農業工程學報 第41卷第4期
中華民國84年12月出版

Journal of Chinese Agricultural Engineering
Vol. 41. No. 4, December 1995

通風畜舍中氣懸粉塵動態之機率與微分模式關係

Relationship between Probability and Differential Models of Airborne Dust Dynamics in Ventilated Livestock Housing

國立台灣大學農業工程學系副教授

廖 中 明

Chung-Min Liao

摘 要

本文顯示描述通風畜舍中氣懸粉塵動態行為之微分模式與一機率模式相一致。系統模式可藉一以系統傳輸矩陣特性化之微分方程表示，亦可藉一以時變轉換機率矩陣特性化之機率公式表達。由此兩者模式間關係可導得一結構式鑑定演算法則。此演算法則不僅可檢視一多空間系統其輸入／輸出量測之內在結構並可決定需何種實驗以獲取其內在關聯性。由機率模式之物理及數學分析可導出通風空間中氣懸粉塵之停留時間模式。因此，應用一微分方程或一機率公式對參數鑑定或物理定義之解釋皆為最適用途。最後舉一典型通風畜舍中氣懸粉塵分佈之模擬例以說明此演算法則之應用。

關鍵詞：氣懸粉塵，機率模式，微分模式，停留時間，通風畜舍。

ABSTRACT

This paper shows the differential model describes the dynamics of airborne dust in ventilated livestock housing is consistent with a probability description. The system model can be represented either by a differential equation characterized by a system transport matrix or a probability formulation characterized by a time-dependence transition probability matrix. A structural identification algorithm is derived from the relationship between both models. The algorithm is useful not only to get insight into the internal structure of a multi-airspace system from input/output measurements but to determine what experiments are necessary to obtain uniquely the internal couplings. From the physical and mathematical analysis of the probability model leads to a residence time model of airborne dusts in a ventilated airspace. Therefore, either a differential equation or a probability formulation for parameter identification or for physical interpretation can suit the purpose. To illustrate this procedure, the algorithm is applied to simulate the airborne dust distribution in typical ventilated livestock housing.

Keywords : Airborne dust, Probability model, Differential model, Residence time, Ventilated livestock housing.

INTRODUCTION

From the viewpoint of air quality, the main objective of a ventilation system in agricultural structures is to limit animals, plants, or crops exposure to polluted air. This implies at least for a steady air pollutant source, that its spread and residence time within the agricultural structures shall be minimized. The spread of a pollutant and pollutant itself and the distribution of the supplied air stated above must be quantified. That is to say, the flow field occurring must be characterized.

The characterization of flow patterns in terms of residence times permits treatment of continuous flow systems which is independent of specific mixing mechanisms.This type of approach seems suitable for use in characterizing flows in ventilated agricultural structures.

A deterministic theory of a lumped-parameter model representing the average behavior of airborne dust in ventilated animal housing has been dealt with extensively by Liao and Feddes (1990). The equation used to describe the dynamics of airborne dust is a differential equation which describes the behavior of airborne dust undergoing turbulent diffusion deposition and gravitational sedimentation at any location within a ventilated airspace.

On the other hand, the air movements in a ventilated airspace are turbulent and the predominant and distinctive feature of turbulence is its randomness. An inevitable consequence of the randomness is that the airborne dust concentration field (n(t)) is also random. The results of repeated experiments on the dispersion of airborne dust with the same conditions at different times, will be different from one another. Therefore, the concentration field must be described in a statistical sense, i. e., in terms of mean concentration (E[n(t)]), variance(Var[n(t)]), and the third central moment (M_3 [n (t)]). Very little has been published on the statistical viewpoint of multi-airspace systems describing fluctuation from such ideal behavior.

A ventilated agricultural structure is a multiport system having several supply and exhaust terminals. It is of vital importance to know where an air contaminant goes within a ventilated airspace. Thereasse and Sine (1974) in their study of ventilation for livestock buildings interpreted the results of tracer gas experiments in terms of a concept related to residence time.

The purposes of this paper are: (1) to develop a probability model for a multi-airspace system with a time -dependent input and to include inputs from several sources. The first three central moments of the concentration fields as random variables will also be calculated by the method of generation function, (2) to reveal a probability description between the system transport matrix in a linear dynamic equation and the system transition probability. Thus, the differential-equation based parameters may be determined from the corresponding set of probability parameters and vice versa, and (3) to use the relationships between both models to interpret parameters from the statistical point of view. This will lead to the interpretation of residence time concept of airborne dust in a ventilated airspace. A structural identification algorithm will also be derived.

ASSUMPTIONS AND DEFINITIONS

Before the model development, the following definitions and assumptions are made.

1. The concentration vector of airborne dust ({ n_i (t)}) presents in a multi-airspace (or multi-lump) system at time t is a random variable, each value of the random

variable represents a different state of the system.

2. The probability of the transition of one airborne dust in a system from one state to another depends only on the present state of the system and not on its past history.

3. Random variables are statistically independent.

4. There are four probabilities defined as follows.

(1) $P_{j,k}(t) \equiv p[n_j(t)=k]$: the probability that k airborne dusts are presented in lump j at time t.

(2) $\mu_j dt$: the probability that any given airborne dust exits the lump j in the interval (t, t+dt).

(3) $\alpha_{ji}\mu_i dt$: the probability that a given airborne dust presents in lump i at time t enters lump j in the interval of time (t,t+dt), in which α_{ji} can be defined as the fraction of airborne dusts leaving lump i that enter directly lump j, with $\sum_j \alpha_{ji} \le 1$.

(4) $\xi_j(t)dt$: the probability that one airborne dust enters lump j in the interval(t, t+dt).

