農業工程學報 第40卷 第1期
中華民國83年3月出版

Journal of Chinese Agricultural Engineering
Vol. 40, No. 1, March 1994

泥砂滲入孔隙介質之序率模式研究(一) 理論推演部份

Stochastic Modeling of Sediment Infiltration into Porous Media Phase I: Theory Development

國立臺灣大學水工試驗所副研究員

吳 富 春

Fu-Chun Wu

摘 要

由於泥砂滲入孔隙介質之過程是一種非恆定、非均勻之過程，一名爲非均質波桑程序模式之序率模式適合用以描述具這種特性之過程。然而在應用這種序率模式時，須確知兩個參函數 λ_1 和 λ_2。本研究之第一部份乃推演決定兩個參函數 λ_1 和 λ_2 之理論。根據這理論， 本研究之第二部份乃以實驗方法決定 λ_1 和 λ_2，並將所得之結果加以討論及分析以供進一步之應用。

關鍵詞：序率模式，泥砂入滲，孔隙介質

ABSTRACT

A stochastic model, nonhomogeneous Poisson process model, is adopted to describe the physical process of sediment particles infiltration into the porous media because of this model's suitability for the unsteady and nonuniform nature of the sediment infiltration process as well as the random behavior of the sediment particles within the porous matrix. Two parameters of this model, λ_1 and λ_2 , are needed for stochastic modeling of this process. In phase I of this study, the theoretical bases for determination of λ_1 and λ_2 are presented. In phase II, an experimental study based on the theories from phase I is conducted to evaluate λ_1 and λ_2 . The experimental results are discussed and analyzed for further application.

Key words: Stochastic Model, Sediment Infiltration, Porous Media

INTRODUCTION

The infiltration of sediment particles into porous media has gained considerable attention from engineers and researchers involved in environmental protection issues over the past decades. Investigations on this problem have been carried out with a variety of approaches such as field studies, laboratory experiments, and numerical modeling. Generally speaking, the physical process of sediment particles infiltration into porous media is of important engineering concern in the aspects of 1) Intrusion of fine sediment into the spawning gravel bed substrate: Intrusion of sediment fines several inches into the gravel bed can entomb fish or greatly restricts transport of dissolved oxygen into and within the bed substrate and reduces the ability of interstitial water flow to remove metabolic wastes, which could have adverse effects on fish production and development (Diplas and Parker, 1985; Alonso et al., 1985; Lisle, 1989); 2) Contamination of streambeds: Chemical contaminants adsorbed on sediment particles will contaminate the river bed when the sediment particles are transported by the fluid force and deposited into the gravel bed substrate (Cerling et al., 1990; Berndtsson, 1990); 3) Groundwater recharge: The use of granular filter at groundwater recharge site has shown a good performance in terms of sediment trap efficiency and maintaining the permeability of the underlying aquifer (Yim and Sternberg, 1987); 4) Design of protective filters: Preventing the penetration of fine material through the coarse matrix is one of the major objectives for designing the protective filters. Examples are the inverted filter used for the embankment dam to protect the base soil from piping as well as the filter blanket underlying the bank protection facility to inhibit the erosion of bank material (Sherard et al., 1963; FHWA, 1989).

Sakthivadivel (1966) developed a mathematical model to describe the filtration mechanism of non-colloidal particles through the porous media. Joy et al. (1993) presented a stochastic model for the steady-state particulate transport in porous media under turbulent flow conditions. However, no universally accepted quantitative model to describe the processes of sediment infiltration into and resulting deposition within the porous medium is yet available primarily due to the fact that so many factors govern the complicated behavior of the sediment particles inside of the porous matrix. These factors include the sizes and shapes of the sediment particles and the media; the sediment load; the hydrodynamics in the porous media; and the paths for sediment particles to travel; among many others.

