

# 蒸煮擠壓系統內米穀粉滯留時間分佈模式之推導

## Modeling Residence Time Distribution for Extrusion Cooking System of Rice Flour

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### 摘 要

米穀粉被用來研究蒸煮擠壓機(APV Baker MPF 50/25)內物料滯留時間分佈之模式。本研究為 $3 \times 3 \times 3$ 複因子實驗設計，操作變數為螺軸組成、進料速率以及螺軸轉速等。

最適合之滯留時間分佈模式已被推導出，本實驗之最適合模式為綜合Tanks-in-series模式與修正後的Wolf and White模式兩者而成。此模式可以滿意地預估物料在擠壓機內滯留時間分佈之情形。而且，此種滯留時間分佈模式之推導技術可以成功地應用到其它各種型式之擠壓系統。

關鍵詞：滯留時間分佈，蒸煮擠壓系統，米穀粉。

### ABSTRACT

Residence time distribution (RTD) modeling for a cooking extruder (APV Baker MPF 50/25) was studied using rice flour as the feed material. The experiment was  $3 \times 3 \times 3$  factorial design, and the process variables were screw profile, feed rate, and screw speed.

A best fit TRD model was developed. The combination RTD model (Tanks-in-Series model and modified Wolf and White model) developed in this study can satisfactorily predict the residence time distributions in the extrusion cooking system. The RTD modeling technique can also be extended to other extruder experiments.

Keywords: Residence time distribution, Extrusion cooking system, Rice flour.

## INTRODUCTION

Extrusion is a high-temperature short-time (HTST) process which combines several unit operations including mixing, kneading, shearing, cooking, shaping and forming. It has been used in many industries, such as the plastic, food, rubber, metal, carbon and ceramic industries (Fellows, 1988; Harper, 1979 and 1981; Mercier et al., 1989; Valentas et al., 1991). A number of well-known food products, such as baby food, roast flavors, biscuits, puffed rice, cereal flakes, glass or fish noodles, pet food, and chocolate, can be made with an extruder instead of the conventional machines (Wiedman and Strobel, 1987).

An extruder is a machine which shapes the materials by the process of extrusion. Single-screw cooking extruders were developed in the 1940s to make puffed snacks from cereal flours or grits (Mercier et al., 1989). The use of twin-screw extruders for food processing started in the 1970s, with an expanding number of applications in the 1980s (Mercier et al., 1989).

The most important factors affecting extrusion systems are the operating conditions of the extruder and the rheological properties of the food (Fellows, 1988). The residence time distribution data are most useful in diagnosing axial mixing phenomena in extruders, providing the basis for scale up, and providing guidance in improvements of equipment (Bruin et al., 1978; Janssen, 1978; Todd, 1975a, b). Bruin et al. (1978) and Lin and Armstrong (1988) also reported that the residence time distribution (RTD) of the feed material in an extruder is one of the most important parameters in characterizing mixing and chemical kinetics, and is a useful tool in determining the optimal operating conditions for blending, dispersing, and polymerization applications.

Residence time distributions of the material in an extruder can be determined by using radioactive-tracer and dye-tracer techniques, which can be ex-

pressed by E curves and F curves. From the literature studied, the feed composition, feed rate (or throughput), feed moisture content, screw speed, screw profile, barrel temperature, and die size (or die opening) are known as important process variables affecting the residence time distributions in extrusion cooking systems.

The flow pattern (RTD model) in the extruder can be derived from the theoretical velocity profile or can be determined experimentally from the residence time distributions (Bounie, 1988; Jassen, 1978; Martelli, 1983). RTD models, such as perfect plug flow, perfect mixing, laminar pipe flow, tanks-in-series, axial dispersion, and the Wolf and White model, have been widely tested for single-screw extruders. Owing to the incomplete knowledge of the exact flow behavior and the complexity of the mathematical models, RTD models have not been widely investigated for the twin-screw extruder (Altomare and Anelich, 1988; Altomare and Ghossi, 1986; Bounie, 1988; Janssen, 1978; Janssen et al., 1979; Onwulata, 1991; Todd, 1975a and b; van Zuilichem et al., 1973, 1983, 1988a, b, c; Wolf et al., 1986). Thus, a further study on the residence time distributions models in a twin-screw extruder are necessary. Therefore, the objectives of this research were to develop a RTD model to optimize the process variables and satisfactorily predict the residence time distribution for the extrusion cooking system.

## LITERATURE REVIEW

### RESIDENCE TIME DISTRIBUTION

One of the most important properties in the extruder's performance is the residence time distribution (RTD) of the food material during its passage through the apparatus. The residence time distribution can be used to characterize the overall mixing in an extruder (Janssen, 1978). The theory of the RTD was developed by chemical engineers, and the E curve and F curves were used to summarize the RTD. The E curve

is the response of the system to a pulse-like injection of a tracer at the inlet, and the F curve is the response of the system when a stepwise change in inlet concentration (e. g. a tracer) is made (Altomare and Anelich, 1988; Bruin et al., 1978; Dan-ckwerts, 1953; Fogler, 1986; Levenspiel, 1972; Lin and Armstrong, 1988; Smith, 1981).

The E(t) function is obtained by injecting an instantaneous pulse (tracer) into the system and determining the output concentration of tracer, and the time it takes to exit from the extruder. Since it is difficult to insure that the same amount of tracer is used in all experiments, it is common to normalize the tracer concentrations at each point in time by dividing them by the total amount of tracer passed through the system. Therefore, the E curve gives the exit age distribution and is plotted as normalized concentration E(t) versus residence time (t). The E(t) is given by Levenspiel (1972) and Smith (1981):

$$E(t) = \frac{C}{\int_0^\infty C dt} = \frac{C}{\sum_0^t C \Delta t} \dots\dots\dots (1)$$

and

$$\int_0^\infty E(t) dt = \int_0^\infty \left( \frac{C}{\int_0^\infty C dt} \right) dt = \int_0^\infty \frac{C}{M} dt = 1 \dots\dots\dots (2)$$

where C: Concentration  
t: Time

The F(t) function is the system response in output to a step change of tracer to the inlet feedstock, and it represents the cumulative tracer concentration in the exit stream at any time. Therefore, it is obtained by integrating the E(t) function (Levenspiel, 1972; Smith, 1981).

$$F(t) = \int_0^t E(t) dt = \frac{\int_0^t C dt}{\int_0^\infty C dt} = \frac{\sum_0^t C \Delta t}{\sum_0^\infty C \Delta t} \dots\dots\dots (3)$$

The F curve, which is plotted as the cumulative E(t), i. e., F(t), versus normalized time ( $\theta$ ), is usually used for the comparison of RTD models. The normalized time ( $\theta$ ) is defined by:

$$\theta = \frac{t}{\bar{t}} \dots\dots\dots (4)$$

where  $\bar{t}$  is mean residence time

### RESIDENCE TIME DISTRIBUTION MODELS

It is very important to thoroughly understand the flow behavior in an extruder when producing extruded foods or designing extruders.

Bounie (1988) reported that the modeling of flow conditions may be used for the following purposes:

- (1) To design and scale-up extruders.
- (2) To optimize the type of sequence of screw elements.
- (3) To compare the flow characteristics and the efficacy of different extruders.
- (4) To predict the extent of a continuous chemical reaction and to control the physical and chemical changes occurring during the different stages of extrusion.
- (5) To quantify the effect of each screw configuration on the flow behavior under different process conditions.

The flow pattern may be derived from the theoretical velocity profile for the food mix in the extruder, and this approach has been used extensively for single-screw extruder (Bigg and Middleman, 1974; Bounie, 1988; Bruin et al., 1978; Harper, 1981; Janssen, 1978; Lidor and Tadmor, 1976; Tadmor and Klein, 1970). However, this approach is still unreliable in the general case of twin-screw extruders because of the complexity of flow patterns in the intermeshing zone and through the different leakage gaps. The other approach, which determines the flow behavior by experimental measuring the residence time distribution, is recommended for twin-screw extruders (Bounie, 1988; Janssen, 1978).

Numerous flow models (RTD models), such as perfect plug flow (PFR-plug flow reactor), perfect mixing (CSTR-continuous stirred tank reactor), perfect mixing (CSTRs-in-series), axial dispersion, and

Wolf and White models, have been tested for single-screw extruders (Bruin et al., 1978; Davidson et al., 1983; van Zuilichem et al., 1973; Wolf and White, 1976). Different studies examined RTDs in twin-screw extruders (Altomare and Anelich, 1988; Altomare and Ghossi, 1986; Bounie, 1988; Jager, 1989; Jager et al., 1991; Janssen, 1978; Janssen et al., 1979; Kao and Allison, 1984; Lin and Armstrong, 1988; Mange et al., 1986; Olkku et al., 1980; Onwulata, 1991; Todd, 1975a and b; Todd and Irving, 1969; van Zuilichem et al., 1973, 1983, 1988a, b, c; Wolf et al., 1986), but only a few of these suggested realistic flow models. The reasons were due to the incomplete knowledge of the exact flow behavior in twin-screw extruders and also the complexity of the mathematical methods. These RTD models are described below (Fogler, 1986; Levenspiel, 1972; Middleman, 1977; Miles and Briston, 1965; Smith, 1981).