PROBABILITY CONCENTRATION FIELDS

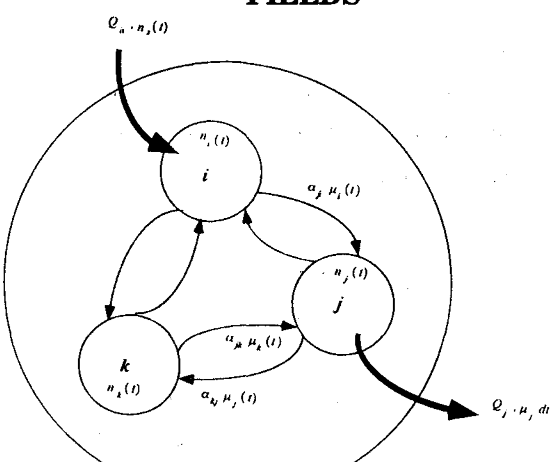

Fig. 1. A multiple airspaces system shows the pathways of airborne dust among each airsapce

As shown in Figure 1, the probability that an airborne dust enters airspace (or, lump) j from airspace i in the same interval of time is:

$$\sum_{k=0}^{\infty} kP_{i,k}(t)\alpha_{ji}\mu_i dt = \alpha_{ji}\mu_i E[n_i(t)]dt$$

The expectation $E[n_i(t)]$ satisfies the following relations (Feller, 1968):

$$E[n_i(t)] = \sum_{k=0}^{\infty} kP_{i,k}(t)$$

Therefore, the probability of an airborne dust entering lump j from any other lump in the same time interval becomes:

$$\sum_i \alpha_{ji}\mu_i E[n_i(t)]dt \quad (1)$$

Above expression indicates that the probability of an airborne dust entering lump j from any other lump depends on an expression involving the expectations of the random variable of the $n_i(t)(E[n_i(t)])$, and not on their probability distributions.

In order to synthesize a multi-lump system to describe the concentration field in a sense of statistical representation, a generating function concept may be introduced. The generating function of a random variable (n(t)) for a single lump can be expressed as (see Appendix):

$$G_n(s,t) = g(s)[1-(1-s)\exp(-\mu t)] \cdot \exp\{-(1-s)\cdot \xi(t) * \exp(-\mu t)\} \quad (2)$$

where g(s) is the generating function of n(0), and the convolution integral term defined as:

$$\int_0^t \exp(-\mu(t-\tau))\xi(\tau)d\tau = \xi(t) * \exp(-\mu t)$$

In terms of a multi-lump system, equation (2) can be rewritten with μ_j instead of μ, and with equation (1) instead of $\xi(t)dt$, thus;

$$G_{n_j}(s,t) = g_j(s)[1-(1-s)\exp(-\mu_j t)] \cdot \exp\left\{-(1-s)\cdot \sum_i \alpha_{ji}\mu_i \cdot E[n_i(t)] * \exp(-\mu_j t)\right\} \quad (3)$$

or,

$$G_{n_j}(s,t) = G_{y_j}(s,t)\prod_i G_{z_{ji}}(s,t) \quad (4)$$

where

$$G_{y_j}(s,t) = g_j(s)[1-(1-s)\exp(-\mu_j t)] \quad (5)$$

$$G_{z_{ji}}(s,t) = \exp\left\{-(1-s)\sum_i \alpha_{ji}\mu_i \cdot E[n_i(t)] * \exp(-\mu_j t)\right\} \quad (6)$$

$G_{Y_i}(s,t)$ and $G_{Z_{ji}}(s,t)$ are the generating functions of random variables of $Y_i(t)$ and $Z_{ji}(t)$, respectively.

Therefore, concentrations of airborne dust presented in lump j ($n_j(t)$) can be expressed in terms of another two random variables of $Y_j(t)$ and $Z_{ji}(t)$ as:

$$n_j(t) = Y_j(t) + \sum_i Z_{ji}(t) \tag{7}$$

where $Y_j(t)$ = the concentration of airborne dust in lump j if no new airborne dusts enter it, i. e., $\sum_i \alpha_{ji} = 0$; and $Z_{ji}(t)$ = the concentration of airborne dust in lump j if it were initially empty and if only lump i has airborne dust enters lump j.

$Y_i(t)$ and $Z_{ji}(t)$ are statistically independent. In measuring of particle number concentrations, number concentration is obtained by dividing the total number of particles collected by the total gas volume sampled. The standard deviation of the count is usually assumed has a Poisson distribution (Hinds, 1982). Therefore, $Z_{ji}(t)$ and $Y_i(t)$ may be assumed have a Poisson distribution. Thus, the first three central moments of $n_j(t)$ are (Feller, 1968):

$$E[n_j(t)] = E[Y_j(t)] + \sum_i \alpha_{ji}\mu_i E[n_i(t)] * \exp(-\mu_j t) \tag{8}$$

$$Var[n_j(t)] = Var[Y_j(t)] + \sum_i \alpha_{ji}\mu_i E[n_i(t)] * \exp(-\mu_j t) \tag{9}$$

$$M_3[n_j(t)] = M_3[Y_j(t)] + \sum_i \alpha_{ji}\mu_i E[n_i(t)] * \exp(-\mu_j t) \tag{10}$$

When the concentration of airborne dust in lump j at time t=0 is determined, i. e. $n_j(t=0)=n_{j0}$, the generating function of $n_j(t=0)$ becomes, $g_j(s)=S^{n_{j0}}$. The first three central moments of $Y_i(t)$ are (Feller, 1968):

$$E[Y_j(t)] = n_{j0}\exp(-\mu_j t) \tag{11}$$

$$Var[Y_j(t)] = n_{j0}\exp(-\mu_j t)(1-\exp(-\mu_j t)) \tag{12}$$

$$M_3[Y_j(t)] = n_{j0}\exp(-\mu_j t)(1-\exp(-\mu_j t))(1-2\exp(-\mu_j t)) \tag{13}$$

