data Record Period and Location

Station	Latitude	Longitude	Period of Record		
River mouth	25.174°N	121.41°E	1/1/2020	to	31/12/2020
Tutikungpi	25.128°N	121.4494°E	1/1/2020	to	31/12/2020
Bailing Bridge	25.098°N	121.502°E	1/1/2020	to	31/12/2020
Taipei Brige	25.0736°N	121.499°E	1/1/2020	to	31/12/2020
Xinhai Bridge	25.0364°N	121.448°E	1/1/2020	to	31/12/2020
Zhongzheng Bridge	25.024°N	121.506°E	1/1/2020	to	31/12/2020

For harmonic analysis, ten tidal constituents were used. The period of tidal constituent shown on **Table 2** below.

Table 2. Tidal Constituents Period

Tidal constituents	Period (sec/cycle)	Period (hour/cycle)
M_2	44714.1644	12.42
S_2	43200.0000	12.00
N_2	45570.0535	12.66
K_1	86164.0908	23.93
O_1	92949.6300	25.82
M_4	22357.0822	6.21
M_6	14904.7214	4.14
MS_4	21972.0214	6.10
MN_4	22569.0254	6.27
MK_3	29437.7049	8.18

2.2. Methodology

2.2.1. Harmonic Analysis of Tide

The harmonic method is commonly used to study tidal dynamics and asymmetry in estuarine and coastal systems. The amplitudes and phase of the tidal component combination are calculated from the recorded tidal data series [3]. The harmonic model's equation is shown below.

$$h(x,t) = h_0 + \sum_{j=1}^{m} f_j H_j \cos\left(\omega_j t + u_j - k_j^* \right) \tag{1}$$

where t is the time in hours, $h(x,t)$ is the predicted water level, f_j is the lunar node factor for constituent, H_j is the amplitude for constituent, h_0 the mean water level in that location, u_j is the nodal phase for constituent, $\kappa_j{}^*$ is the phase of constituent, ω_j is the frequency of constituent, and m is the number of constituents.

The tidal response of shallow estuarine systems is often characterized by the formation of tidal distortion and asymmetry in the estuary as a result of tidal harmonic interactions. The M_2 and M_4 tides, as the most important astronomical and shallow-water constituents, respectively, possess a mathematical relationship which is always used as an indicator of the degree of tidal distortion. The M_4 to M_2 sea-surface amplitude ratio, A_r, and the sea-surface phase of M_4 relative to M_2, G_r, defined by Aubrey and Speer [1985] are calculated in this study to investigate the characteristic of tidal asymmetry, the expressions of which are as follows [1][2][10]

$$A_r = \frac{a_{M4}}{a_{M2}} \tag{2}$$

$$G_r = 2\theta_{M2} - \theta_{M4} \tag{3}$$

where a_{M4} and a_{M2} are the amplitudes of M_4 and M_2; θ_{M4} and θ_{M2} are the relative phases of M_4 and M_2; $A_r > 0{,}01$ indicates that the distortion of the tidal wave is significant; $0° < G_r < 180°$ indicate that the location tends to flood-dominance while $180° < G_r < 360°$ indicate the location tends to ebb-dominance. Similarly, using triad constituent the value of G_r and determination of tidal asymmetry same as the two constituents with G_r is equal to [1][5]:

$$G_r = \theta_1 + \theta_2 - \theta_3 \tag{4}$$

2.2.2. Tidal Skewness

Tidal skewness is a statistical approach based on calculation of the probability density function of water level proposed by Nedzieko (2010) and extended by Song et.al (2011) [13][5]. This methodology can estimate how much each combination of tidal constituents contributes to total tidal asymmetry. The tidal skewness resulting from combination of two tidal constituents is given as follow [5]:

$$\gamma_2 = \frac{\frac{3}{4} a_1^2 \omega_1^2 a_2 \omega_2 \sin(2\theta_1 - \theta_2)}{\left[\frac{1}{2}(a_1^2\omega_1^2 + a_2^2\omega_2^2)\right]^{\frac{3}{2}}} \tag{5}$$

The skewness resulting from combination of three tidal constituents is given as follow [5]:

$$\gamma_3 = \frac{\frac{3}{4} a_1 \omega_1 a_2 \omega_2 a_3 \omega_3 \sin(\theta_1 + \theta_2 - \theta_3)}{\left[\frac{1}{2}(a_1^2\omega_1^2 + a_2^2\omega_2^2 + a_3^2\omega_3^2)\right]^{\frac{3}{2}}} \tag{6}$$

