

## 閘門潛式水躍之水動力特性研究

### Study on Hydrodynamic Characteristics of Submerged Hydraulic Jumps with Sluice Gate

工業技術研究院研究員

羅慶瑞  
Ching-Ruey Luo

#### 摘要

二維性非均勻流之潛式水躍特性是在水工構造物中常見之流況。有關該潛式水躍之水動力特性在定床上的解析解<sup>(1)</sup>，包括流速分佈、紊流剪力分佈、紊流動能分佈及能量消散率等特性已被廣泛分析，並且與數值解及實驗值作一比較印証，其適用性甚廣( $Fr_{(0)}=0.90\sim 8.19, S_j=0.24\sim 0.85$ )。本論文中，將進一步，求解運用在水躍長度及在動床上流場解析解，並與數值解及實驗解比較。數值解敏感度中有關參數之分析亦將一併討論。

關鍵詞：紊流剪力，動床，水躍長度。

#### ABSTRACT

Two-dimensional free-surface open channel flows in non-uniform single channels, such as convergent, divergent, and abruptly varied channels with or without hydraulic jumps are typical and common ones. The hydrodynamic characteristics for fixed bed<sup>(1)</sup> with sluice gate submerged hydraulic jumps have been discussed. In this study, the analytical length of hydraulic jumps and the velocity field of two-dimensional vertical situation are solved and the comparisons among analytical, numerical and experimental results are made. The sensitivity analysis for each parameters in numerical models is also discussed in this study.

Keywords : Turbulent shear stress, Movable bed, Length of hydraulic jumps.

## 1. INTRODUCTION

River engineering works require good understanding of river characteristics and their actions on their geometries. Water bodies of river can be mastered by

understanding the self-form geometric shapes and their response to changes in nature and human interference. Naturally, the river geometries are always non-uniform not only in the lateral direction but also in the depth direction. Eventhough, the man-made canals are non-

uniform for specific purpose due to constructing hydraulic engineering constructions.

Fluid flow, no doubt, in natural river channels or man-made constructions can be best described by 3-D mathematical model. Due to lacking of sufficient data to be used in calibrating this 3-D model and in some situation the flow field in one of the 3-D flow is quite uniform, the problem may be simplified to a 2-D situation and solved by 2-D model to obtain the very suitable solution and show major phenomena with the magnitudes of the necessary information.

The idea of plane turbulent wall jets, which possessed very strong turbulence, is used to solve analytical solutions of primary velocity and turbulent shear stress profiles, which are then used to derive the profile of turbulent eddy viscosity. The turbulent kinetic energy is derived from equation of motion, which is then used in determining the energy dissipation rate.

## 2. METHODOLOGY AND ANALYTICAL EXPRESSIONS

As mentioned above, the wall jets ideas proposed by Rajaratnam<sup>(2X3)</sup> (1965, 1976) are used to derive the primary velocity and turbulent shear stress profiles, which are then used to express the turbulent eddy viscosity. The basically theoretical considerations on 2-D continuity equation and equation of motion are used to derive the secondary velocity and turbulent kinetic energy profiles. Finally, the 2-D two-equation model presented by Rodi<sup>(4)</sup> (1980) is used to obtain the energy dissipation rate. The main turbulent quantities of the analytical expressions are as follows:

### 2-1 Velocity Profiles

(a) Foreward Flows:

$$\frac{U}{U_{max}} = \text{Exp} [ -0.905(\eta - 0.125)^2 ] \dots\dots\dots (1)$$

$$\frac{U_{max}}{V_0} = 5.395 \left[ \frac{X'}{h_0} + 11.2 \right]^{-0.555} \dots\dots\dots (2)$$

$$\frac{\delta}{h_0} = 0.0678 \left[ \frac{X'}{h_0} + 11.2 \right] \dots\dots\dots (3)$$

where:  $\eta$  = dimensionless boundary layer displacement =  $\frac{Z}{\Delta}$ ;  $-2.00 \leq \eta \leq 2.50$ ;