**Plug Flow Reactor (PFR)**

The characteristics of a plug flow reactor are uniform velocity profile and no axial mixing, and it requires a constant residence time, i. e.,  $t=V/Q=\text{constant}$ .

For a step-function input (shown in Figure 1),

$$F(t) = 0, t < \frac{V}{Q} \dots\dots\dots (5)$$

$$F(t) = 1, t \geq \frac{V}{Q} \dots\dots\dots (6)$$

For a pulse input, the input and response curve would correspond to narrow peaks at  $t=0$  and  $t=V/Q$ , as shown in Figure 2.

**Perfect Mixing (CSTR-Continuous Stirred Tank Reactor)**

For an ideal continuous stirred tank reactor,  $F(t)$  or  $(C/C_0)_{t=0}$  can be calculated by writing a mass balance equation:

$$C_0 Q \Delta t - C Q \Delta t = V \Delta C \dots\dots\dots (7)$$

where V: Volume of reactor (extruder)

Q: Volumetric flow rate

C: Concentration at time=t (output, prod-

uct)

$C_0$ : Concentration at time=0 (input, feed)

$\Delta C$ : The change in concentration of tracer in the reactor during  $\Delta t$ .

Dividing by  $V \Delta t$  and taking the limit as  $\Delta t \rightarrow 0$

gives

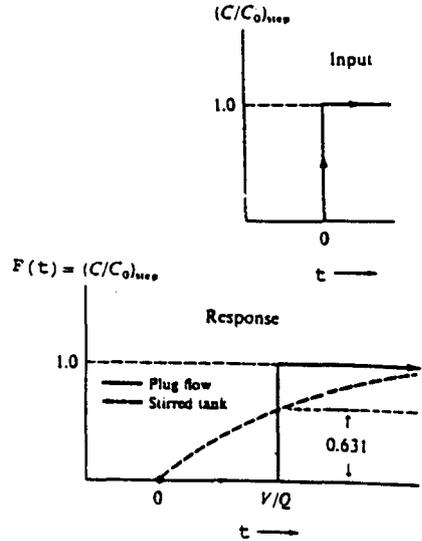


Figure 1. Response curves to a step input for ideal reactors

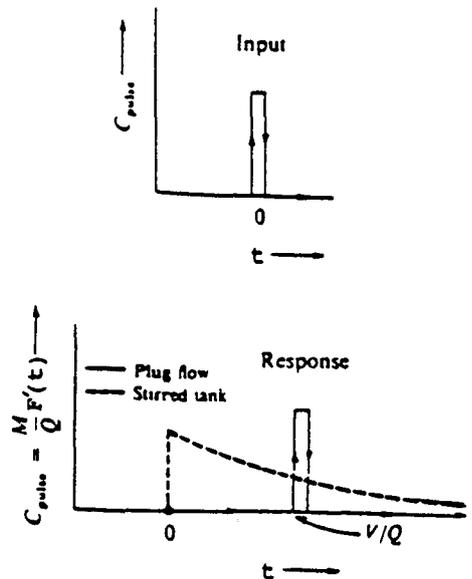


Figure 2. Response curves to a pulse input for ideal reactors

$$\frac{dC}{dt} = \frac{Q}{V}(C_0 - C) = \frac{1}{\bar{t}}(C_0 - C) \dots \dots \dots (8)$$

With the initial condition  $C=0$  at  $t \leq 0$ , the solution of equation (8) is

$$\left(\frac{C}{C_0}\right)_{s.t.e.p} = F(t) = 1 - e^{-(t/\bar{t})} \dots \dots \dots (9)$$

The response curve described by equation (9) is shown in Figure 1.

At the mean residence time  $t=\bar{t}=V/Q$ ,  $F(t)=0.631$ . It means 63.1% of the effluent stream has a residence time less than the mean value.

For pulse input, the total molecules marked in CSTR is

$$M = C_0 Q \Delta t_0 \dots \dots \dots (10)$$

Since  $C$  is the concentration of marked molecules at time  $t$ , the number of such molecules leaving the reactor in the time period  $t$  to  $t+dt$  will be  $CQdt$ . All the marked molecules in the effluent will have a residence time  $t$  to  $t+dt$  because they were added only at  $t=0$ . By definition, the fraction of the effluent stream consisting of such molecules will be  $dF(t)$  or  $F'(t)dt$ . So, the number of such molecules will be  $MF'(t)dt$ . Equating the two expressions for the number of marked molecules gives

$$CQdt = MF'(t)dt \dots \dots \dots (11)$$

or

$$F'(t) = \frac{(C)_{pulse} Q}{M} \dots \dots \dots (12)$$

$M$  can be determined from the area of the response curve, that is,

$$M = Q \int_0^\infty (C)_{pulse} dt \dots \dots \dots (13)$$

Therefore, equation (12) becomes

$$F'(t) = \frac{(C)_{pulse}}{\int_0^\infty (C)_{pulse} dt} \dots \dots \dots (14)$$

Integral  $F(t)$ , is as follows

$$F(t) = \int_0^t F'(t) dt = \frac{\int_0^t (C)_{pulse} dt}{\int_0^\infty (C)_{pulse} dt} \dots \dots \dots (15)$$

By definition,  $F'(t)$  is the slope of step function, so

$$F'(t) = \frac{d}{dt} \left( \frac{C}{C_0} \right)_{s.t.e.p} \dots \dots \dots (16)$$

Substituting equations (16) and (10) to equation (12), gives

$$\frac{d}{dt} \left( \frac{C}{C_0} \right)_{s.t.e.p} = \frac{(C)_{pulse} Q}{C_0 Q \Delta t_0} \dots \dots \dots (17)$$

Finally,

$$\frac{(C)_{pulse}}{C_0} = \Delta t_0 \frac{d}{dt} \left( \frac{C}{C_0} \right)_{s.t.e.p} \dots \dots \dots (18)$$

Thus, the response curve to a pulse input is proportional to the derivative of the response curve for a step input.

From equations (12) and (16),

$$(C)_{pulse} = \frac{M}{Q} F'(t) \dots \dots \dots (19)$$

that is,

$$(C)_{pulse} = \frac{M}{Q} \left[ \frac{d}{dt} \left( \frac{C}{C_0} \right)_{s.t.e.p} \right] \dots \dots \dots (20)$$

For CSTR, from equation (9), taking the first derivative, gives

$$F'(t) = \frac{1}{\bar{t}} e^{-(t/\bar{t})} = \frac{1}{\bar{t}} e^{-\theta} \dots \dots \dots (21)$$

Substituting equation (21) to equation (19), gets

$$(C)_{pulse} = \frac{M}{Q} \frac{1}{\bar{t}} e^{-(t/\bar{t})} \dots \dots \dots (22)$$

or from equation (18),

$$(C)_{pulse} = C_0 \Delta t_0 \frac{d}{dt} \left( \frac{C}{C_0} \right)_{s.t.e.p} = C_0 \Delta t_0 \frac{1}{\bar{t}} e^{-(t/\bar{t})} \dots \dots \dots (23)$$

Equations (22) and (23) show that  $(C)_{pulse}$  will be greatest at  $t=0$  and will continually decrease toward zero as  $t$  increases. Such a distribution curve, given as the dashed line in Figure 2.

### Tanks-in-Series (CSTRs-in-Series)

In the series of stirred-tanks model, Smith (1981) reported that the actual reactor is simulated by  $N$  ideal stirred tanks in series. The total volume of the tanks is the same as the volume of the actual reactor. Thus, for a given flow rate, the total mean residence time is also the same. Levenspiel (1972) also pointed out this model has been widely used to represent the nonideal flow. Figure 3 describes the situation.

According to Figure 3(a),

$$V_1 = V_2 = V_3 = \dots = V_j = \dots = V_N \dots \dots \dots (24)$$

$$V_t = NV_j \dots\dots\dots (25)$$

$$\bar{t}_t = \frac{NV_j}{Q} = \frac{V_t}{Q} \dots\dots\dots (26)$$

The objective is to find the value of N for which the response curve of the model would best fit the response curve for the actual reactor. To do this, the relation between  $(C/C_0)_{step}$  and N should be developed.