Where a_n, θ_n, ω_n are amplitude, phase, and frequency of the tidal constituent. The contribution of different combination to the total tidal asymmetry is obtained as follow [5]:

For combination of two constituents

$$\beta_2 = \gamma_2 \left(\frac{a_1^2\omega_1^2 + a_2^2\omega_2^2}{\sum_{i=1}^{N} a_i^2 \omega_i^2}\right)^{\frac{3}{2}} \tag{7}$$

For combination of three constituents

$$\beta_3 = \gamma_3 \left(\frac{a_1^2\omega_1^2 + a_2^2\omega_2^2 + a_3^2\omega_3^2}{\sum_{i=1}^{N} a_i^2 \omega_i^2}\right)^{\frac{3}{2}} \tag{8}$$

Hence, the total skewness can be obtained by the summation of β

$$\gamma_N = \Sigma\beta \tag{9}$$

The direction of tidal asymmetry is determined by the sign of the total skewness γ_N. The positive value of γ_N refer to flood dominance while the negative value refers to ebb dominance of tidal asymmetry [5].

3. Result and Discussion

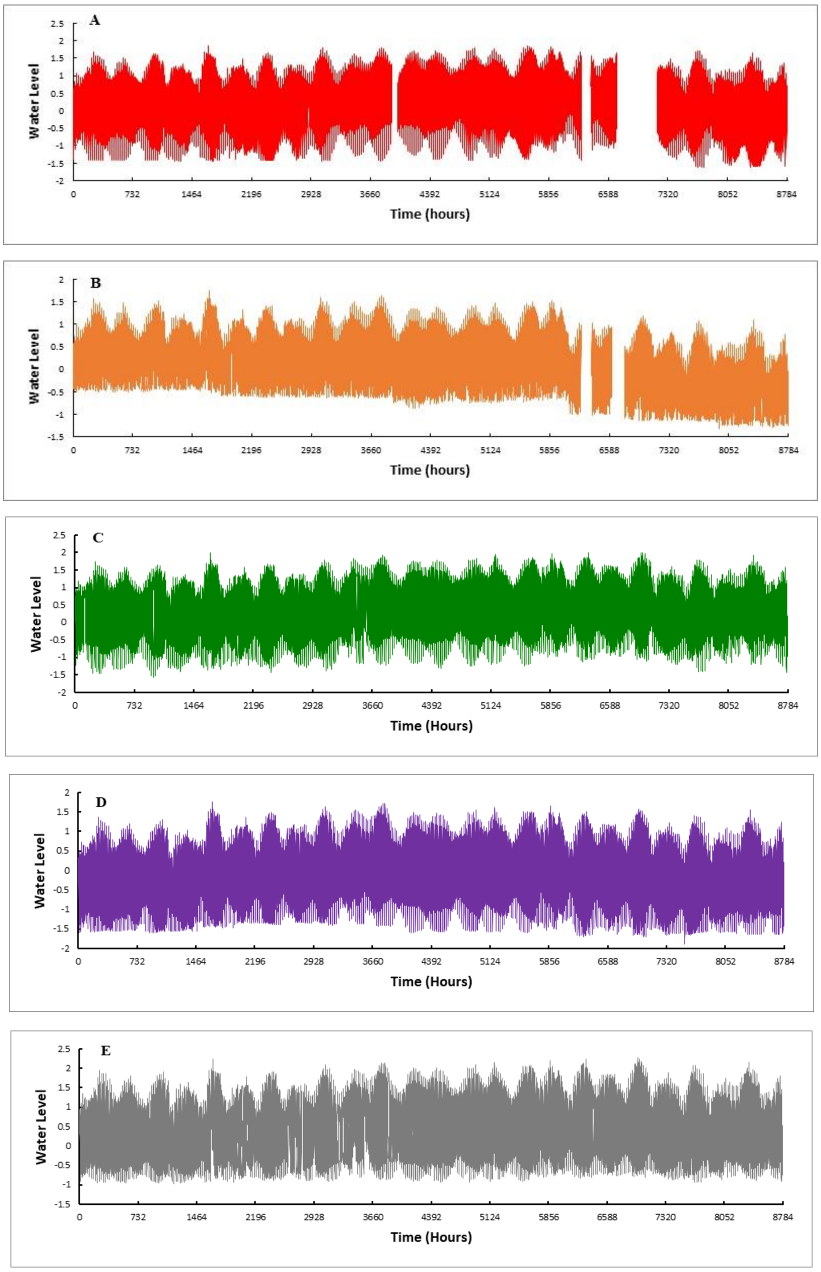
3.1. Validation of Water Surface Elevation