$V_0$  = the inflow velocity at the sluice gate (m/s);  
 $\tan(2 \theta) = 0.0678$ ;

The above equations are suitable for the following range of Reynolds number,

$$R_{e_0} = \frac{V_0 h_0}{\nu} = 13,000 \sim 50,000 \dots\dots\dots (4)$$

$$\text{and } X = X_0 + X' \dots\dots\dots (5)$$

$$X_0 = \left[ \frac{h_0}{2} \right] \cot \theta \dots\dots\dots (6)$$

Where  $X_0$  is the length of translating the flow from developing to developed;  $X'$  is the location to be analyzed. Generally  $X_0$  is neglected for calculating velocity values, while it included for concerning with calculation of the length of submerged hydraulic jumps. The definitions of the submerged hydraulic jumps are

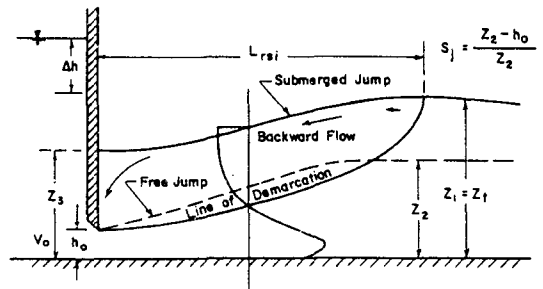


Fig 1. Submerged jump (from N. Rajaratnam, 1965)

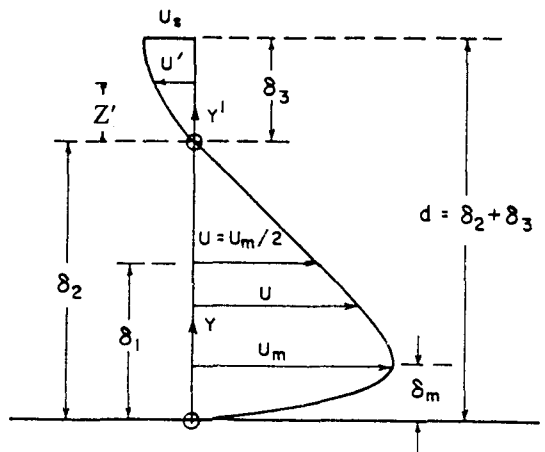


Fig 2. Forward and Backward Flow in the Submerged jump (from N. Rajaratnam, 1965)

shown in Fig. 1, Fig. 2, and Fig. 3.

The length of the longitudinal reference,  $L_{rsj}$ , could

be expressed as the relationship with

$$L'_{rsj} = L'^{rsj} + X_0 \quad \dots\dots\dots (7)$$

where  $L'_{rsj}$  is

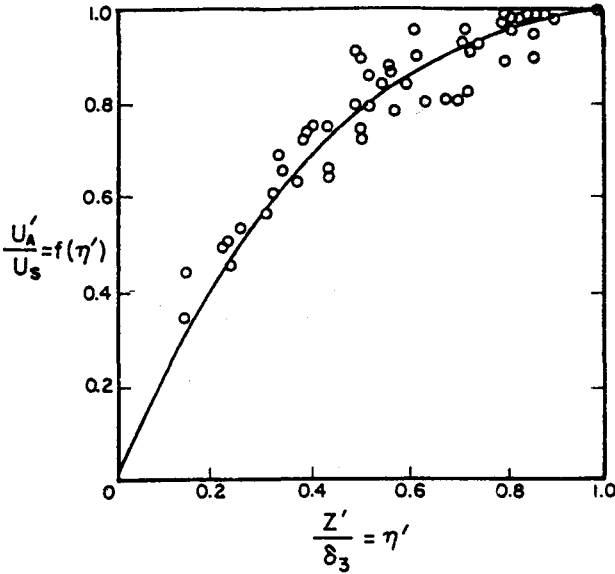


Fig 3. Velocity Distribution in the Backward Flow (from N. Rajaratnam, 1965)