The expectation of $Z_{ji}(t)$ can be calculated by using the theory of generating functions(Parzen, 1960) based on equation (6):

$$E[Z_{ji}(t)] = \lim_{s\to 1} \partial G_{Z_{ji}}(s,t)/\partial s$$

$$= \alpha_{ji}\mu_i E[n_i(t)] * \exp(-\mu_j t)$$

Therefore,

$$G_{Z_{ji}}(s,t) = \exp\{-(1-s)E[Z_{ji}(t)]\}$$

Thus,

$$\prod_i G_{Z_{ji}}(s,t) = \exp\left\{-(1-s)\sum_i E[Z_{ji}(t)]\right\}$$

$$= \exp\left\{-(1-s)E\left[\sum_i Z_{ji}(t)\right]\right\} \tag{14}$$

By definition (7),

$$\prod_i G_{Z_{ji}}(s,t) = \exp\{-(1-s)E[n_j(t)-Y_j(t)]\} \tag{15}$$

Since $E[n_j(t)-Y_j(t)] = E[n_j(t)] - E[Y_j(t)]$, thus; equation (14) becomes:

$$\prod_i G_{Z_{ji}}(s,t) = \exp\{-(1-s)E[n_j(t)]\}/\exp\{-(1-s)E[Y_j(t)]\} \tag{16}$$

Equation (16) can be rewritten in accordance with equation (4) as:

$$G_{n_j}(s,t)/G_{Y_j}(s,t)$$

$$= \exp\{-(1-s)E[n_j(t)]\}/\exp\{-(1-s)E[Y_j(t)]\} \tag{17}$$

Thus, if the distribution of a random variable at time t=0 is known, its distribution at any time can be computed through equation (17) by operating only on its expectation.

PROBABILITY AND DIFFERENTIAL MODELS

The dynamic behavior of an airborne dust undergoing turbulent diffusive and gravitational depositions in a ventilated airspace can be represented by a linear dynamic matrix equation (Liao and Feddes, 1990):

$$\{dn(t)/dt\} = -[B]\{n(t)\} + [V]^{-1}\{G(t)\},$$

$$\{n(0)\} = \{n_0\} \tag{18-1}$$

where [B] is a system transport matrix, contains the essential dynamic characters of the system being studied, $[V]^{-1}$ is a inverse system volume matrix, and {G(t)} is a dust generation rate vector. Matrix [B] may be defined as (Liao and Feddes, 1990):

$$[B]=([H']^{-1}+[V]^{-1}[S']+[T']) \tag{18-2}$$

where $[H']=U_s(r)[H]^{-1}$, $[H]^{-1}=[V]^{-1}[A]$, $[T']=[V]^{-1}[Q]$, and $[S']=(D(r)+\in)/\delta[S]$; in which [A] = cross-section area matrix of lump, [Q] = square airflow matrix ([Q]=Q[β], Q=total system volumetric flow rate, and [β]=entrainment ratio matrix), [S] = wall surface area matrix of lump, $U_s(r)$ = particle terminal settling velocity (cms^{-1}), D(r) = molecular diffusion coefficient (cm^2s^{-1}), $\in$ = eddy diffusion coefficient (cm^2s^{-1}), and δ = thickness of concentration boundary layer(mm).

The general solution of equation (18) is:

$$\{n(t)\}=\exp(-[B]t)\{n(0)\}$$
$$+\int_0^{\infty}\exp(-[B](t-\tau))[V]^{-1}\{G(t-\tau)\}d\tau \tag{19}$$

The random state variable at time (t+h) ({n(t+h)}) can be transformed nonsingularly via a time-dependent transition probability matrix ([p(h)]) by any other random variables at previous time({n(t)}), and may be expressed as (Liao and Feddes, 1990):

$$\{n(t+h)\}=[P(h)]\{n(t)\} \tag{20}$$

The element $P_{ij}(h)$ of [P(h)] represent a proportion or percentage of airborne dust in airspace j that is transferred to airspace i during a specified time period h. It is assumed that these percentages remain constant over the time range during which sample data are collected (say,2 hrs). Equation (20) may be referred to as a probability model.

As can be seen from equation (19), the solution of {n(t+h)} is:

$$\{n(t+h)\}=\exp(-[B](t+h))\{n_0\}$$
$$+\int_0^t\exp(-[B](t+h-\tau))[V]^{-1}\{G(t+h-\tau)\}d\tau$$
$$=(\exp(-[B]t)\cdot\exp(-[B]h))\{n_0\}$$
$$+\int_h^t\exp(-[B](t-\tau')[V]^{-1}\{G(t-\tau')\}d\tau' \tag{21}$$

where $\tau'=\tau-h$. If h is very small, a combination of equations (21) and (19) yields:

$$\{n(t+h)\}=\exp(-[B]h)\{n(t)\} \tag{22}$$

In view of equations (22) and (20), an important relationship between system transport matrix ([B])and transition probability matrix ([p(h)]) may be obtained as:

$$[P(h)]=\exp(-[B]h) \tag{23}$$

On the other hand, equation (23) may be expanded as follows:

$$[P(h)]=\exp(-[B]h)=[I]-h[B]+h^2[B]^2/2!-\ldots \tag{24}$$

that for small h, it becomes:

$$[P(h)]\approx[I]-h[B] \tag{25}$$

A matrix $[M(t)]\equiv[[I]-[P(t)]^{-1}$ exists and is equal to the series $\sum_{m=0}^{\infty}[P(mt)]$ (Maki and Thompson, 1973). Matrix [M(t)] is known as the fundamental matrix corresponding to [P(t)]. Therefore,

$$[M(t)]=[[I]-[P(t)]]^{-1}=[[I]-([I]-t[B])]^{-1}=t^{-1}[B]^{-1} \tag{26}$$

Let $P_{ij}(t)$ denotes the element of [P(t)]. Then, in an infinitestimal interval (t, t+dt), the mean time the process is in lump i, having started in lump j, before entering a lump, is $P_{ij}(t)dt$. The total mean time the process stays in lump i, having started in lump j,before entering a lump, is the (i, j)th element of:

$$[T]=\int_0^{\infty}[P(t)]dt \tag{27}$$

Above integration may be obtained using the function of a matrix representation (Zadeh and Desoer, 1963):

$$\int_0^{\infty}[P(t)]dt=\int_0^{\infty}\exp(-[B]t)dt$$
$$=\int_0^{\infty}\left(1/(2\pi i)\oint_C\exp([\lambda]t)([\lambda][I]+[B])^{-1}d\lambda\right)dt$$
$$=1/(2\pi i)\oint_C\left(\int_0^{\infty}\exp([\lambda]t)dt\right)([\lambda][I]+[B])^{-1}d\lambda$$
$$=1/(2\pi i)\oint_C-[\lambda]^{-1}([\lambda][I]+[B])^{-1}d\lambda$$
$$=[B]^{-1} \tag{28}$$

where [λ] = distinct eigenvalue matrix of [B], C = the boundary of a domain containing the eigenvalues [λ] of [B], consist of a finite number of closed rectifiable Jordan curves. In the last step, the contour C in the left hand plane was chosen, so that exp ([λ]t) approaches zero as t approaches infinity.

Therefore, the row sum in [T], say the ith row, is equal to mean residence time of airborne dust in lump i:

$$\{\bar{t}\} = [T]\{1\} = [B]^{-1}\{1\} \tag{29}$$

Employing the Riemann sum, for small increments of time h, to approximate the integral in equation (27), it becomes:

$$[T] \approx \sum_{m=0}^{\infty}[P(mt)]t = t[M(t)] \tag{30}$$

Notice that the approximation (30), along with equation (27) yields equation (26). Equation (20) also reveals that the element of [M(t)], say $m_{ij}(t)$, represents the mean number of hour of airborne dust in lump i having started initially from the lump j.

It is an interesting numerical exercise to determine the [B] corresponding to a time-invariant fundamental matrix [M]. A relationship between matricess [B] and [M] can be expressed as (Callier and Desoer, 1991):

$$[B] = g([M]), \quad where \quad g([\lambda]) = \ln([I]-[\lambda]^{-1}) \tag{31}$$

Using the Lagrange interpolation method (Zadeh and Desoer, 1963), matrix [B] can be obtained from [M] as:

$$[B] = \sum_{k=1}^{n}\left[\left(g(\lambda_k)/\prod_{\substack{i=1\\i\neq k}}^{n}(\lambda_k-\lambda_i)\right)\prod_{\substack{i=1\\i\neq k}}^{n}([M]-\lambda_i[I])\right] \tag{32}$$

where λ_i is eigenvalues of [M].

Therefore, it develops that these matrices ([B], [P(t)], and [M(t)]) are related by fairly simple formulas which permit computation of one from the other.

RESIDENCE TIME MODEL

1. Relationship between $\bar{t}_p$ and U_p

The inverse system transport matrix ($[B]^{-1}$) may be expressed in terms of a local purging flow rate matrix ([U]) and a transition probability ([P]) as (Liao and Feddes, 1990):

$$[B]^{-1} = [U]^{-1}[P][V] \tag{33}$$

Therefore, the residence time of an airborne dust finally can be expressed in terms of the matrices of local purging flow rate and transition probability via equations (29) and (29) as:

$$\{\bar{t}\} = [U]^{-1}[P][V]\{1\} \tag{34}$$

In view of equation (29) that the sum elements in an arbitrary row p in $[B]^{-1}$ is equal to the mean residence time of airborne dusts in airspace p, ($\bar{t}_p$):

$$\bar{t}_p = 1/U_p\sum_{j=1}^{N}V_jP_{pj} = (V_p/U_p) + 1/U_p\sum_{\substack{j=1\\(j\neq p)}}^{N}V_jP_{pj} \tag{35}$$

It is true that $p_{pj} \leq 1$ and therefore the following upper bound of the local mean residence time in airspace p can be obtained as:

$$\bar{t}_p \leq 1/U_p\sum_{j=1}^{N}V_j = V/U_p \tag{36}$$

or,

$$U_p\bar{t}_p \leq V \tag{37}$$

Relation (37) connects two important quantities, the local purging flow rate and local residence time of the airborne dust. Equation (37) may be rewritten as:

$$U_p \leq (\tau_n/\bar{t}_p)Q \tag{38}$$

Where $\tau_n \equiv V/Q$ = system nominal residence time. When $\bar{t}_p < \tau_n$, i. e., when local mean residence time of an airborne dust is less than system mean residence time at airspace p, relation (38) gives rise to the following restriction to the local purging flow rate:

$$U_p > Q \quad (when \quad \bar{t}_p < \tau_n) \tag{39}$$

In the case of several extract airspaces there are two possibilities, in each extract airspace is: $\bar{t}_p = \tau_n$, or $\bar{t}_p < \tau_n$; while $\bar{t}_p > \tau_n$ in at least one airspace.

2. Relationship between $\bar{t}_p$ and $\mu_p dt$

From the statistical point of view, the mean residence time of an airborne dust in lump p can be written as (Feller, 1968):

$$E[T_p] = \int_0^\infty \tau n_p(\tau)d\tau / \int_0^\infty n_p(\tau)d\tau = \bar{t}_p \quad (40)$$

where T_p is a continuous random variable represents the mean residence time in a lump p, and can be taken as any nonnegative value.

Recall that $\mu_p dt$ is the probability of a given airborne dust present in lump p at time t leaves within the interval dt. If a probability that an airborne dust is present in lump p at time t is defined as $q_p(t)$, then $q_p(t)\mu_p dt$ is the probability that an airborne dust leaves the lump p in the interval (t, t+dt). Thus, a mass balance applied to a single lump yields:

$$1 = \int_{t_0}^{t} q_p(\tau)\mu_p d\tau + q_p(t), \quad and \quad q_p(t) = 0 \quad at \quad t < t_0$$

where t_0 is the time of entry of the airborne dust into lump p.