Before proceeding with a deeper analysis, the observation data is validated by comparing it to the results of a time series analysis considering 10 tidal constituents (M2, S2, N2, K1, O1, M4, M6, MS4, MN4, MK3). Observation data in 2020 was collected every hour from January 1, 2020 to December 31, 2020 at 6 locations along the Danshuei River from downstream to upstream (**Figure 2**). **Figure 3** shows the comparison between computational results and observations. In general, the computer analyses provide water levels that are relatively close to those observed. The inaccuracy created by the computational results of the water level surface is calculated in **Table 3** using root mean square (RMS) and energy ratio (ER) with equation below.

$$\mathrm{RMS} = \left(\frac{1}{\mathrm{N}}\sum_{\mathrm{i}=1}^{\mathrm{n}}\left[\mathrm{H}(\mathrm{t_i}) - \mathrm{H_p}(\mathrm{t_i})\right]\right)^{\frac{1}{2}} \tag{10}$$

$$\mathrm{ER} = \frac{\Sigma\left[H_p(t_i) - \bar{H}\right]^2}{\Sigma\left[H(t_i) - \bar{H}\right]^2} \times 100\% \tag{11}$$

with H = observed water elevation, H_p = simulated water elevation, $\bar{H}$= mean surface elevation



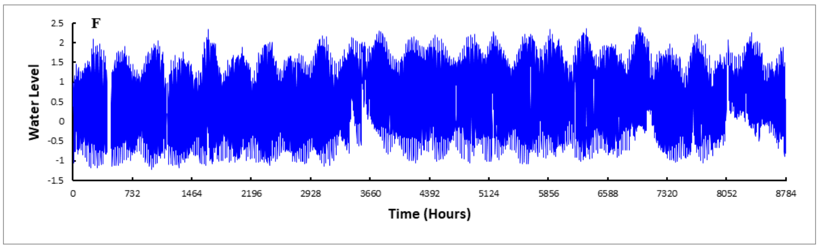
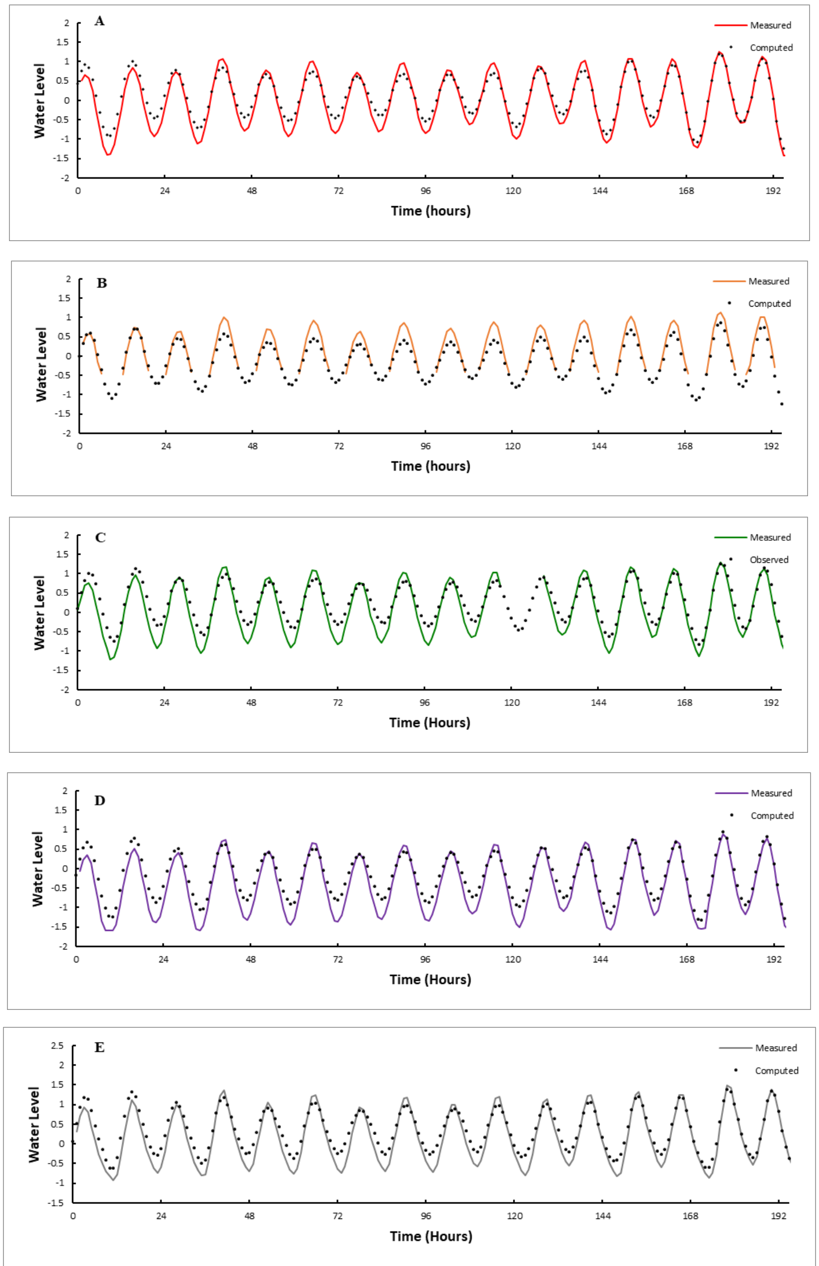