$$\frac{L'_{rsj}}{Z_c} = \frac{3.31}{\left\{ \left( \frac{Z - Z_3}{Z_3} \right) + \left( \frac{1}{F_{r0}} \right) \right\}^{0.885}} \quad \dots\dots\dots (8)$$

and  $Z_c = \sqrt[3]{\frac{(v_0 h_0)^2}{g}}$  ;  $F_{r0} = \frac{v_0}{\sqrt{gh_0}}$  ..... (9)

(b) Backward Flows:

$$\frac{U'}{U_s} = f(\eta) = 1.122(\eta')^2 + 2.098\eta' \quad \dots\dots\dots (10)$$

with  $\eta' = Z' / \delta_3$  ;  $\delta_3 = d - \delta_2 = h - 2.58$  ;

$$\frac{U_s}{V_0} = -0.27 \sin(\pi\alpha) ; h = \text{water depth};$$

and  $z'$  in Fig. 2.

2-2 Turbulent Shear Stress

$$\tau_{xz} / \rho = u^2 \left[ 1 - \frac{z}{h} \right] \quad \dots\dots\dots (11)$$

and  $u^2 = 0.027 \left\{ \frac{\nu}{UR} \right\}^{\frac{1}{4}} \quad \dots\dots\dots (12)$

where R=hydraulic radius;

$\nu$  =fluid kinematic viscosity;

2-3 Turbulent Eddy Viscosity

$$v_{tz} = \delta^4 \left\{ x \frac{\nu}{h_0} + 11.2 \right\}^{0.555} \left\{ \frac{u_*^2 (h + 0.37\delta)}{9.3Z^3 h V_0} \frac{u_*^2}{9.3Z^2 h V_0} \right\} \quad \dots\dots\dots (13)$$

2-4 Turbulent Kinetic Energy

$$\begin{aligned} K = & \frac{1}{2} \left\{ 0.6h_0 V_0^2 \left[ \frac{\nu h}{RV_0 h_0} \right]^{\frac{1}{4}} \left\{ \left( 2.9 \frac{5}{h_0} \right) \right. \right. \\ & p_*^{-0.334} \left. \left. + \left[ 1. \frac{5}{h_0} \frac{6Z}{hh_0} \right] p_*^{-0.666} + \right. \right. \\ & + \left[ \frac{30Z}{h_0^2} - \frac{101Z}{h_0^2} - \frac{118Z^2}{hh_0^2} \right] p_*^{-1.666} + \\ & \left. \left. \left[ \frac{542Z^2}{h_0^3} \right] p_*^{-2.666} - \left[ \frac{11596Z^3}{h_0^4} \right] p_*^{-3.666} \right. \right. \\ & \left. \left. + \left[ \frac{93224Z^4}{h_0^5} \right] p_*^{-4.666} \right\} - U^2 \quad \dots\dots\dots (14) \end{aligned}$$

where  $p_* = \left[ \frac{X}{h_0} + 11.2 \right]$  ; R=hydraulic radius;

2-5 Energy dissipation Rate

$$\bar{\epsilon} = C_\mu \left[ \frac{k^{-2}}{v_{12}} \right] \quad \dots\dots\dots (15)$$

and  $C_\mu = 0.09$

3. COMPARISONS AND DISCUSSION

The situations of submerged hydraulic jumps due to sluice gate with and without jumps are compared among analytical, numerical and experimental results from Tran<sup>(5)</sup> (1991) and Long<sup>(6)(7)</sup> (1990, 1991)