In differentiating above equation with respect to time yields:

$$-\dot{q}_p(t) = \mu_p q_p(t), \quad q_p(t_0) = 1$$

Hence,

$$q_p(t) = \exp(-\mu_p(t - t_0)), \quad t \geq t_0$$

The generating function of T_p $(G_{T_p}(s))$ can be given by the integral (Parzen, 1960):

$$G_{T_p}(s) = \int_0^\infty p_{t,p} s^t dt \quad (41)$$

where $p_{t,p}$ dt is the probability that t < T_p < t+dt.

From the definition of the probability of $q_p(t)\mu_p dt$, the following relations is true:

$$p_{t,p} dt = q_p(t)\mu_p dt \quad (42)$$

Therefore,

$$G_{T_p}(s) = \int_{t_0}^\infty \mu_p \exp(-\mu_p(t - t_0)) s^t dt$$

$$= s^{t_0} / (1 - 1/\mu_p \ln|s|) \quad (43)$$

When the time of entry of an airborne dust is taken as the initial time, i.e., $t_0 = 0$, the generating function becomes:

$$G_{T_p}(s) = \int_0^\infty \mu_p \exp(-\mu_p t) s^t dt$$

$$= 1 / (1 - 1/\mu_p \ln|s|) \quad (44)$$

As in the case of the discrete probability distribution, the first three central moment of T_p may be calculated using the formula presented by Parzen (1960):

$$E[T_p] = \lim_{s \to 1} \partial G / \partial s = \mu_p^{-1}$$

$$Var[T_p] = \lim_{s \to 1} [\partial^2 G / \partial s^2 + \partial G / \partial s - (\partial G / \partial s)^2] = \mu_p^{-2} \quad (45)$$

$$M_3[T_p] = \lim_{s \to 1} [\partial^3 G / \partial s^3 + 3\partial^2 G / \partial s^2 (1 - \partial G / \partial s) + 2(\partial G / \partial s)^3$$

$$-3(\partial G / \partial s)^2 + \partial G / \partial s] = 2\mu_p^{-3}$$

Therefore, an important relationship among t_p, μ_p, and the system transport matrix ([B]) can be obtained as:

$$\{\bar{t}\} = \{\mu^{-1}\} = [B]^{-1}\{1\} \quad (46)$$

Equation (46) can also be extended to the time-dependent case.

MODEL IMPLEMENTATION

A model implementation will be studied in some details to give insight into the meaning of the concepts introduced. The example will illustrate the procedure and the implementation of the structural identification algorithm to show the relationship between probability and differential models. A two-lump model of a typical livestock housing unit with a negative pressure ventilation system will be considered (Figure 2). The geometric and system parameters used in the model implementation are listed in Table 1. A structural identification algorithm for the dynamics of airborne dusts in a ventilated airspace is illustrated in Figure 3.

It is assumed that an indoor disturbance occurs resulted from animal activities in the building. As a result, the indoor airborne dust source ($G_2(t)$) undergoes a sudden pulse change with a specific generation rate of $5x10^4$ particles/min and lasts for about 5 seconds. That is to say,initially lump 2 had 4170 particles of airborne dust and lump 1 was empty.

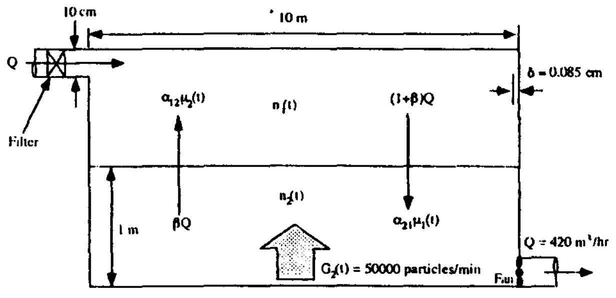

Fig. 2. A two-lump model of a typical livestock housing with a negative pressure ventilation system used in model implementation

Table 1. Input parameters used in the model implementation

Geometric parameters[a]
System volume = 120 m^3 (10x6x2 m)
System surface = 64 m^2
System height = 2 m
System parameters[a]
Ventilation airflow rate during cold weather = 420 m^3/hr
Entrainment ratio = 5.0 (slot width = 10 cm)
Temperature = 20°C, RH = 30-40% (1 atm)
Average particle radius = 2.5 µm
Reynolds number = 2000-3000
Particle settling velocity = 0.0776 cm/sec
Effective diffusive coefficient = 0.00375 cm^2/sec
Concentration boundary layer thickness = 0.085 cm

[a]All parameters are adapted from Liao and Feddes (1993).