Figure 2. Time Series Raw Data of Hourly Water Level along Danshuei River at (A) River Mouth (B) Tutikungpi (C) Bailing Bridge (D) Taipei Bridge (E) Xinhai Bridge (F) Zhongzheng Bridge



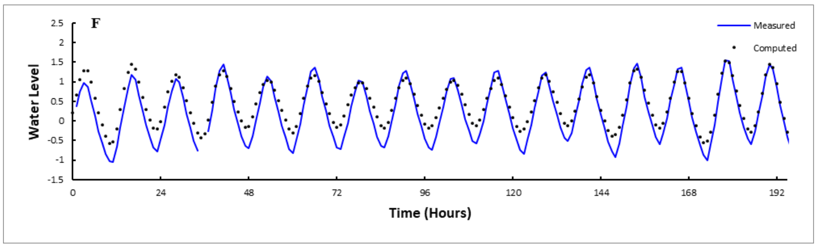


Figure 3. Comparison of Water Level between Model Prediction and Observation at (A) River Mouth (B) Tutikungpi (C) Bailing Bridge (D) Taipei Bridge (E) Xinhai Bridge (F) Zhongzheng Bridge

Table 3. RMS and ER Value for Validating the Simulated Result

	River Mouth	Tutikungpi	Bailing Bridge	Taipei Bridge	Xinhai Bridge	Zhongzheng Bridge
RMS (m)	0.18413	0.4865	0.26682	0.20877	0.20479	0.2408
ER (%)	95.02	83.86	93.44	93.96	96.73	95.98

The result indicates that the RMS has a low value and the energy ratio approaches 1.0. As a consequence, the simulated results closely approximate observed water surface elevation [11].

3.2. Tidal Harmonic

A computer model was developed to estimate the value of the water level in the Danshuei River region based on the tidal constituent set. This study employs ten tidal constituents to generate the tidal range shown in **Table 4**.

Table 4. Tidal Range of Danshuei River

Station	Max High Tide	Min Low Tide	Tidal Range
River mouth	1.83	-1.68	3.51
Tutikungpi	1.45	-1.73	3.18
Bailing Bridge	1.84	-1.42	3.26
Taipei Brige	1.54	-1.92	3.46
Xinhai Bridge	2.11	-1.17	3.28
Zhongzheng Bridge	2.19	-1.11	3.3

This value indicates a discontinuity from downstream to upstream. Like understand, the greater the distance from the estuary, the lower the tidal astronomical impact, causing high and low tides to have less influence on water level. However, in this case, the reduction in the value of high and low tide occurs at arbitrary. **Table 5** displays various amplitude and phase values for the 10 tidal constituents (M2, S2, N2, K1 O1, M4, M6, MS4, MN4, MK3). The harmonic analysis results demonstrate that the tide character is being identified using the tidal form number constituent for amplitude O1, K1, M2, S2. This connection is denoted by the letter $F = \frac{a_{O1}+a_{K1}}{a_{M2}+a_{S2}}$. The value of F is shown in the **Table 6** [6].