A mass balance on the jth reactor for step-input gives

$$C_{j-1}Q - C_jQ = V_j \frac{dC_j}{dt} \dots\dots\dots (27)$$

Then

$$\frac{dC_j}{dt} + \frac{Q}{V_j} C_j = \frac{Q}{V_j} C_{j-1} \dots\dots\dots (28)$$

The mean residence time is given from equation (26), so

$$\frac{Q}{V_j} = \frac{N}{\bar{t}_t} \dots\dots\dots (29)$$

Substituting equation (29) to equation (28), gets

$$\frac{dC_j}{dt} + \frac{N}{\bar{t}_t} C_j = \frac{N}{\bar{t}_t} C_{j-1} \dots\dots\dots (30)$$

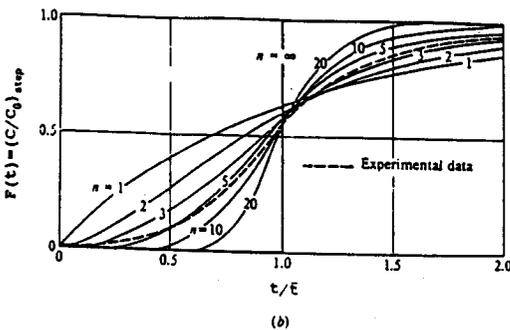
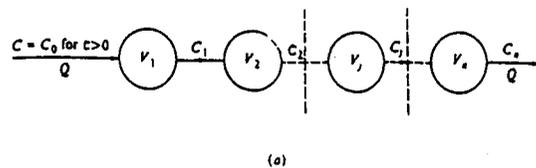


Figure 3. (a) Series of ideal stirred-tank reactors  
(b) Response curves for tanks-in-series model

Equation (30) is a first-order linear differential equation. It is similar to the form:

$$y' + p(x)y = r(x) \dots\dots\dots (31)$$

The solution of equation (31) for y(x) is

$$y(x) = e^{-h} \left[ \int e^h r dx + c \right]$$

$$h = \int p(x) dx \dots\dots\dots (32)$$

The initial condition for tanks-in-series model is  $C_j=0$  at  $t=0$ . Thus, equation (30) can be solved by using the method of equation (32). The solution is

$$C_j = \frac{N}{\bar{t}_t} e^{-Nt/\bar{t}_t} \int_0^t C_{j-1} e^{Nt/\bar{t}_t} dt \dots\dots\dots (33)$$

For the first stage,  $C_{j-1}=C_0$ , then

$$C_1 = \frac{N}{\bar{t}_t} e^{-Nt/\bar{t}_t} \int_0^t C_0 e^{Nt/\bar{t}_t} dt \dots\dots\dots (34)$$

Integrating the right hand side of equation (34) gets

$$\left(\frac{C_1}{C_0}\right)_{step} = 1 - e^{-Nt/\bar{t}_t} \dots\dots\dots (35)$$

For the second stage,  $C_{j-1}=C_1$ , then

$$C_2 = \frac{N}{\bar{t}_t} e^{-Nt/\bar{t}_t} \int_0^t C_0 (1 - e^{-Nt/\bar{t}_t}) e^{Nt/\bar{t}_t} dt \dots\dots (36)$$

or

$$\left(\frac{C_2}{C_0}\right)_{step} = 1 - e^{-Nt/\bar{t}_t} \left(1 + \frac{Nt}{\bar{t}_t}\right) \dots\dots\dots (37)$$

For the third stage,

$$\left(\frac{C_3}{C_0}\right)_{step} = 1 - e^{-Nt/\bar{t}_t} \left[1 + \frac{Nt}{\bar{t}_t} + \frac{1}{2} \left(\frac{Nt}{\bar{t}_t}\right)^2\right] \dots\dots (38)$$

For the fourth stage,

$$\left(\frac{C_4}{C_0}\right)_{step} = 1 - e^{-Nt/\bar{t}_t} \left[1 + \frac{Nt}{\bar{t}_t} + \frac{1}{2} \left(\frac{Nt}{\bar{t}_t}\right)^2 + \frac{1}{3!} \left(\frac{Nt}{\bar{t}_t}\right)^3\right] \dots (39)$$

Therefore, for N stages,

$$\left(\frac{C_N}{C_0}\right)_{step} = F_N(t) = 1 - e^{-Nt/\bar{t}_t} \left[1 + \frac{Nt}{\bar{t}_t} + \frac{1}{2!} \left(\frac{Nt}{\bar{t}_t}\right)^2 + \dots + \frac{1}{N-1!} \left(\frac{Nt}{\bar{t}_t}\right)^{N-1}\right] \dots (40)$$

Figure 3(b) is a plot of equation (40) for various values of N. The case of  $N=\infty$  shows the PFR model.

### Wolf and White Model

The actual conditions in the extruder are mostly

neither that of perfect mixing (CSTR) nor of plug flow (PFR). Wolf and White (1976) proposed a new model which was a combination of plug flow and perfect mixing.

The model is

$$\left(\frac{C}{C_0}\right)_{t \leq p} = F(t) = 1 - e^{-\left(\frac{1-p}{t}\right)\left(\frac{t}{\tau} - p\right)}; \quad \frac{t}{\tau} \geq p \dots\dots (41)$$

$$\left(\frac{C}{C_0}\right)_{t \leq p} = F(t) = 0; \quad 0 < \frac{t}{\tau} < p \dots\dots\dots (42)$$

Where P is the fraction of material in plug flow, and (1-P) is the fraction of material in perfect mixing.

In the Wolf and White model, P=1.0 represents plug flow and P=0.0 represents perfect mixing (as shown in Figure 4).

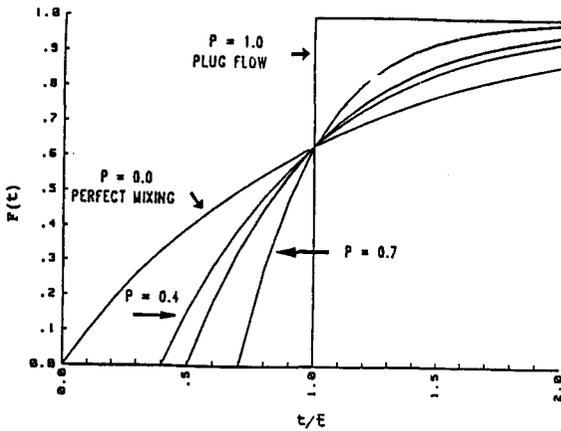


Figure 4. The Wolf and White model

**MATERIALS AND METHODS**

**Materials**

Rice flour (RL-100, RIVLAND, Stuttgart, AK) with moisture content of 20.0% (wet basis) was used as the feed material. Red dye (FD&C, #40, Warner Jenkinson, St. Louis, MO) was chosen as a tracer to measure the residence time distributions of the rice flour in a twin-screw extruder.

**Extruder**

A co-rotating and intermeshing APV Baker MPF 50/25 twin-screw extruder (APV Baker Inc., Grand Rapids, MI), as shown in Figure 5, was used in this study. The power of this machine is 28.0 KW (kilowatt) and the total length: diameter ratio is 25:1. The barrel diameter is 50mm. there are nine temperature controlled barrel sections, and six sections were used (Hisih et al. 1990). The barrel temperature was set at 26.7°C (feeding zone), 51.7°C, 93.3°C, 121.1°C, and 121.1°C (80, 125, 200, 250, and 250°F) respectively throughout the experiments. The maximum screw speed of this machine was 500 rpm. The die size was 3.175mm (1/8 in.). Rice flour was fed into the extruder with a K-tron Type T-35 twin-screw volumetric feeder with a Series 6300 controller (K-tron Corp., Pitman, NJ). The adjustable cutter with four blades was operated at 325 rpm. A computer data acquisition system was used to record feed rate, barrel and product temperatures, die pressure and temperature, % torque, screw speed and cutter speed. It took about 15 minutes for the extruder to reach te steady state after start-up.

**Experimental design**

A 3x3x3 factorial experiment with two replicati-

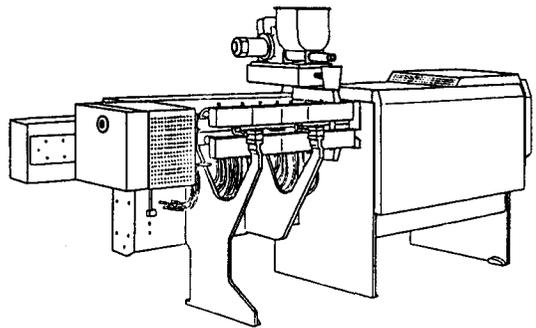


Figure 5. The APV Baker MPF 50/25 twin-screw extruder

ons was designed for this study. The process variables chosen for modeling residence time distributions were screw profile (7:5, 9:3, and 11:1 i. e., the number of forward screw paddles to reverse paddles in the extruder metering zone), feed rate (30, 40, and 50 kg/hr), and screw speed (200, 300, and 400 rpm). Thus, 27 treatments were conducted throughout this research. For each treatment, 33 samples were collected in 6 minute intervals to determine the RTDs of rice flour in the extruder. A colorimetric method developed (Peng et al., 1991) was used for RTD modeling.