Series	$h_0$ (m)	$V_0$ (m/s)	$Z_t$	$F_{r0}$	$S_j$	$Z_3$ (m)	$Z_c$ (m)
1	0.025	1.58	0.187	3.19	0.85	0.163	0.054
2	0.025	2.72	0.299	5.49	0.63	0.238	0.078
3	0.015	3.14	0.206	8.19	0.24	0.135	0.061

Series	$X_0$ (m)	$L_{rsj}^*$ (m)	$L_{rsj}^{**}$ (m)	$L_{rsj}^{***}$ (m)
1	0.369	0.73	0.75	0.75
2	0.369	0.91	1.00	1.00
3	0.224	0.54	0.72	0.72

### 3-1 Length of Submerged Hydraulic Jump

Based on the analytical results in Eq. (7), the comparison with numerical results of Long(1990,1991) are listed:

where  $Z_3$  and  $Z_1, h_0$  and  $V_0, X_0$  and  $S_j$  are known values. The  $L_{rsj}$  is the analytical result and  $L_{rsj}$  and  $L_{rsj}$  are experimental and numerical results, respectively. For series1 and 2, the prediction of analytical method is quite good and economical from the view of real application. When  $F_{r0} > 6$ , we need some condition on adding safe factor for analytical results.

### 3-2 Sensitivity Analysis of Parameters in Numerical Model

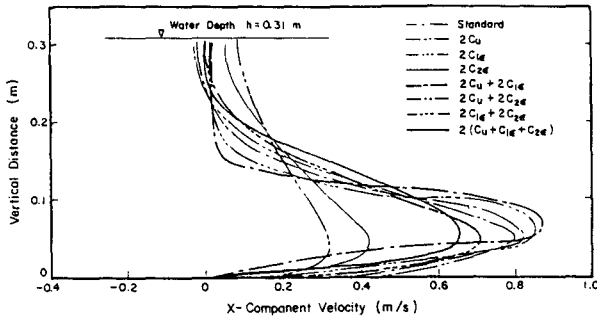


Fig 4. Effect of Model Parameters on Computed Velocity, Width-Averaged Hydrodynamic Model with Sluice Gate, at Section  $x = 0.40m$

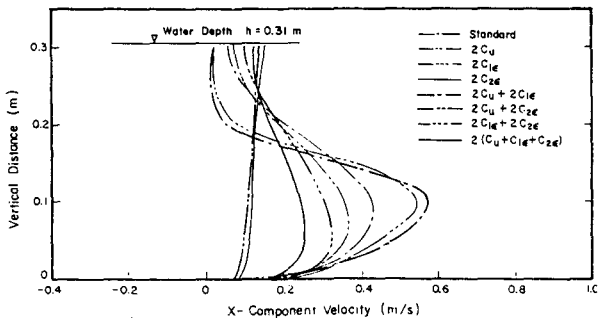


Fig 5. Effect of Model Parameters on Computed Velocity, Width-Averaged Hydrodynamic Model with Sluice Gate, at Section  $x=1.0m$

The standard  $k-\epsilon$  equation is widely used.

However, it is also been well documented that this model has the tendency to "underpredict" the size of zones of flow separation and the turbulent quantities. The "algebraic stress model" can improve the weakness of  $k-$

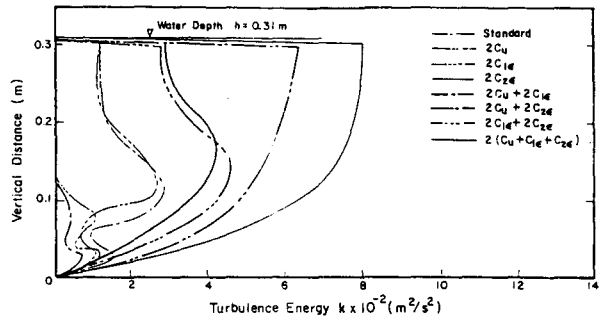


Fig. 6 Effect of Model Parameters on Computed Turbulence Energy, Width-Averaged Hydrodynamic Model With Sluice Gate at Section  $x = 0.40m$