This study concerns the stochastic modeling of sediment infiltration into the porous media. The nonhomogeneous Poisson process (NHPP) model developed by Shen and Todorovic (1971) is adopted because of its suitability for the unsteady and nonuniform nature of the sediment infiltration process and the random behavior of the sediment particles within the porous matrix. Two parameters of this model, λ_1 and λ_2 , are needed for stochastic modeling of this process. In phase I of this study, the theoretical bases for determination of λ_1 and λ_2 are presented. In phase II, an experimental study based on the theories from phase I is conducted to evaluate λ_1 and λ_2 . The experimental results are analyzed and generalized to functional forms in terms of the medium-sediment size ratio, total sediment input, and seepage flow rate by regression for further application.

STOCHASTIC MODELING

Construction of the model describing the physical process of sediment infiltration into the porous media is based on the preliminary observations and understanding of this process. Firstly, it is found that the pattern of particulate infiltration into porous media is dominated by the ratio of media to sediment sizes. McDowell-Boyer et al. (1986) classified three types of filtration mechanism by the particle size dependence and the deposit morphology. Sakthivadivel and Einstein (1970) conducted the experimental study with uniform sediment of size d moving downward through uniform spheres of size D. According to their experiments, the patterns of surface deposition, deep-bed infiltration, and penetration of the sediment particles, as illustrated in Fig. 1, occur respectively for the size ratio $D/d \leq 7$, $7 < D/d \leq 14$, and $D/d > 14$. They also found that the characteristic mechanism of sediment deposition within the porous bed for sediment with the size ratio in the range $7 < D/d \le 14$ is clogging of the pores, which is the phenomenon of retaining fine solid particles in the pores of the medium. In the final stage of clogging, the pores near the bed surface are filled up with fine particles which restrict further downward infiltration of the sediment particles.

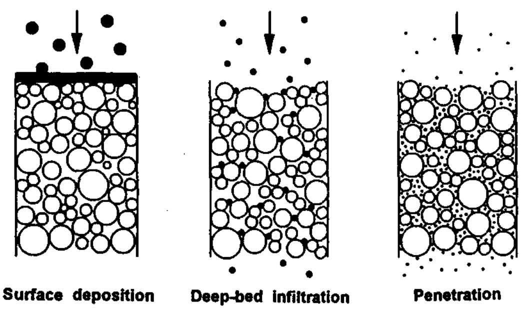

Figure 1. Three infiltration patterns

Secondly, based on a series of preliminary investigations of the sediment infiltration process, it is noticed that the sediment deposition within the void space of the porous matrix varies with both time and depth. The nonhomogeneous Poisson process (NHPP) model developed by Shen and Todorovic (1971) is adopted to describe this physical process because of the unsteady and nonuniform nature of the sediment infiltration process and the random behavior of the sediment particles within the porous matrix.

The NHPP model is an improvement over the homogeneous Poisson process model, which is the stochastic sediment transport model first developed by Einstein (1937) based on the assumption that the step length and the rest period of sediment particles are both homogeneous in time and space. The major result of the Shen-Todorovic's NHPP model is the one-dimensional cumulative probability distribution function (CDF), $F_t(x)$, given by

$$
\begin{aligned}
F_t(x) &= P(X_t \le x) \\
&= \exp\left[-\int_{t_0}^{t} \lambda_1(s)ds\right] \cdot \exp\left[-\int_{x_0}^{x} \lambda_2(s)ds\right] \cdot \\
&\quad \sum_{n=0}^{\infty} \sum_{j=n}^{\infty} \frac{\left[\int_{t_0}^{t} \lambda_1(s)ds\right]^n \left[\int_{x_0}^{x} \lambda_2(s)ds\right]^j}{n!\,j!} \\
&= \exp[-\Lambda_1(t)] \cdot \exp[-\Lambda_2(x)] \cdot \sum_{n=0}^{\infty} \sum_{j=n}^{\infty} \frac{[\Lambda_1(t)]^n [\Lambda_2(x)]^j}{n!\,j!}
\end{aligned} \tag{1}
$$