### Standard Curve Experiment

A standard curve of this study was established by a separate experiment. Rice flour and red dye were mixed in a HOBART N50 mixer (Hobart Canada Inc., North York, Ontario, Canada) to different dye concentrations of 0.0%, 0.01%, 0.025%, 0.05%, 0.075%, 0.1%, 0.15%, 0.2%, 0.4%, and 0.6% (w/w). The mixtures were extruded in a MPF 50/25 twin-screw extruder at 20% moisture (w.b), 40 kg/hr and 300 rpm. As the extruder reached the steady state, a timer was started. Samples for each mixture were collected in one minute intervals from the 4th to 6th minutes. The color values L, a, and b ("L" measures lightness, "a" measures redness, and "b" measures yellowness) of each sample were measured according to the method described below, and the concentrations (g color/100 g rice flour) vs. redness color values were plotted as the standard curve for the experiment.

### Collection of Samples and Color Measurement

In the study of the  $3 \times 3 \times 3$  factorial experiment, samples of each treatment were collected as the extruder reached the steady state. One gram of red dye was suddenly dropped into the extruder as a tracer, and a timer was started at the same time. Meanwhile, the extrudate samples were collected for up to 6 minutes (10 s intervals in the first 30 s, 5 s intervals in the next 60 s, 10 s intervals in the following 90 s, and 20 s intervals in the last 180 s). Therefore, 33 samples were collected for each treatment. Each sample was

ground by a Waring Blender (New Hartford, CT) and passed the Taylor standard screen (No. 8). The color values (L, a, and b) for each ground sample were measured and recorded using a HunterLab D25 colorimeter (Hunter Associates Lab., Inc., Reston, VA). A white tile (standard No. C2-28656) with values of  $L = 91.2$ ,  $a = -0.9$ , and  $b = -0.7$  was used to standardize the colorimeter. For each sample, two petri dishes with ground extrudates were measured, and two color readings (L, a and b) were recorded for each dish. The second reading was obtained after a  $90^\circ$  rotation of the first reading. Therefore, four readings of color values L, a and b, respectively, were recorded for each sample.

### Data analysis

The LOTUS 1-2-3 computer software (Lotus Development Corporation, MA, 1986) was used to calculate the raw data of the color readings for each treatment in the  $3 \times 3 \times 3$  factorial experiment. Fortran 77 programs were written for the standard curve establishment and the residence time distributions modeling. The RTD regression models were derived using Statistical Analysis System (SAS, Release 6.03, Cary, NC, 1989). The F curves of this study were plotted using computer graphics software package (Sigmaplot, version 3.1, Jandel Scientific, 1987).

## RESULTS AND DISCUSSION

The F curves of residence time distributions were plotted as cumulative  $E(t)$ , i. e.,  $F(t)$ , versus normalized time (residence time/mean residence time). Almost all RTD models are represented by the  $F(t)$  function. The  $F(t)$  function was calculated according to equation (3).

Residence time distribution models, such as perfect plug flow, perfect mixing, laminar flow, tanks-in-series, axial dispersion, and Wolf and White models have been examined in twin-screw extruders by many investigators. The power-law flow or newtonian flow model was used to explain the flow behavior in the

twin-screw extruder (Bigg and Middleman, 1974). The axial dispersion model was used by van Zuilichem et al. (1988c) and Altomare and Anelich (1988). The Wolf and White model was used to predict the RTD in a twin-screw extruder by Wolf and White (1976), Wolf et al. (1986), Lin and Armstrong (1988), and Altomare and Anelich (1988). Altomare and Anelich (1988) also used a tanks-in-series model in their study, and they reported that the Wolf and White model was the best fitting model among three models (axial dispersion, tanks-in-series, and Wolf and White models) used. A combination model of plug flow and perfect mixing for twin-screw RTD modeling was presented by Bounie (1988). No model has been reported to explain the flow behavior accurately and exactly.

Therefore, the best approach for twin-screw RTD modeling is to first fit the experimental data with an existing model, and then, to modify the existing model in order to get a best fit. If the experimental results and modified model cannot achieve a good fit, then several models are combined to obtain best fit.

The residence time distribution modeling work of this study was reported follows.

**Wolf and White Model**

The Wolf and White model is widely used in the RTD modeling for twin-screw extrusion cooking because of its best fit among all the RTD models (Wolf and White, 1976; Wolf et al., 1986; Lin and Armstrong, 1988; Altomare and Anelich, 1988). Therefore the first choice for the RTD model in this study was to use the Wolf and White model. In an attempt to get the best fit using the Wolf and White model, the fraction of plug flow (P) for each treatment in the 3x3x3 factorial experiment were optimized and are shown in Table 1. Figures 6 through 8 showed some results using Wolf and White model for the RTD modeling in this study.

The F(t) is equal to zero when the normalized time ( $t/\bar{t}$ ) is less than the fraction of plug flow (P) in the Wolf and White model, and F(t) increases suddenly when the normalized time becomes greater than the fraction of plug flow. In this study, the beginning part of F curve, however, was slightly increased with increasing normalized time. Comparing with the tanks-in-series model, the beginning part of F curve of this study was similar to the beginning part of the former. Therefore, the tanks-in-series model was also tried for RTD modeling the beginning part of the F curve in order to achieve better fitting.

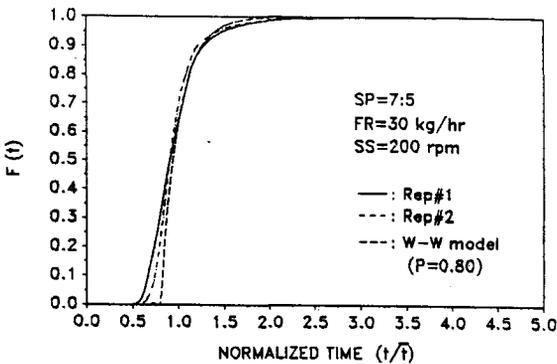


Figure 6. The best fit using Wolf and White model at 7:5 screw profile, 30 kg/hr feed rate, and 200 rpm screw speed

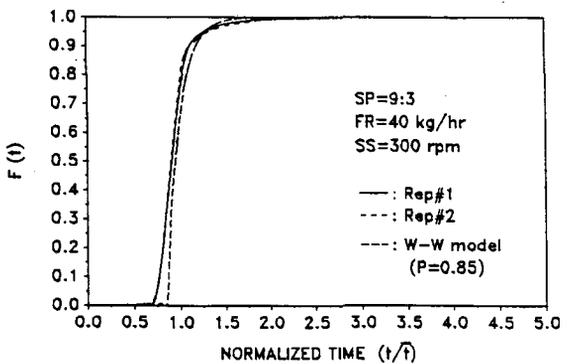


Figure 7. The best fit using Wolf and White model at 9:3 screw profile, 40 kg/hr feed rate, and 300 rpm screw speed

Table 1. The fraction of plug flow (P) in the Wolf and White model obtained from the experimental results (F-curves) for the best fit

Screw profile	Feed rate (kg/hr)		Screw speed (rpm)			Sub-average
			200	300	400	
7 : 5	30	Rep1	0.80	0.78	0.80	0.79
		Rep2	0.80	0.80	0.75	
		Ave	0.80	0.79	0.78	
	40	Rep1	0.85	0.80	0.80	0.83
		Rep2	0.85	0.83	0.83	
		Ave	0.85	0.82	0.82	
	50	Rep1	0.85	0.78	0.75	0.79
		Rep2	0.85	0.78	0.75	
		Ave	0.85	0.78	0.75	
	Sub-average			0.83	0.80	0.78
9 : 3	30	Rep1	0.80	0.78	0.75	0.78
		Rep2	0.80	0.78	0.75	
		Ave	0.80	0.78	0.75	
	40	Rep1	0.80	0.80	0.85	0.81
		Rep2	0.80	0.80	0.80	
		Ave	0.80	0.80	0.88	
	50	Rep1	0.85	0.85	0.85	0.85
		Rep2	0.85	0.85	0.85	
		Ave	0.85	0.85	0.85	
	Sub-average			0.82	0.81	0.81
11 : 1	30	Rep1	0.80	0.85	0.85	0.83
		Rep2	0.78	0.85	0.85	
		Ave	0.79	0.85	0.85	
	40	Rep1	0.83	0.85	0.85	0.85
		Rep2	0.83	0.85	0.85	
		Ave	0.83	0.85	0.85	
	50	Rep1	0.85	0.85	0.85	0.84
		Rep2	0.80	0.85	0.85	
		Ave	0.83	0.85	0.85	
	Sub-average			0.82	0.85	0.85
			Overall-average			0.82