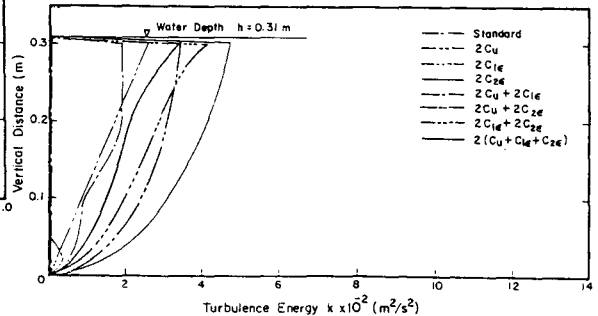


Fig 7. Effect of Model Parameters on Computed Turbulence Energy, Width-Averaged Hydrodynamic Model with Sluice Gate at Section  $x=1.0m$

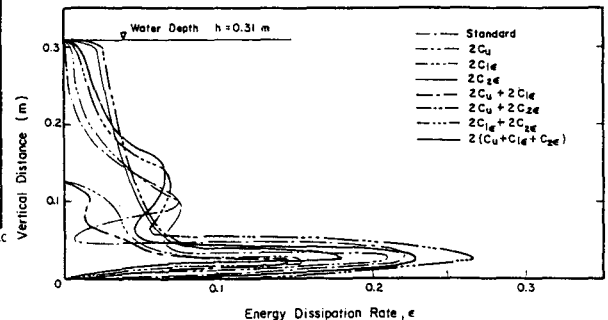


Fig 8. Effect of Model Parameters on Computed Energy Dissipation Rate, Width-Averaged Hydrodynamic Model with Sluice Gate at Section  $x = 0.40m$

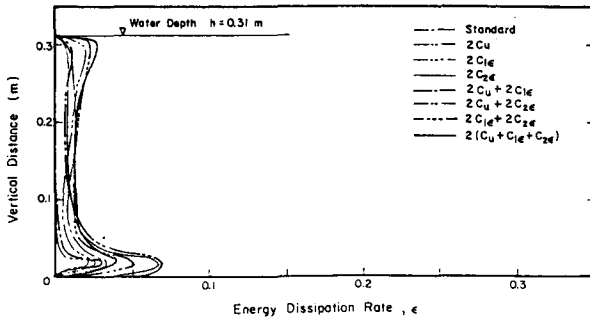


Fig 9. Effect of Model Parameters on Computed Energy Dissipation Rate, Width-Averaged Hydrodynamic Model with Sluice Gate at Section  $x=0.0m$

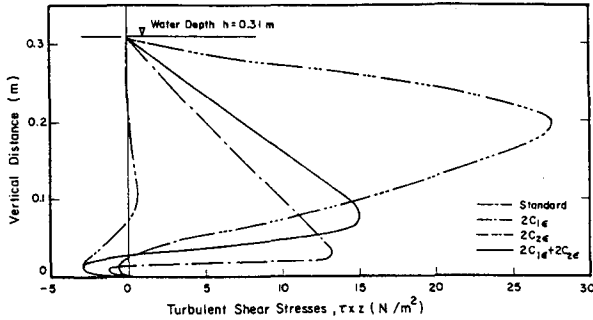


Fig 10. Effect of Model Parameters on Computed Turbulent Shear Stresses Width-Averaged Hydrodynamic Model with Sluice Gate at Section  $x = 0.40m$

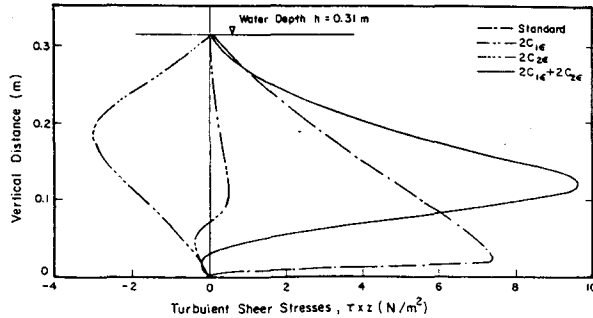


Fig 11. Effect of Model Parameters on Computed Turbulent Shear Stresses, Width-Averaged Hydrodynamic Model with Sluice Gate at Section  $x = 1.0m$