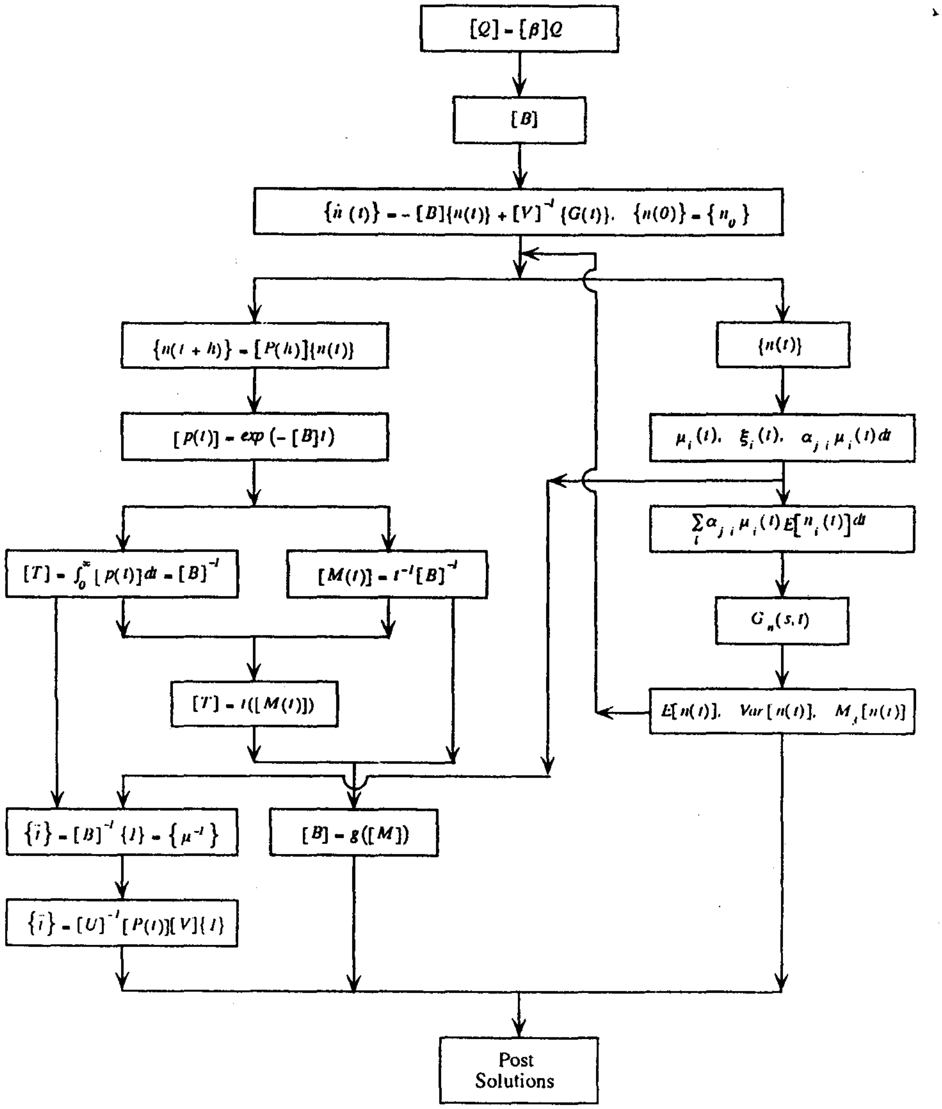

Fig 3. The structural identification algorithm of airborne dust dynamics in a ventilated airsapce

1. [Q], [B], and $[B]^{-1}$

The airflow matrix [Q] may be expressed as [Q]=Q [β]:

$$[Q]=\begin{bmatrix} Q_{11} & -Q_{12} \\ -Q_{21} & Q_{22} \end{bmatrix}=[\beta]Q$$

$$=Q\begin{bmatrix} \beta+1 & -\beta \\ -(\beta+1) & \beta+1 \end{bmatrix}$$

$$=11.3\begin{bmatrix} 6 & -5 \\ -6 & 6 \end{bmatrix}(m^3 \min^{-1})$$

Transport matrix ([B]) is expressed based on equation (18-2) as,

$$[B]=[H']^{-1}+[V]^{-1}[S']+[V]^{-1}[Q]:$$

$$[B]=\begin{bmatrix} 2.266 & -1.888 \\ -2.266 & 2.266 \end{bmatrix}(\min^{-1})$$

The inverse of transport matrix ($[B]^{-1}$) can be calculated as:

$$[B]^{-1}=\begin{bmatrix} 2.65 & 2.30 \\ 2.65 & 2.65 \end{bmatrix}(\min)$$

Therefore, the steady-state mean residence time of airborne dust in lumps1 and 2 may be computed from equation (29),

$\{\bar{t}(\infty)\}=[B]^{-1}\{1\}$, as:

$$\bar{t}_1(\infty) = 2.65 + 2.30 = 4.95 \text{ min},$$

$$\bar{t}_2(\infty) = 2.65 + 2.65 = 5.30 \text{ min}.$$

2. {μ}

A tracer gas technique usually applied to monitor the local mean residence time distribution at lumps 1 and 2 (Himmelblau and Bischoff, 1968, Sandberg and Blomqvist, 1985). It is assumed that the runs gave the following linear regression relationship between transient mean residence time distribution($\bar{t}$(t)}) and running time (t):

$$\bar{t}_1(t)=t+1, \quad and \quad \bar{t}_2(t)=1.25(t+1) \tag{47}$$

From equation (46), the probability that any given airboune dust exits the lump j (μ_j) is represented as: {μ}= {t^{-1}}, therefore,

$$\mu_1=1/(t+1), \quad and \quad \mu_2=0.8/(t+1) \tag{48}$$

3. [P(t)]

The time-dependent transition probability matrix ([P(t)]) may be calculated from equation (23), [P(t)]=exp(-[B]t) as: $P_{12}(t)$ = exp(1.888)t = 6.6t, and P_{21} (t) = exp (2.266)t = 9.6t. It is assumed that the transient response times of transition probability were very short, say less than 3 seconds, before the steady-state values approached. Thus, $P_{12}(\infty)$ = 0.375, and $P_{21}(\infty)$ = 0.4 were taken.Identically, $P_{12}(\infty)$ and $P_{21}(\infty)$ may be seen as $\alpha_{ji}(\infty)$, the fraction of airborne dust leaving lump i that enter directly lump j.

Therefore, the probability that a given airborne dust presents in lump i at time t enters lump j ($\alpha_{ji}\mu_i$) are:

$$\alpha_{12}(\infty)\mu_2=(0.375)0.8/(t+1)=0.3/(t+1),$$

$$\alpha_{21}(\infty)\mu_1=(0.4)1/(t+1)=0.4/(t+1).$$

4. E $[n_i(t)]$, Var $[n_i(t)]$, and $M_3[n_i(t)]$

Initially, lump 1 was empty and lump 2 had pulse input with a strength of 4170 airborne dust particles, the boundary conditions (i. e., the generating function of) $n_j(t=0)$: $g_j(s)=s^{n_j(t=0)}$) for the probability model are then:

$$g_1(s)=s^0=1, \quad and \quad g_2(s)=s^{4170}.$$

The corresponding concentration expectation equations for the system are writtem based on equation (1):

$$E[\dot{n}_1(t)]=-1/(t+1)E[n_1(t)]+0.3/(t+1)E[n_2(t)],$$

$$E[\dot{n}_2(t)]=0.4/(t+1)E[n_1(t)]-0.8/(t+1)E[n_2(t)]. \tag{49}$$

With initial conditions of $n_1(0)$=0 and $n_2(0)$=4170 .