Table 2. The number of tanks (N) in the tanks-in-series model obtained from the experimental results (F-curves) for the best fit

Screw profile	Feed rate (kg/hr)		Screw speed (rpm)			Sub-average
			200	300	400	
7 : 5	30	Rep1	10	15	15	12.50
		Rep2	12	15	8	
		Ave	11	15	11.5	
	40	Rep1	25	15	12	17.83
		Rep2	25	15	15	
		Ave	25	15	13.5	
	50	Rep1	25	15	10	16.67
		Rep2	25	15	10	
		Ave	25	15	10	
	Sub-average			20.33	15.00	11.67
9 : 3	0	Rep1	13	15	15	14.67
		Rep2	15	15	15	
		Ave	14	15	15	
	40	Rep1	25	25	25	23.33
		Rep2	25	25	15	
		Ave	25	25	20	
	50	Rep1	25	25	18	23.00
		Rep2	25	25	20	
		Ave	25	25	19	
	Sub-average			21.33	21.67	18.00
11 : 1	0	Rep1	25	25	25	24.17
		Rep2	20	25	25	
		Ave	22.5	25	25	
	40	Rep1	25	25	25	25.00
		Rep2	25	25	25	
		Ave	25	25	25	
	50	Rep1	25	25	20	22.50
		Rep2	20	25	20	
		Ave	22.5	25	20	
	Sub-average			23.33	25.00	23.33
			Overall-average			19.96

**Tanks-in-Series Model**

The tanks-in-series model was used for RTD modeling of a twin-screw extruder (Altomare and Anelich, 1988). The number of tanks (N) should be given when using the tanks-in-series model for RTD modeling. The number of tanks in the tanks-in-series model can be experimentally calculated by the following equation (Fogler, 1986; Levenspiel, 1972; Smith, 1981):

$$N = \frac{\overline{t}^2}{\sigma^2} \dots\dots\dots (4)$$

where N: The number of tanks in tanks-in-series model

$\overline{t}$ : The mean residence time (s)

$\sigma$ : The spread of the residence time distribution (s)

In an attempt to get the best fit for the beginning part using the tanks-in-series model, the optimum

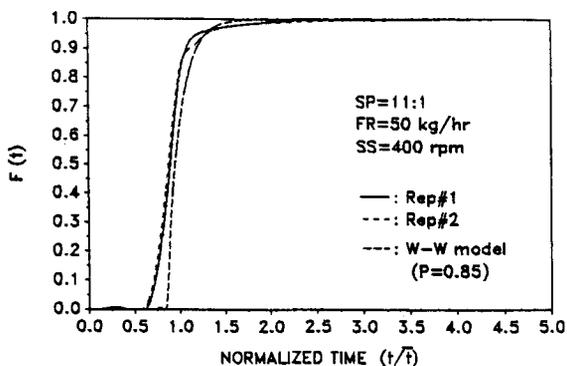


Figure 8. The best fit using Wolf and White model at 11:1 screw profile, 50 kg/hr feed rate, and 400 rpm screw speed

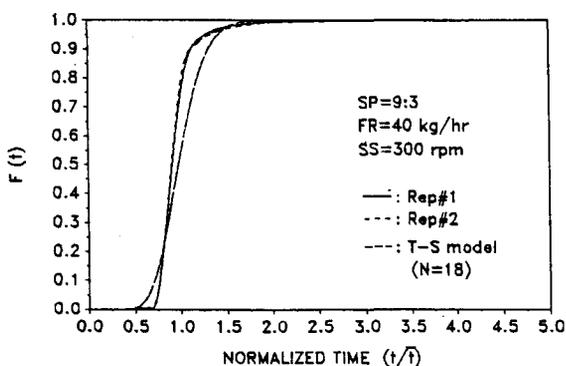


Figure 10. The best fit for the beginning part using tanks-in-series model at 9:3 screw profile, 40 kg/hr feed rate, and 300 rpm screw speed

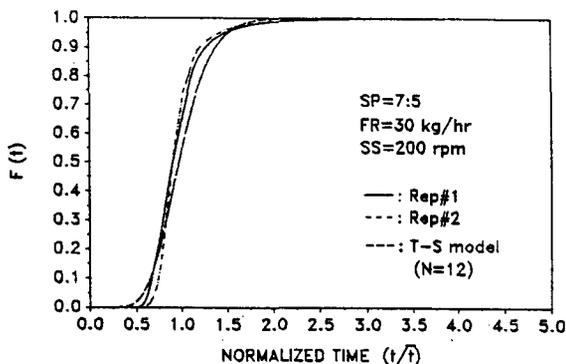


Figure 9. The best fit for the beginning part using tanks-in-series model at 7:5 screw profile, 30 kg/hr feed rate, and 200 rpm screw speed

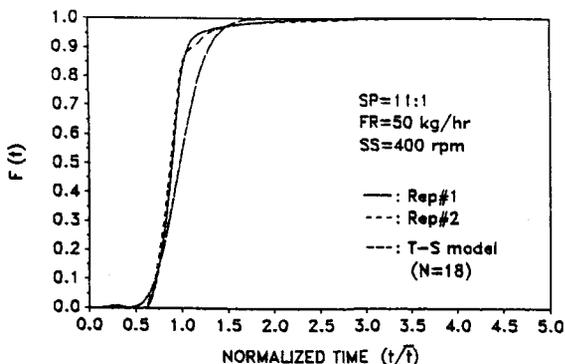


Figure 11. The best fit for the beginning part using tanks-in-series model at 11:1 screw profile, 50 kg/hr feed rate, and 400 rpm screw speed

number of tanks (N) for each treatment of the  $3 \times 3 \times 3$  factorial experiment were estimated and are shown in Table 2. Figures 9 through 11 show some results using the tanks-in-series model for the RTD modeling.

Considering the RTD modeling in this study, neither the Wolf and White model nor the tanks-in-series model could achieve the best fit for the  $3 \times 3 \times 3$  factorial results (Figures 12 through 14).

### Combination of Tanks-in-Series and Wolf and White Models

In order to achieve a better fitting RTD model in this study, the tanks-in-series model was used to fit the beginning part of the F curve and the Wolf and White model was applied to fit the rest of the curve.

The combination F curve for the tanks-in-series and Wolf and White model was plotted following the procedures described below:

1. First, the intersection point of the tanks-in-series and Wolf and White model F curves was determined. A Fortran 77 program was written to obtain the intersection point. The intersection point was represented as percentage of the normalized time.

2. The tanks-in-series model was used for the first part when the normalized time was less than at

the intersection point, and the Wolf and White model was applied to the second part when the normalized time was equal or greater than at the intersection point. A Fortran 77 program was written to obtain the F curve data set for the combination RTD model.

Some examples of the combination RTD modeling are shown in Figures 15 through 17. The beginning part appeared to fit very well by the tanks-in-series model. But, the second part was not fitted well by the Wolf and White model. It was found that the

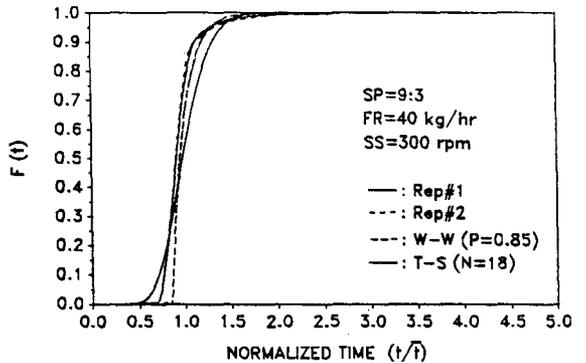


Figure 13. The RTD modeling of tanks-in-series model and Wolf and White model at 9:3 screw profile, 40 kg/hr feed rate, and 300 rpm screw speed

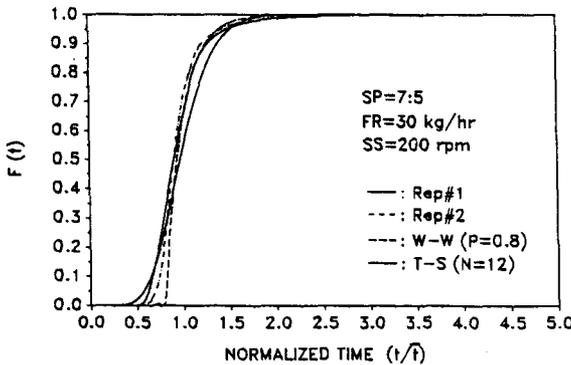


Figure 12. The RTD modeling of tanks-in-series model and Wolf and White model at 7:5 screw profile, 30 kg/hr feed rate, and 200 rpm screw speed