Table 5. Amplitude (m) and Phase (degrees) for the Tidal Constituents Used in the Analysis

Tidal Constituent	River Mouth		Tutikungpi		Bailing Bridge		Taipei Brige		Xinhai Bridge		Zhongzheng Bridge	
	Amp.	Phase	Amp.	Phase	Amp.	Phase	Amp.	Phase	Amp.	Phase	Amp.	Phase
M_2	1.0478	84.3	1.2048	96.128	0.98964	117.79	1.0765	105.75	0.99782	121.49	1.0221	121.71
S_2	0.29972	351.8	0.34078	8.3155	0.27034	33.076	0.287	20.394	0.258	39.323	0.26959	40.154
N_2	0.20773	226.38	0.24423	239.47	0.1981	264.23	0.20375	251.76	0.18285	266.64	0.17913	268.9
K_1	0.22057	243.05	0.27874	253.08	0.18332	260.41	0.19148	255.24	0.15311	264.19	0.15904	264.63
O_1	0.18034	339.49	0.22829	353.64	0.14463	355.41	0.15236	350.11	0.11761	358.54	0.12998	355.52
M_4	0.02856	153.26	0.10708	13.294	0.044672	178.42	0.046326	185.04	0.14105	193.94	0.11205	192.12
M_6	0.002972	251.05	0.079526	287.48	0.014704	30.198	0.024397	358.85	0.012961	355.21	0.026015	13.782
MS_4	0.020984	59.783	0.039098	301.83	0.031221	85.868	0.030893	89.854	0.086206	111.25	0.072175	106.87
MN_4	0.010964	296.02	0.026841	163.12	0.019051	323.71	0.020618	322.1	0.054472	339.53	0.047664	336.41
MK_3	0.005504	49.116	0.05736	175.92	0.018498	347.46	0.022064	351.14	0.052256	356.86	0.041053	356.79

Table 6. The Classification of the tides based on F-scale Ratio [6]

Form number $F = (K_1+O_1)/(M_2+S_2)$	Type of tide
$0 < F < 0.25$	Purely semi-diurnal
$0.25 < F < 1.5$	Mixed, mainly semi-diurnal
$1.5 < F < 3.0$	Mixed, mainly diurnal
$F > 3.0$	Purely diurnal

The tidal nature of the Danshuei river classified as mixed, mostly semi-diurnal, as determined by the F value, with the specifics of the F value at each station shown in **Table 7**.

Table 7. The Classification of the Danshuei River Tide

Station	F	Type of Tide
River mouth	0.30	Mixed, mainly semi-diurnal
Tutikungpi	0.25	Mixed, mainly semi-diurnal
Bailing Bridge	0.26	Mixed, mainly semi-diurnal
Taipei Brige	0.26	Mixed, mainly semi-diurnal
Xinhai Bridge	0.22	Purely semi-diurnal
Zhongzheng Bridge	0.22	Purely semi-diurnal

Among the main tidal constituents used in the analysis, the largest was M2 in all station. The second important constituent was S2, followed by K1, N2 and O1. The maximum amplitudes of the principal tidal constituents (M2, S2, N2, K1, O1) were observed in the Tutikungpi Station. However, the main tidal constituent varies irregularly from downstream to upstream. This occurrence is further confirmed by erratic fluctuations in the tidal range. This is because the M4, M6, MS4, MN4, and MK3 components, which are shallow water constituents, have a significant influence on the upstream, particularly after passing over the Taipei Bridge station. In the other hand, the most significant shallow water component was M4 in the Xinhai Bridge followed by MS4, MN4, MK3 and M6. The M4 amplitude is greatly increase when there is great difference in geometry of the river which is from Taipei Bridge and start entering Xinhai bridge. This great increase made the increase in tidal range even though far away from estuary.