Solution of equation (49) gives $E[n_1(t)]$ and $E[n_2(t)]$. Since $E[Y_2(t)]$ = 4170exp{-0.8t/(t+1)} (equation (11)), $E[Z_2(t)]$ may be computed immediately (equation (7)). Furthermore, Var $[n_2(t)]$ and $M_3[n_2(t)]$ can be computed from equations (9) and (10). Since E $[Y_1(t)]$ =0, lump 1 has a Possion distribution and as a consequence, $E[n_1(t)] = Var[n_1(t)] = M_3[n_1(t)]$.

Equation (49) was solved numerically using the 4th-order Runge-Kutter subroutine and the rest of the analysis was done in double precision of Fortran 77.

Results were graphically illustrated in Figure 4.

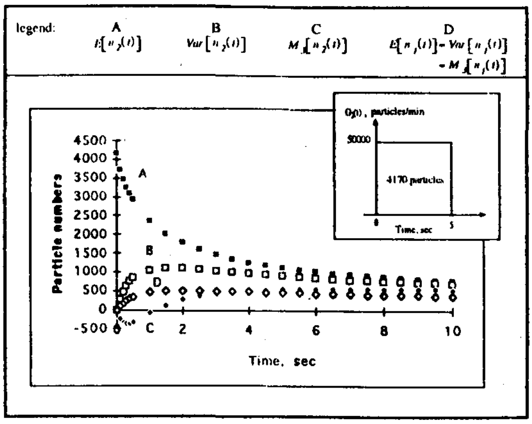

Fig. 4. Moments of airborne dust profiles in a two lump model of a typical ventilated livestock housing undergoses a pulse dust generation rate of 5x10 [4] particles/min

The model example shows that the correspondence between the probability mean and the deterministic value is established in case of time-dependence. It is also shown how the consequence of this can be utilized to compute the distributions and the moments of airborne dust concentration at any location within a ventilated airspace. Therefore, either a differential lumped-parameter equation of a probability formulation for concentration fields, either for parameter identification or for physical interpretation as best suits the purpose.

CONCLUSIONS

The system equation of airborne dust concentrations at any location within a ventilated airspace may be represented either by a differential equation characterized by a system transport matrix ([B]) as, $\{\dot{n}(t)\} = -[B]\{n(t)\} + [V]^{-1}\{G(t)\}$; or a probability formulation characterized by a timedependence transition probability matrix ([P(h)] as, $\{n(t+h)\} = [P(h)]\{n(t)\}$. The dynamics of airborne dust concentration fields may also be expressed by its expectation (E[n(t)]) as, $\sum_i \alpha_{ji}\mu_i E[n_i(t)]dt$, where $\alpha_{ji}\mu_i dt$ is the probability that a given airborne dust presents in lump i at time t enters lump j in the time interval of (t, t+dt).

The identification algorithm is useful not only to get insight into the internal structure of a multi-lump system from input/output measurements but to determine what experiments are necessary to obtain uniquely the internal couplings. A probability model for a multi-airspace system not only to determine the first three central moments of the concentration fields but to obtain the interpretation of residence time concept of airborne dusts in a ventilated airspace.

The mean residence time vector of airborne dusts ($\{\bar{t}\}$) in a ventilated airspace can be calculated either from an inverse system transport matrix ($[B]^{-1}$) or a probability that any given airborne dust exists the airspace ($\{\mu\}$) as, $\{\bar{t}\} = [B]^{-1}\{1\} = \{\mu^{-1}\}$. The time-dependence residence time of airflow in a ventilated airspace may also be interpreted by the characteristics of local purging flow rate matrix ([U]) and transition probability matrix ([P(t)]) as, $\{\bar{t}(t)\} = [U]^{-1}$ [P(t)][V] {1},where [V] is an air volume matrix.

REFERENCES

1. Callier, F. and Desoer, C. A. 1991. Linear System Theory. Springer-Verlag,New York, NY.
2. Feller, W. 1968. An Introduction to Probability Theory and Its Applications.Vol. I. John Wiley & Sons, New York, NY.
3. Hinds, W. C. 1982. Aerosol Technology. John Wiley & Sons, New York, NY.
4. Himmelblau, D. M. and Bischoff, K. B. 1968. Process Analysis and Simulation.Deterministic Systems. John Wiley & Sons, New York, NY.
5. Liao, C. M. and Feddes, J. J. R. 1990. Mathematical analysis of a lumped-parameter model for describing the behavior of airborne dust in animal housing. Appl. Math. Modelling 14(5): 248-257.
6. Liao, C. M. and Feddes, J. J. R. 1993. Design of an airborne dust control systems in ventilated animal housing. Appl. Math. Modelling 17(3): 133-140.
7. Maki, D. P. and Thompson, M. 1973. Mathematical Models and Applications. Prentic-Hall, Englewood Cliffs, NJ.

8. Parzen, E. 1960. Modern Probability Theory and Its Applications. John Wiley& sons, New York, NY.

9. Sandberg, M. and Blomqvist, C. 1985. A quantitative estimation of the accuracy of tracer gas methods for the determination of the ventilation flow rate in buildings. Building and Environ. 20(3):139-150.

10. Therasse, R. and Sine, L. 1974. Determination d'une l' air daus les batiments. Bull. Res. Agron. Fembloux 9(2): 165-187.