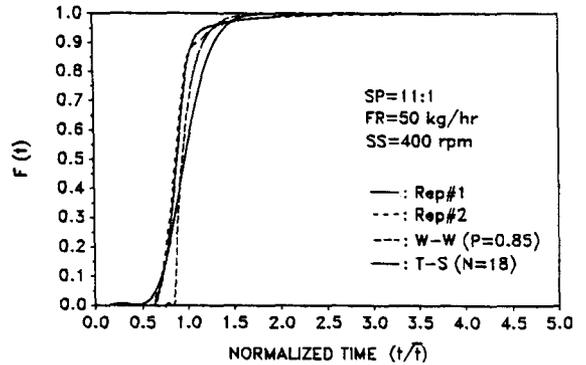


Figure 14. The RTD modeling of tanks-in-series model and Wolf and White model at 11:1 screw profile, 50 kg/hr feed rate, and 400 rpm screw speed

second part of the experimental F curves had a significant relationship to the Wolf and White model. Therefore, it was necessary to modify the Wolf and White model in order to get the best fit of the RTD model using a combination model.

### Modified Wolf and White Model

By trial and error method, the best modified Wolf and White model to be used in this study was obtained by moving the Wolf and White model parallel to the left 0.05 normalized time. A Fortran 77 program was written to get the F curve data for the modified Wolf and White model.

The modified Wolf and White model is

$$F(t) = 0; \quad 0 < \frac{t}{\bar{t}} < (P - 0.05)$$

$$F(t) = 1.0 - e^{-\left(\frac{t}{\bar{t}}\right)^{0.5 - p}}; \quad \frac{t}{\bar{t}} \geq (P - 0.05) \dots (44)$$

In order to get the best fit using the modified Wolf and White model, the fractions of the plug flow for the modified Wolf and White model for each treatment of the  $3 \times 3 \times 3$  factorial experiment were optimized and are shown in Table 3.

### Combination of Tanks-in-Series and Modified Wolf and White Models

Following the same procedure described above,

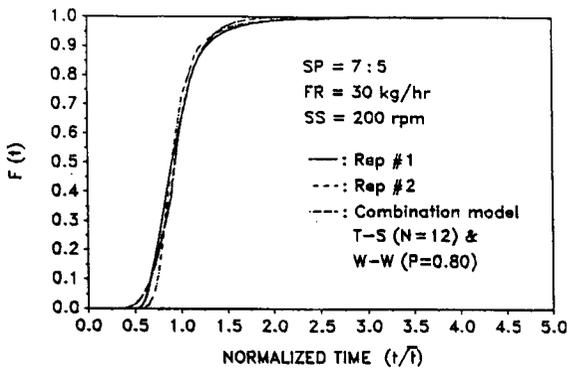


Figure 15. Combination of tanks-in-series and Wolf and White model at 7:5 screw profile, 30 kg/hr feed rate, and 200 rpm screw speed

a Fortran 77 program was written to find the intersection point of the F curves between the tanks-in-series model and the modified Wolf and White model. Also, the other Fortran 77 program was written to obtain the F curve data set for the combination RTD model, which was using the tanks-in-series model as the first part when the normalized time was less than at the intersection point, and using the modified Wolf and White model as the second part when the normalized

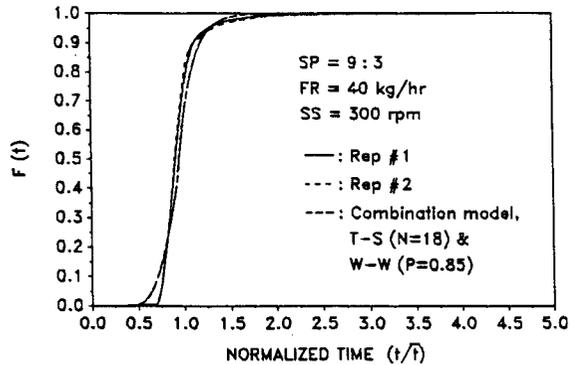


Figure 16. Combination of tanks-in-series and Wolf and White models at 9:3 screw profile, 40 kg/hr feed rate, and 300 rpm screw speed

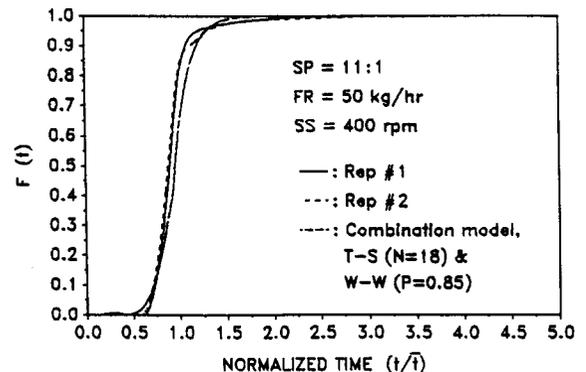


Figure 17. Combination of tanks-in-series and Wolf and White models at 11:1 screw profile, 50 kg/hr feed rate, and 400 rpm screw speed

Table 3. The fraction of plug flow (P) in the modified Wolf and White model obtained from the experimental results (F-curves) for the best fit

Screw profile	Feed rate (kg/hr)		Screw speed (rpm)			Sub-average
			200	300	400	
7 : 5	30	Rep1	0.80	0.78	0.80	0.79
		Rep2	0.80	0.80	0.75	
		Ave	0.80	0.79	0.78	
	40	Rep1	0.85	0.75	0.75	0.80
		Rep2	0.85	0.83	0.75	
		Ave	0.85	0.79	0.75	
	50	Rep1	0.83	0.75	0.75	0.78
		Rep2	0.83	0.75	0.75	
		Ave	0.83	0.75	0.75	
	Sub-average			0.83	0.78	0.76
9 : 3	30	Rep1	0.75	0.78	0.75	0.76
		Rep2	0.75	0.78	0.75	
		Ave	0.75	0.78	0.75	
	40	Rep1	0.80	0.83	0.85	0.81
		Rep2	0.80	0.83	0.75	
		Ave	0.80	0.83	0.70	
	50	Rep1	0.83	0.83	0.83	0.83
		Rep2	0.83	0.83	0.83	
		Ave	0.83	0.83	0.83	
	Sub-average			0.79	0.81	0.79
11 : 1	30	Rep1	0.78	0.85	0.83	0.82
		Rep2	0.75	0.85	0.85	
		Ave	0.77	0.85	0.84	
	40	Rep1	0.80	0.83	0.85	0.83
		Rep2	0.80	0.83	0.85	
		Ave	0.80	0.83	0.85	
	50	Rep1	0.83	0.85	0.85	0.82
		Rep2	0.78	0.80	0.83	
		Ave	0.80	0.83	0.84	
	Sub-average			0.80	0.84	0.84
Overall-average					0.80	

time was equal or greater than at the intersection point. The figures of the same treatments are shown in Figures 18 through 20.

Comparing Figures 15 through 17 with Figures 18 through 20, the combination of the tanks-in-series and the modified Wolf and White model was much better than the combination of the tanks-in-series and the Wolf and White models for the RTD modeling of twin-screw extrusion cooking. Therefore, the combination tanks-in-series and modified Wolf and White RTD model was the best fitting model in this study. The F curves of this combination model were close to the F curves of the experimental results.

#### Prediction of the RTD Combination model

According to the results shown above, the combination tanks-in-series and modified Wolf and White RTD model was the best fitting model for the APV Baker MPF 50/25 twin-screw extrusion cooking system. The process variables in this study were screw profile, feed rate, and screw speed. It was important to know whether the combination RTD model could satisfactorily predict the actual extrusion system when screw profile, feed rate, or screw speed changed. Owing to get the prediction models, the following functions should be determined.

$$N=f(SP,FR,SS) \dots\dots\dots (45)$$

$$P=f(SP,FR,SS) \dots\dots\dots (46)$$

where N: The number of tanks in the tanks-in-series model

P: The fraction of plug flow for the modified Wolf and White model

SP: Screw profile (the number of forward paddles in the extruder metering zone; 7, 9, or 11)

FR: Feed rate (30, 40, or 50 kg/hr)

SS: screw speed (200, 300, or 400 rpm)

In order to determine the above regression functions, the number of tanks (N) in the tanks-in-series model optimized from the experimental results (Table 2), and the fraction of plug flow (P) in the modified Wolf and White model also optimized from the experimental results (Table 3), were introduced to the General Linear Model procedure in SAS (1985). Also, a Stepwise backward procedure in SAS was conducted to remove the insignificant terms in the regression equation.