where

$$\Lambda_1(t) = \int_{t_0}^{t} \lambda_1(s)ds \tag{2}$$

$$\Lambda_2(x) = \int_{x_0}^{x} \lambda_2(s)ds \tag{3}$$

X_t denotes x-direction displacement of a sediment particle at time t. λ_1 and λ_2 are called intensity functions (or density functions) in time and space domain respectively. The physical interpretation of λ_1 is the inverse of the average rest period, and λ_2 is the inverse of the average step length. n and j are integers represent the number of steps a sediment particle takes in (t_0, t) and (x_0, x) respectively. Hung and Shen (1972) claimed that Einstein's homogeneous model is a special case of the NHPP model by putting $\lambda_1(s) = k_1$, $\lambda_2(s) = k_2$, $t_0 = 0$, and $x_0 = 0$ in Eqn (1), where k_1 and k_2 are constants.

Given the cumulative probabilities at two locations x_1 and x_2 at time t, the probability of a sediment particle being in the interval (x_1, x_2) at time t is known by

$$P(x_1 < X_t \le x_2) = F_t(x_2) - F_t(x_1) \tag{4}$$

To apply Eqn (1) in evaluating the distribution of sediment infiltration along the depth of the porous matrix at various times, the two parameters λ_1 and λ_2 have to be determined.

THEORY DEVELOPMENT

1. Determination of λ_1

It is noticed that in Eqn (3), integration of $\lambda_2(s)$ from x_0 to x_0, which physically means integration at the bed surface as shown in Fig. 2, gives $\Lambda_2(x_0) = 0$, then Eqn (1) becomes

$$
\begin{aligned}
F_t(x_0) &= \exp[-\Lambda_1(t)]\cdot\exp(0)\cdot\sum_{n=0}^{\infty}\sum_{j=n}^{\infty}\frac{[\Lambda_1(t)]^n}{n!}\frac{(0)^j}{j!} \\
&= \exp[-\Lambda_1(t)]\cdot(1)\cdot \\
&\left\{\sum_{j=0}^{\infty}\frac{[\Lambda_1(t)]^0}{0!}\frac{(0)^j}{j!}+\sum_{j=1}^{\infty}\frac{[\Lambda_1(t)]^1}{1!}\frac{(0)^j}{j!}+\right. \\
&\left.\sum_{j=2}^{\infty}\frac{[\Lambda_1(t)]^2}{2!}\frac{(0)^j}{j!}+\cdots\right\} \\
&= \exp[-\Lambda_1(t)]\cdot\left\{\frac{[\Lambda_1(t)]^0}{0!}\sum_{j=0}^{\infty}\frac{(0)^j}{j!}+\right. \\
&\left.\frac{[\Lambda_1(t)]^1}{1!}\sum_{j=1}^{\infty}\frac{(0)^j}{j!}+\frac{[\Lambda_1(t)]^2}{2!}\sum_{j=2}^{\infty}\frac{(0)^j}{j!}+\cdots\right\} \\
&= \exp[-\Lambda_1(t)]\cdot\left\{\frac{[\Lambda_1(t)]^0}{0!}\cdot\left[\frac{(0)^0}{0!}+\frac{(0)^1}{1!}+\cdots\right]+\right. \\
&\left.\frac{[\Lambda_1(t)]^1}{1!}\cdot\left[\frac{(0)^1}{1!}+\frac{(0)^2}{2!}+\cdots\right]+\cdots\right\} \\
&= \exp[-\Lambda_1(t)]\cdot\{(1)\cdot(1+0+\cdots)+0+0+\cdots\} \\
&= \exp[-\Lambda_1(t)]
\end{aligned}
\tag{5}
$$