3.3. Tidal Asymmetry

Tidal Asymmetry can be created by the combination of two or more constituents, both astronomical or mixed between astronomical and shallow water constituents. Song et.al (2011) examined the tidal wave deformation in 335 sea-level stations around the world. He reported that, in the mixed mainly semi-diurnal tidal system, the most significant triad interaction that contribute tidal asymmetry were M2/M4, O1/K1/M2, M2/S2/MS4, K1/M2/MK3, M2/N2/MN4, M2/M4/M6 [5]. The result indicated that the relative phase relation between M2 and M4 $(2\theta_{M2} - \theta_{M4})$ and the distortion of tidal wave shown on **Table 8**. All phase differences at any location result are between 0 and 180 degrees. Hence, according to Aubrey and Speer [1985], Danshuei River exhibit a flood dominance behavior if only the M2-M4 interaction is taken into account [2]. If using three tidal constituents, the result will not consistent in particularly on K1/M2/MK3 and M2/M4/M6. It can be concluded that there are two locations that have ebb dominance for a short length of time, but the overall dominance is flood. According to Lu et.al (2015), the M4/M2 amplitude ratio that is greater than 0.01 indicates the significant tidal distortion in the tidal system. In Danshuei River, the ratio between 0.027 until 0.141 and greatly increase when start entering Xinhai bridge. The evolution of this ratio depends on the behavior of M2 and M4 but mostly caused by M4. The spatial distribution ratio is highly independent on the local geometry of the river cross section.

Table 8. Phase Degree and Tidal Distortion Value on Every Station at Danshuei River

	Tidal Combination	River Mouth	Tutikungpi	Bailing Bridge	Taipei Brige	Xinhai Bridge	Zhongzheng Bridge
phase degree	M2/M4	15.34	178.962	57.16	26.46	49.04	51.3
		flood dominance	flood dominance	flood dominance	flood dominance	flood dominance	flood dominance
	O1/K1/M2	138.24	150.592	138.03	139.6	141.24	138.44
		flood dominance	flood dominance	flood dominance	flood dominance	flood dominance	flood dominance
	M2/S2/MS4	16.317	162.6135	64.998	36.29	49.563	54.994
		flood dominance	flood dominance	flood dominance	flood dominance	flood dominance	flood dominance
	K1/M2/MK3	278.234	173.288	30.74	9.85	28.82	29.55
		ebb dominance	flood dominance	flood dominance	flood dominance	flood dominance	flood dominance
	M2/N2/MN4	14.66	172.478	58.31	35.41	48.6	54.2
		flood dominance	flood dominance	flood dominance	flood dominance	flood dominance	flood dominance
	M2/M4/M6	346.51	181.942	266.012	291.94	320.22	300.048
		ebb dominance	ebb dominance	ebb dominance	ebb dominance	ebb dominance	ebb dominance
Tidal distortion	M2/M4	0.027	0.089	0.045	0.043	0.141	0.110

Tidal asymmetry was investigated further by the tidal skewness approach. The result of tidal skewness on every station can be seen on **Table 9**. Positive value of total tidal skewness was observed at all station, indicating a longer period of falling tide than rising tide called flood dominance. The highest value of tidal skewness was recorded at Xinhai Bridge Station, indicating nonuniform behavior. Decreasing tidal skewness in Taipei Bridge caused by increasing local geometry that causing ebb duration increase. **Figure 4** illustrate the contributor of tidal constituent due to tidal asymmetry. It can be shown that the effect of the M2/M4 combination has the greatest influence on tidal asymmetry in the Danshuei River. Moreover, when it reaches the upstream, where it is Xinhai Bridge, the M4 factor, a shallow water constituent, has a major effect on an increase in tidal skewness. This is owing to the area's local geometry, which is smaller than the main river. On the second contributor, the M2/S2/MS4 interaction play a fundamental role in the tidal asymmetry. This is also confirmed in the research of Song et.al (2011) that this tidal combination is the main contributor to the triad combination that affects tidal skewness [5].