Thus, the final regression models were acceptable for the prediction of N and P. The R<sup>2</sup> for N and P were 0.87 and 0.76, respectively, at the significance level of 5%. The final third order Stepwise regression

model parameters and estimates for N and P are shown in Table 4.

The estimated N and P values from the experimental results, and the predicted N and P values from the SAS regression models, are listed in Table 5. A student T-test was conducted with the SAS program to compare the estimated and predicted N and P. The results showed there were no significant differences

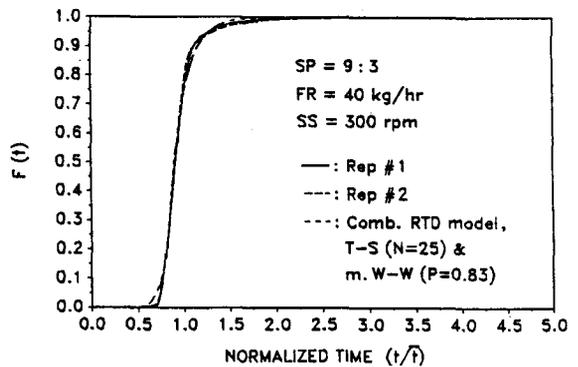


Figure 19. Combination of tanks-in-series and modified Wolf and White models at 9:3 screw profile, 40 kg/hr feed rate, and 300 rpm screw speed

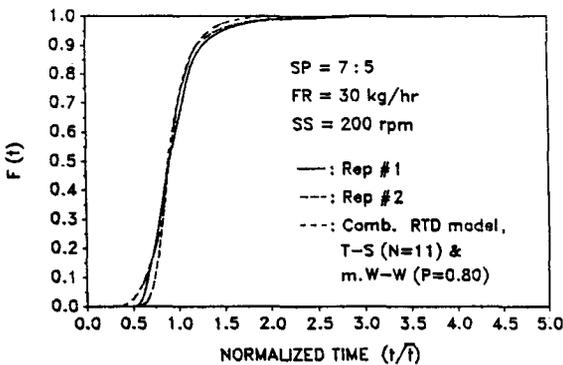


Figure 18. Combination of tanks-in-series and modified Wolf and White models at 7:5 screw profile, 30 kg/hr feed rate, and 200 rpm screw speed

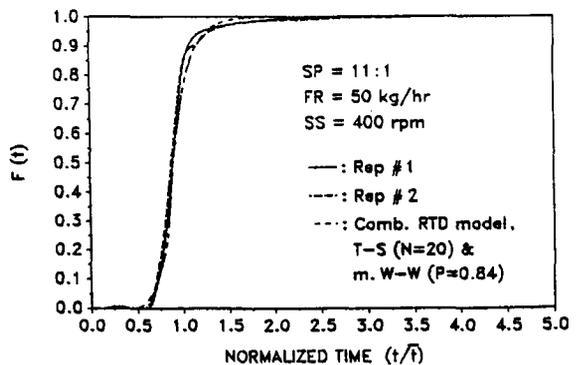


Figure 20. Combination of tanks-in-series and modified Wolf and White models at 11:1 screw profile, 50 kg/hr feed rate, and 400 rpm screw speed

Table 4. The parameters and estimates for the third order N and P regression models

Parameter	Estimate	
	N	P
Intercept	114.23136979	2.59943022
SP	-48.15708751	-0.61023569
FR	1.11985859	0.01158285
SP × SS	0.01422975	0.00077495
FR × SS	-0.00913513	-0.00019645
SP <sup>2</sup>	3.01969328	0.02662281
FR <sup>2</sup>	-0.03243218	-0.00037710
SS <sup>2</sup>	0.00041407	-
SP <sup>2</sup> × FR	-0.07896455	-0.00063432
FR <sup>2</sup> × SS	-	7.65571E-07
SP × SS <sup>2</sup>	-5.62284E-05	-1.02593E-06
FR × SS <sup>2</sup>	-	2.13357E-07
SP × FR × SS	0.00075707	-

N : The number of tanks in the tanks-in-series model

P : The fraction of plug flow in the modified Wolf and White molel

SP: Screw profile (the number of forward paddles in the extruder metering zone)

FR: Feed rate (kg/hr)

SS: Screw speed (rpm)

R<sup>2</sup>: 0.87 and 0.76 for N and P, respectively

Table 5. Comparison of the estimated and predicted N and P

Screw profile	Feed rate (kg/hr)	Screw speed (rpm)	N		P	
			Est	Pred	Est	Pred
7	30	200	11	13	0.80	0.81
7	30	300	15	12	0.79	0.79
7	30	400	12	12	0.78	0.76
7	40	200	25	22	0.85	0.84
7	40	300	15	18	0.79	0.79
7	40	400	14	14	0.75	0.76
7	50	200	25	25	0.83	0.83
7	50	300	15	17	0.75	0.76
7	50	400	10	9	0.75	0.75
9	30	200	14	15	0.75	0.75
9	30	300	15	16	0.78	0.79
9	30	400	15	15	0.75	0.77
9	40	200	25	24	0.80	0.82
9	40	300	25	23	0.83	0.81
9	40	400	20	20	0.80	0.80
9	50	200	25	27	0.83	0.83
9	50	300	25	24	0.83	0.81
9	50	400	19	19	0.83	0.82
11	30	200	23	22	0.77	0.76
11	30	300	25	25	0.85	0.84
11	30	400	25	24	0.84	0.84
11	40	200	25	25	0.80	0.80
11	40	300	25	27	0.83	0.85
11	40	400	25	26	0.85	0.85
11	50	200	23	22	0.80	0.80
11	50	300	25	23	0.83	0.83
11	50	400	20	21	0.84	0.85
Average			20	20	0.80	0.80

N: The number of tanks in the tanks-in-series model

P: The fraction of plug flow in the modified Wolf and White model

Est: The estimated number from the experimental results

Pred: The predicted number from the SAS regression equations

between the estimated  $N$  and predicted  $N$ , and estimated  $P$  and predicted  $P$ . It was proven that the regression model could be used satisfactorily to predict the residence time distributions for the APV Baker MPF 50/25 twin-screw extruder.

The brief procedures for prediction of the residence time distributions using the combination RTD model (tanks-in-series model and modified Wolf and White model) and regression models (third order regression equations) are presented below.

(1) Input any particular screw profile, feed rate and screw speed (in the range of this study) into the regression models shown in Table 4 to predict the number of tanks ( $N$ ) in the tanks-in-series model and the fraction of plug flow ( $P$ ) in the modified Wolf and White model. A Fortran 77 program is provided to predict the  $N$  and  $P$ .

(2) Determine the intersection point of the  $F$  curves for the tanks-in-series model (input the predicted  $N$ ) and modified Wolf and White model (input the predicted  $P$ ). A Fortran program is used to determine the intersection point.

(3) The tanks-in-series model is used for the first part of the predicted  $F$  curve where the normalized time is less than at the intersection point, and the Wolf and White model is applied to the second part of the predicted  $F$  curve where the normalized time is equal or greater than at the intersection point. The data set of the predicted  $F$  curve can be determined using the Fortran program.

(4) Plot the predicted  $F$  curve as  $F(t)$  versus  $t/\bar{t}$  using Sigmaplot software.

Figures 21 through 23 are some examples of this experiment. Comparing the experimental results with the combination RTD model predicted curve for each figure, it was found that the combination RTD model ( tanks-in-series model and modified Wolf and White model ) satisfactorily predicted the residence time distributions in the twin-screw extrusion cooking system. Therefore, the combination RTD model appeared

to be suitable for the APV Baker MPF 50/25 twin-screw extruder, and the RTD modeling technique in this study can also be extended to other twin-screw extruder experiments.