Eqn (5) can be written as

$$\Lambda_1(t) = -\ln[F_t(x_0)] \tag{6}$$

$F_t(x_0)$ is the probability of a sediment particle staying at the bed surface at time t. If M_T is the total amount of sediment at bed surface that starts infiltrating into the porous matrix at the initial time t_0, and $m_{x_0}(t)$ is the amount staying at the bed surface at time t, as shown in Fig. 2, then

$$F_t(x_0) = \frac{m_{x_0}(t)}{M_T} \tag{7}$$

Substitution of Eqn (7) into Eqn (6) results in

$$\Lambda_1(t) = -\ln\left[\frac{m_{x_0}(t)}{M_t}\right] \tag{8}$$

Given the values of $m_{x_0}(t)$ at various time t, Λ_1 curve is obtained by fitting the points calculated from Eqn (8). Then according to the definition given by Eqn (2), the intensity function λ_1 is determined from the differentiation of the Λ_1 curve with respect to t.

2. Determination of λ_2

λ_2 is approached by the first-order finite-difference approximation described in this section. By Taylor's expansion, Eqn (9) can be written down for small Δx,

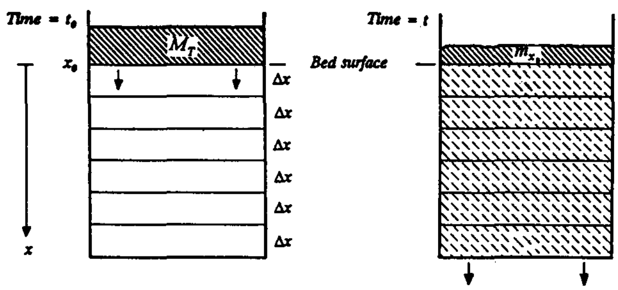

Figure 2. Porous matrix configurations at time t_0 and time t

$$
\begin{aligned}
\Lambda_2(x+\Delta x) &= \Lambda_2(x)+(\Delta x)\Lambda_2'(x)+O(\Delta x^2) \\
&\approx \Lambda_2(x)+(\Delta x)\Lambda_2'(x)
\end{aligned}
\tag{9}
$$

According to Eqn (3), Eqn (9) can be expressed as

$$\Lambda_2(x+\Delta x) = \Lambda_2(x)+(\Delta x)\lambda_2(x) \tag{10}$$

Similarly, for small Δx, substitution of Eqn (10) leads to the following,

$$
\begin{aligned}
[\Lambda_2(x+\Delta x)]^j &= [\Lambda_2(x)+(\Delta x)\lambda_2(x)]^j \\
&= [\Lambda_2(x)]^j + j[\Lambda_2(x)]^{j-1}(\Delta x)\lambda_2(x)+ \\
&\quad O(\Delta x^2) \\
&\approx [\Lambda_2(x)]^j + j[\Lambda_2(x)]^{j-1}(\Delta x)\lambda_2(x)
\end{aligned}
\tag{11}
$$

With the first-order approximations given in Eqn (10) and Eqn (11) as well as the relation defined for the derivation of Shen-Todorovic's NHPP model, it can be shown that Eqn (12) is achieved through some mathematical operation (Wu, 1993),

$$\frac{F_t(x+\Delta x)}{F_t(x)} = \exp[-(\Delta x)\lambda_2(x)]\cdot \left\{1+(\Delta x)\lambda_2(x)\left[\frac{P(E_0^{t_0,t})}{P(X_t \le x)} + \frac{\sum_{n=1}^{\infty} P(X_{n-1} \le x)\cdot P(E_n^{t_0,t})}{P(X_t \le x)}\right]\right\} \quad (12)$$

in which $P(E_0^{t_0,t})$ and $P(E_n^{t_0,t})$ are the probabilities of the random events that a sediment particle makes exactly 0 step and n steps respectively, X_{n-1} is x-direction displacement of a sediment particle after $(n-1)$ steps. Since the last term in Eqn (12)