Table 9. Tidal Skewness Value on Every Station at Danshuei River

		River Mouth	Tutikungpi	Bailing Bridge	Taipei Brige	Xinhai Bridge	Zhongzheng Bridge
β	M2/M4	0.00460	0.00163	0.02080	0.01342	0.07273	0.05765
	O1/K1/M2	0.00511	0.00411	0.00386	0.00352	0.00216	0.00264
	M2/S2/MS4	0.00215	0.00562	0.00891	0.00662	0.02223	0.02053
	K1/M2/MK3	-0.00014	0.00025	0.00019	0.00009	0.00044	0.00037
	M2/N2/MN4	0.00061	0.00104	0.00322	0.00267	0.00775	0.00735
	M2/M4/M6	-0.00007	-0.00122	-0.00215	-0.00377	-0.00386	-0.00865
total skewness γ_N		0.0123	0.0114	0.0348	0.0226	0.1014	0.0799
		flood dominance	flood dominance	flood dominance	flood dominance	flood dominance	flood dominance

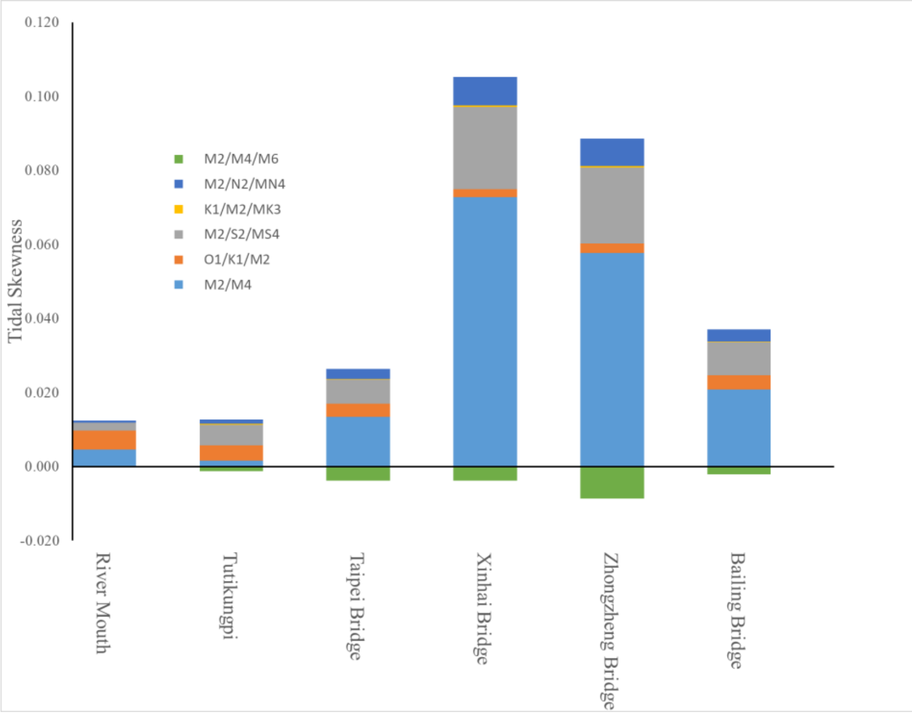


Figure 4. Visualization of Tidal Combination Contribution on Tidal Skewness

4. Conclusion

In this study, the water level measurement indicates that the tidal wave experience ununiform propagation from downstream to upstream. This is due to the fact that the astronomical constituent cannot adjust for the shallow water constituent when entering the tributary, which has a smaller geometry, resulting in the water level not being accompanied regularly from downstream to upstream. The maximum tidal range measured at the coastal station was 3.51 m, with a nonuniform decrease in the estuary's upper reaches. The results of the harmonic analysis indicated that the amplitude of the principal M2 and M4 give the highest impact on every station.

Six combinations of tidal constituents were evaluated. The main contributor to the tidal wave deformation coincides with the interaction of M2 and M4 at all station, followed by M2-S2-MS4 represent three constituents. Both the harmonic approach and the tidal skewness method revealed that the Danshuei River had a flood dominance nature of tidal asymmetry in the mixed, mostly semi-diurnal tidal characteristic. When the tidal skewness results in a large disparity with the main river, nonuniform behavior is observed at Xinhan Bridge Station. This significant difference caused by the local geometry of the tributary of Danshuei River that relatively small compared to the main river. It also proved in the shallow water constituent M4 as major factor which causes this high increase.

It is worth mentioning that the results obtained are mainly based on the water level measurement and focused on the tidal duration asymmetry due to the lack of flow velocity data. So, it is highly recommended to investigate the velocity asymmetry that is directly correlated with the sediment dynamic and morphology evolution in coastal system.

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