11. Zadeh, L. A. and Desoer, C. A. 1963. Linear Systems Theory. McGraw-Hill, New York, NY.

APPENDIX: Derivation of equation (2)

A mass balance applied to a single airspace (Figure 1) for two or more airborne dusts cross the boundary of the system during the same interval:entering, leaving, or both; thus, there are three transition probabilities, P{X(t+△t)=i | X(t)=j} for the random variable X(t) from j to i dust concentration in the interval (t+△t). Define $P_i(t)$ as the probability that X(t)=i, with $\sum_{i=0}^{\infty} P_i(t)=1$, and $P_i(t)=0$ for i<0. For $i = 0, 1, \cdots$,

$$P\{X(t+\Delta t)=i \mid X(t)=i+1\}=(i+1)\mu\Delta t$$

$$P\{X(t+\Delta t)=i \mid X(t)=i\}=1-i\mu\Delta t-\xi(t)\Delta t$$

and for $i = 1, 2, \cdots$,

$$P\{X(t+\Delta t)=i \mid X(t)=i-1\}=\xi(t)\Delta t$$

Combining these three statements, the following expression can be obtaibed as:

$$P_i(t+\Delta t)=P_{i-1}(t)\cdot P\{X(t+\Delta t)=i \mid X(t)=i-1\}$$
$$+P_i(t)\cdot P(X(t+\Delta t)=i \mid X(t)=i\}$$
$$+P_{i-1}\cdot P\{X(t+\Delta t)=i \mid X(t)=i+1\}$$
$$=P_{i-1}(t)\xi(t)\Delta t+(i+1)P_{i+1}(t)\mu\Delta t$$
$$+P_i(t)\{1-i\mu\Delta t-\xi(t)\Delta t\},\quad i=0,1,\ldots$$

above expression may be rewritten as:

$$(P_i(t+\Delta t)-P_i(t))/\Delta t=\xi(t)P_{i-1}(t)$$
$$-(i\mu+\xi(t))P_i(t)+(i+1)\mu P_{i+1}(t)$$

If △t → 0, above equation becomes:

$$dP_i(t)/dt=\xi(t)P_{i-1}(t)$$
$$-(i\mu+\xi(t))P_i(t)+(i+1)\mu P_{i+1}(t),\quad i=0,1,\ldots \tag{A1}$$

A generating function of a random variable X(t) is defined as (Parzen,1960):

$$G_x(s,t)=\sum_{i=0}^{\infty}P_i(t)s^i,\quad (|s|<1)$$

Therefore,

$$\partial G_x/\partial t=\sum_{i=0}^{\infty}(dP_i(t)/dt)s^i$$

and

$$\partial G_x/\partial s=\sum_{i=0}^{\infty}iP_i(t)s^{i-1}$$

Multiplying both sides in equation (A1) by s^i and taking the sum of all such equations:

$$\sum_{i=0}^{\infty}dP_i(t)/dts^i=\xi(t)\sum_{i=0}^{\infty}P_{i-1}(t)s^i$$

$$-\sum_{i=0}^{\infty}\{i\mu+\xi(t)\}P_i(t)s^i+\mu\sum_{i=0}^{\infty}(i+1)P_{i+1}(t)s^i$$

or,

$$\partial G_x/\partial t=\xi(t)sG_x-\mu s\partial G_x/\partial s-\xi(t)G_x+\mu\partial G_x/\partial s$$

Rearranging above equation becomes:

$$-\partial G_x/\partial t+\mu(1-s)\partial G_x/\partial s=(1-s)\xi(t)G_x \tag{A2}$$

Following Lagrang's method, this partial differential equation may be solved by first solving the auxiliary ordinary differential equations:

$$dt/(-1)=ds/(\mu(1-s))=dG_x/((1-s)\xi(t)G_x)$$

From the first auxiliary equation:

$$-\mu dt=ds/(1-s)$$

It obtains:

$$-\mu t+\ln(1-s)=constant \tag{A3}$$

thus,

$$(1-s)=C_1\exp(\mu t)$$

From the second auxiliary equation:

$$-dt=dG_x/((1-s)\xi(t)G_x)$$

It obtains:

$$dG_x/G_x=-(1-s)\xi(t)dt=-C_1\exp(\mu t)\xi(t)dt$$

The integral of above equation is:

$$\ln|G_x|=-C_1\int_0^t\exp(\mu\tau)\xi(\tau)d\tau+C_2$$

Substituting back the value of C_1 from equation (A3):

$$\ln|G_x|=-(1-s)\exp(-\mu t)\cdot\int_0^t\exp(\mu\tau)\xi(\tau)d\tau+C_2$$

The general integral of equation (A2) is: $C_2=\eta(C_1)$; where η is an arbitrary function of its argument. Therefore,

$$\ln|G_x|+(1-s)\exp(-\mu t)\int_0^t\exp(\mu\tau)\xi(\tau)d\tau=\eta\{(1-s)\exp(-\mu t)\}$$

Thus,

$$|G_x(s,t)|=\exp\{\eta[(1-s)\exp(-\mu t)]\}\cdot\exp\left[-(1-s)\int_0^t\exp(-\mu(t-\tau))\xi(\tau)d\tau\right]=G_x(s,t) \quad \text{(A4)}$$

the product on the right hand side of equation (A4) is always positive.

Let the convolution integral defined as:

$$\int_0^t\exp(-\mu(t-\tau))\xi(\tau)d\tau=\xi(t)*\exp(-\mu t)$$

If the initial condition is determined as:

$$G_x(s,0)=g(s)=\sum_{i=0}^{\infty}P_i(0)s^i$$

and

$$g(s)=\exp(\eta(1-s))$$

Let 1-s = r, therefore, $\eta(r)=\ln g(1-r)$.

The generating function of X(t) is finally obtained as:

$$G_x(s,t)=g(s)[1-(1-s)\exp(-\mu t)]\cdot\exp[-(1-s)\cdot\xi(t)*\exp(-\mu t)] \quad \text{(A5)}$$

Equation (2) therefore is derived.

收稿日期：民國84年5月29日
修正日期：民國84年9月26日
接受日期：民國84年10月3日