$\epsilon$  equation but higher order terms occur and move correlation must be done. Based on the view of points on uncertainty and sensitivity analysis, "the more the parameter are, the larger the uncertainty exists" Many sensitivity analyses were done in Tran(1991), Gao<sup>(8)</sup> (1992).  $C_{1\epsilon}$  and  $C_{2\epsilon}$  show the inverse functions on the turbulent quantities. The other parameters, such as  $C_{\mu}$ ,  $\sigma_k$ ,  $\sigma_{\epsilon}$ , have very low sensitivity to the turbulent

quantities. The sensitivity analysis for the parameters is plotted from Fig. 4 to Fig. 11, and we can see the quantities of the hydrodynamic characteristics are affected very much by  $C_{1\epsilon}$  and  $C_{2\epsilon}$ .

From the equation presented in Nezu and Nakagawa<sup>(9)</sup> (1987),  $C_{\mu}$  is,

$$C_{\mu} = 0.09 \left[ 1 - D_1 \exp\left(-\frac{R_t}{D_2}\right) \right] \text{ where } D_1 = 0.95, D_2 = 250, \text{ and } R_t = K^2 / (\nu / \epsilon).$$

In turbulent flows, the value  $R_t$  is always large due to small value of  $\nu$ . Therefore, the variation of  $C_{\mu}$  is quite small, and say,  $C_{\mu} = 0.09$ . This hints the wide applicability and high reliability of the analytical methods and results because  $C_{\mu}$ , with the value 0.09, is the only one parameter used in analytical solutions. The analytically turbulent quantities, such as turbulent shear stress, turbulent kinetic energy and turbulent energy dissipation rate, are always greater than the ones from the standard  $k-\epsilon$  numerical methods, but they have quite close results to the experiment or real field data. This can be used to avoid the underpredict of the result from the numerical

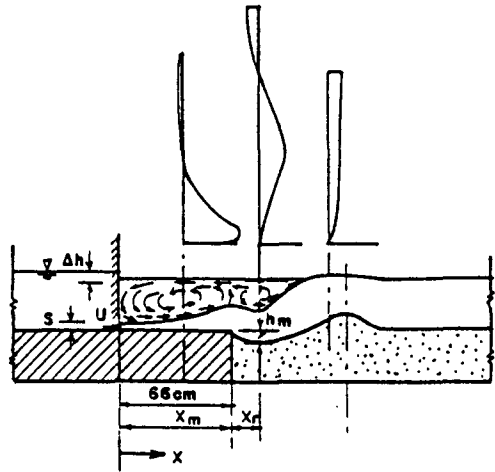


Fig 12. Diffused jet along with Typical Velocity Profiles methods of  $k-\epsilon$  equation with standard parameters, for the stronger turbulence situations.

### 3-3 Flow Field in Movable Bed

From using Eqs. (1)(2), and (3) as in Figs. 12 and 13 for  $7 \leq 0$ , the analytical velocity profile is obtained in Zone 2 of Fig. 14 and compared with the numerical