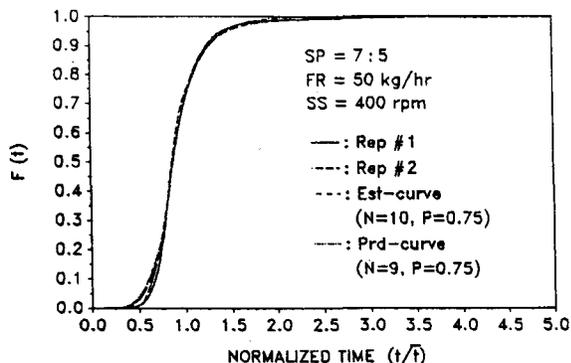


Figure 21. The comparisons of experimental results with the combination RTD model at 7:5 screw profile, 50 kg/hr feed rate, and 400 rpm screw speed

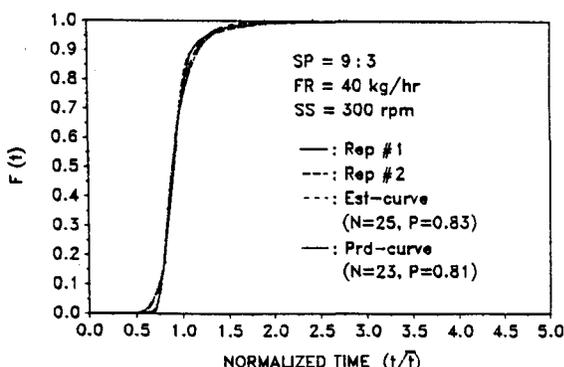


Figure 22. The comparisons of experimental results with the combination RTD model at 9:3 screw profile, 40 kg/hr feed rate, and 300 rpm screw speed

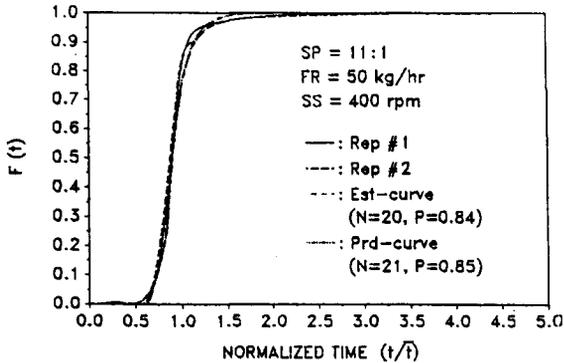


Figure 23. The comparisons of experimental results with the combination RTD model at 11:1 screw profile, 50 kg/hr feed rate, and 400 rpm screw speed

## CONCLUSIONS

A colorimetric method was used to study the residence time distribution (RTD) modeling in an APV Baker MPF 50/25 twin-screw extruder using rice flour as the feed material. The experiment was  $3 \times 3 \times 3$  factorial design, and the process variables were screw profile, feed rate, and screw speed.

A best fit RTD model was developed. The combination RTD model (tanks-in-series model and modified Wolf and White model) developed in this study can satisfactorily predict the residence time distributions in the extrusion cooking system. The RTD modeling technique can also be extended to other extruder experiments.

## ACKNOWLEDGEMENTS

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## REFERENCES

Altomare, R. E. and Anelich, M. 1988. The Effects of Screw Element Selection on the Residence Time Distribution in a Twin screw Cooking Extruder.

- AIChE Annual Meeting #112b.
- Altomare, R. E. and Ghossi, P. 1986. An Analysis of Residence Time Distribution Patterns in a Twin Screw Cooking Extruder. *Biotechnol. Progress.* 2 (3):157.
- Bigg, D. and Middleman, S. 1974. Mixing in a Screw Extruder-A Model for Residence Time Distribution and Strain. *Ind. Eng. Chem. Fundam.* 13(1): 66.
- Bounie, D. 1988. Modelling of the Flow Pattern in a Twin-Screw Extruder Through Residence-Time Distribution Experiments. *J. Food Eng.* 7:223.
- Bruin, S., van Zuilichem D. J., and Stolp, W. 1978. A Review of Fundamental and Engineering Aspects of Extrusion of Biopolymers in a Single-Screw Extruder. *J. Food Process Eng.* 2:1.
- Danckwerts, P. V. 1953. Continuous Flow Systems-Distribution of Residence Time. *Chem. Eng. Sci.* 2(1):1.
- Davidson, V. J., Paton, D., Diosady, L. L., and Spratt, W. A. 1983. Residence Time Distributions for Wheat Starch in a Single Screw Extruder. *J. Food Sci.* 48:1157.
- Fellows, P. 1988. *Food Processing Technology-Principles and Practice.* Ellis Horwood Ltd. Chichester, England.
- Fogler, H. S. 1986. *Elements of Chemical Reaction Engineering.* Prentice-Hall Inc., Englewood Cliffs, NJ.
- Harper, J. M. 1979. Food Extrusion-Critical Review. *Food Sci. Nutr.* 11(2):155-215.
- Harper, J. M. 1981. *Extrusion of Foods-Volume I and II.* CRC Press, Inc. Boca Raton, Florida.
- Hsieh, F., Peng I., and Huff, H. E. 1990. Effects of Salt, Sugar and Screw Speed on Processing and Product Variables of Corn Meal Extruded with a Twin-Screw Extruder. *J. Food Sci.* 55(1):224-227.
- Hunter Associates Laboratory, Inc. Instruction Ma-

- nual for HunterLab D25 Colorimeter.
- Jager, T. 1989. Residence Time Distributions in Twin-Screw Cooking Extruders (Engineering and Food, Vol 3). Elsevier Applied Science. New York.
- Jager, T., van Zuilichem D. J., de Swart, J. G., and van't Riet, K. 1991. Residence Time Distributions in Extrusion Cooking: Part 7-Modelling of a Corotating, Twin-Screw Extruder Fed with Maize Grits. *J. Food Eng.* 14:203-239.
- Janssen, L. P. B. M. 1978. Twin Screw Extrusion. Elsevier Scientific Publishing Co. New York.
- Janssen, L. P. B. M. Hollander, R. W., Spoor, M. W., and Smith, J. M. 1979. Residence Time Distributions in a Plastication Twin Screw Extruder. *J. AIChE.* 25(2):345.
- Kao, S. V. and Allison, G. R. 1984. Residence Time Distribution in a Twin Screw Extruder, *Polym. Eng. Sci.* 24(9):645.
- Levenspiel, O. 1972. Chemical Reaction Engineering. John Wiley & Sons, Inc. New York.
- Lidor, G. and Tadmor, Z. 1976. Theoretical Analysis of Residence Time Distribution Functions and Strain Distribution Functions in Plasticating Screw Extruders. *Polym. Eng. Sci.* 16(6):450.
- Lin, J. K. and Armstrong, D. J. 1988. Residence Time Distributions of Cereals During Extrusion. ASAE Winter Meeting, # 88-6518.
- Mange, C., Boissonnat, P., and Gelus, M. 1986. Extrusion Technology for the Food Industry: Distribution of Residence Times and Comparison of Three Twin-Screw Extruders of Different Size. Elsevier Applied Science Publishers Ltd. New York.
- Martelli, F. G. 1983. Twin-Screw Extruders: A Basic Understanding. Van Nostrand Reinhold Co. Inc., New York.
- Mercier, C., Linko, P., and Harper, J. M. 1989. Extrusion Cooking. American Association of Cereal Chemists, Inc. St. Paul, Minnesota.
- Middleman, S. 1977. Fundamentals of Polymer Procession. McGraw-Hill Book Company. New York.
- Miles, D. C. and Briston, J. H. 1965. Polymer Technology. Chemical Publishing Company, Inc. New York.
- Oikku, J., Antila, J. and Heikkinen, J. 1980. Residence Time Distribution in a Twin-Screw Extruder. *Food Proc. Eng.* 3:791.
- Onwulata, C. I. 1991. Dynamic Characterization and Product Quality Modeling of Twin Screw Extrusion Cooking Process. Ph D. Dissertation. University of Missouri-Columbia. Columbia, MO.
- Peng, J. 1991. RTD Modeling for Twin-Screw Extrusion of Rice Flour. Ph D. Dissertation. University of Missouri-Columbia, MO, USA.
- Peng, J., Hsieh, F., and Huff, H. E. 1991. An RTD Determination Method for a Twin-Screw Extruder. ASAE Mid-central Annual Conference. Paper No. MC91-121.
- SAS/STAT User's Guide. Release 6.03 edition. 1991. SAS Institute Inc., Cary, NC.
- Smith, J. M. 1981. Chemical Engineering Kinetics. McGraw-Hill Book Company. New York.
- Tadmor, Z. and Klein, I. 1970. Engineering Principles of Plastication Extrusion. Van Nostrand Reinhold Company. Toronto, Canada.
- Todd, D. B., 1975a. residence Time Distribution in Twin-Screw Extruders. *Polym. Eng. Sci.* 15(6): 437.
- Todd, D. B., 1975b. Mixing in Starved Twin Screw Extruders. *Chem. Eng. Progress.* 71(2):81.
- Todd, D. B. and Irving, H. F. 1969. Axial Mixing in a Self-Wiping Reactor. *Chem. Eng. Progress.* 65(9):84.
- Valentas, K. J., Levine, L., and Clark J. P. 1991. Food Processing Operations and Scale-Up. Marcel Dekker, Inc. New York.
- van Zuilichem, D. J., de Swart, J. G., and Buisman, G. 1973. Residence Time-Distributions in an Extruder. *Lebbensm-Wiss. U. Techol.* 6 (5):104.
- van Zuilichem, D. J., Jager, T., Stolp, W., and de