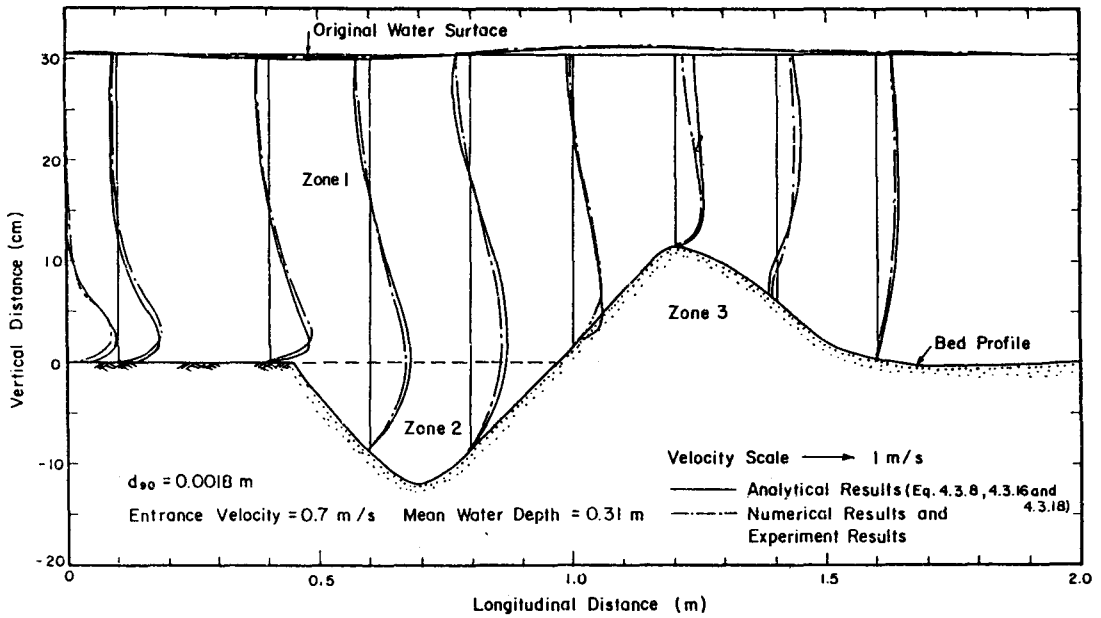


Fig. 14. Comparison of 2-DV Analytical Results and Numerical X - Component Velocities, Width-Averaged Hydrodynamic Model (VEST), with Sluice Gate, Scoured Bed

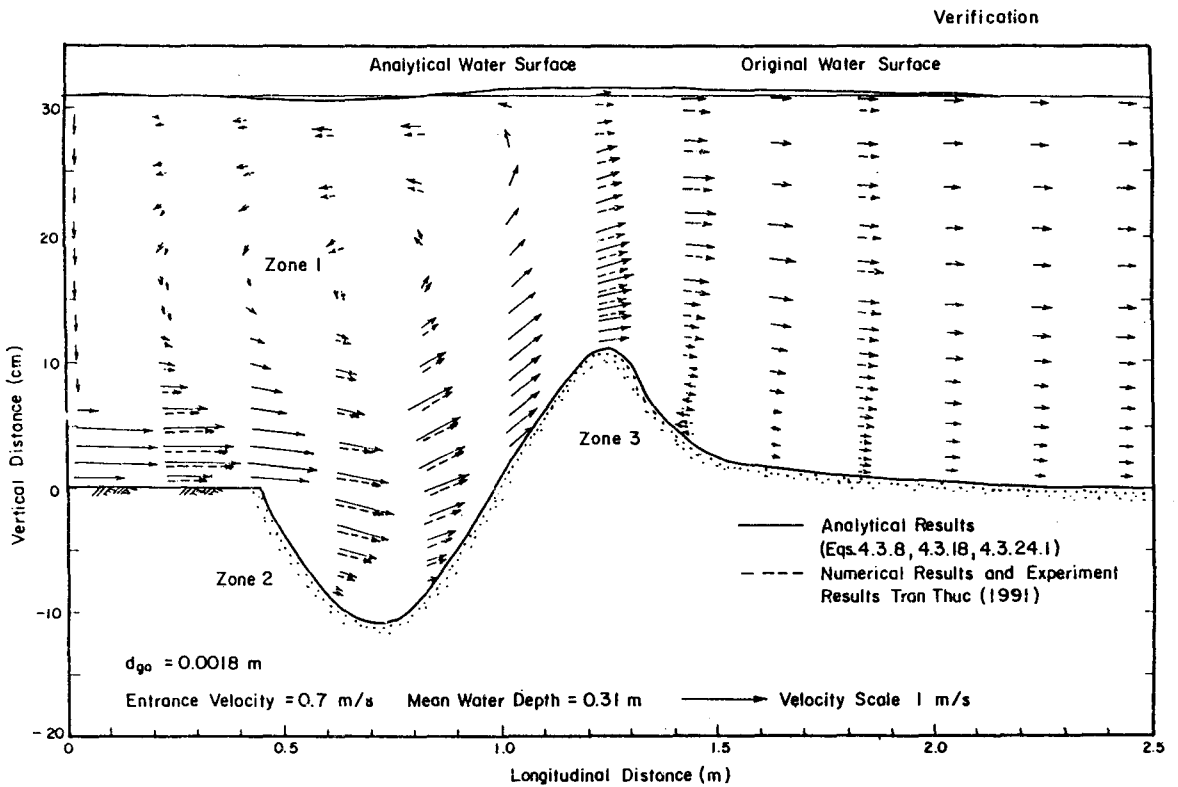


Fig. 15. Analytical Velocity Vector Field, Width-Averaged Hydrodynamic Method with Sluice Gate, Scoured Bed Condition

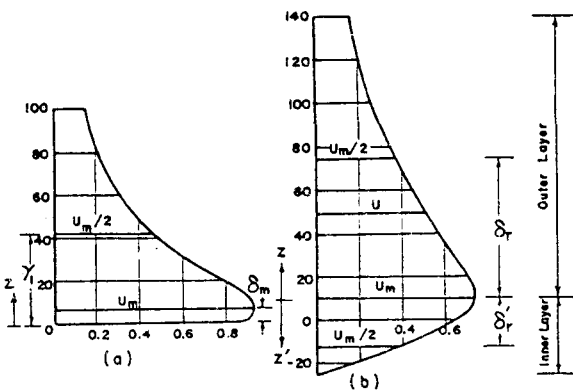


Fig. 13 Typical Mean Velocity Profiles:  
(a) Rigid Apron; (b) at Scour Hole

results of Tran (5). The deceleration open channel flow velocity profile (10) as following is used for Zone 3 of Fig. 14 and also compared with the results of Tran (5) again;

$$\frac{u}{u_*} = \frac{1}{k} \ln \left\{ \frac{ZU_*}{\nu} \right\} + c + \frac{T_d}{k} w(Z/\delta) \dots \dots (16)$$

where  $k=0.40; C=4.90$ ; and

$$T_d = - \left\{ \frac{0.76 + \sqrt{0.58 + 1.68\beta^*}}{0.84} \right\} \dots \dots \dots (17)$$

$$\beta^* = \frac{\delta_*}{\tau_o} r_w \frac{dh}{dx} \dots \dots \dots (18)$$

in which  $\tau_o = \gamma C_v$  the fluid specific weight;  
and  $\delta_* = 1/8 \delta$ ;  $\beta^*$  = the pressure gradient parameter;

By continuity equation, the velocity profile of depth direction can be solved and combined the velocity profile of main flow direction, the flow field are expressed in Fig.15. The good agreement between analytical and numerical results for zone 1(backward flow), zone 2 and zone 3 are obtained in Figs.14 and 15.

#### 4. CONCLUSIONS

After comparisons and discussion, it shows that the very good predictions for the case of submerged hydraulic jumps on velocity profile, and length of submerged hydraulic jumps, are obtained for  $Fr_0 \leq 8.2$  with movable bed. Finally, the analytical results are obtained by using some special function tables and

calculator only with one parameter,  $C_\mu$ . It is more economical than the results by using numerical or experimental methods.

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