$$\frac{\sum_{n=1}^{\infty} P(X_{n-1} \le x)\cdot P(E_n^{t_0,t})}{P(X_t \le x)}$$

is a function of both x and t, when time $t = t_i$, let

$$\frac{\sum_{n=1}^{\infty} P(X_{n-1} \le x)\cdot P(E_n^{t_0,t_i})}{P(X_{t_i} \le x)} \equiv b(t = t_i, x) \equiv b_i(x) \quad (13)$$

and

$$(\Delta x)\lambda_2(x) \equiv k_2(x) \quad (14)$$

Substitution of Eqns (13) and (14) into Eqn (12) leads to Eqn (15) for time $t = t_i$,

$$\frac{F_{t_i}(x+\Delta x)}{F_{t_i}(x)} = \exp[-k_2(x)]\cdot\left\{1+k_2(x)\cdot\left[\frac{P(E_0^{t_0,t_i})}{P(X_{t_i} \le x)} + b_i(x)\right]\right\} \quad (15)$$

In the same manner,

$$\frac{F_{t_i}(x-\Delta x)}{F_{t_i}(x)} = \exp[k_2(x)]\cdot\left\{1-k_2(x)\cdot\left[\frac{P(E_0^{t_0,t_i})}{P(X_{t_i} \le x)} + b_i(x)\right]\right\} \quad (16)$$

$F_{t_i}(x)$ is the amount of sediment that can be found above the depth x of the porous matrix at time t_i, $m_x(t_i)$, divided by the total amount of sediment initially at the bed surface, M_T. That is,

$$F_{t_i}(x) = P(X_{t_i} \le x) = \frac{m_x(t_i)}{M_T} \quad (17)$$

Similarly,

$$F_{t_i}(x+\Delta x) = \frac{m_{x+\Delta x}(t_i)}{M_T} \quad (18)$$

$$F_{t_i}(x-\Delta x) = \frac{m_{x-\Delta x}(t_i)}{M_T} \quad (19)$$

Since $P(E_0^{t_0,t_i})$ is the probability of the random event that a sediment particle makes exactly 0 step in the time interval (t_0, t_i). Therefore, physically

$$P(E_0^{t_0,t_i}) = \frac{m_{x_0}(t_i)}{M_T} \quad (20)$$

Substitution of Eqns (17), (18), (19), and (20) into Eqns (15) and (16) results in the following system of equations

$$\frac{m_{x+\Delta x}(t_i)}{m_x(t_i)} = \exp[-k_2(x)]\cdot\left\{1+k_2(x)\cdot\left[\frac{m_{x_0}(t_i)}{m_x(t_i)} + b_i(x)\right]\right\} \quad (21)$$

$$\frac{m_{x-\Delta x}(t_i)}{m_x(t_i)} = \exp[k_2(x)]\cdot\left\{1-k_2(x)\cdot\left[\frac{m_{x_0}(t_i)}{m_x(t_i)} + b_i(x)\right]\right\} \quad (22)$$

$k_2(x)$ and $b_i(x)$ are obtained by solving the system of equations (21) and (22). Then the value of λ_2 at the depth x is calculated from Eqn (14), which is

$$\lambda_2(x) = k_2(x) / \Delta x \qquad (23)$$

The intensity function λ_2 is determined by fitting the λ_2 points calculated from Eqn (23) at various depth.

DISCUSSION

Two questions arise at this point: 1) Is the more general NHPP model derived for the compound nonhomogeneous stochastic process based on less restrictive assumptions able to describe the unsteady and nonuniform process of sediment infiltration into the porous media? Or in other words, is the stochastic modeling of the process of sediment infiltration into the porous media by the nonhomogeneous Poisson process appropriate? 2) Can the two parameters of the NHPP model, λ_1 and λ_2, be precisely determined from the theories developed in the preceding sections? Or more specifically, will λ_1 and λ_2 obtained from the proposed methodology work when being substituted back into the NHPP model? In order to address the answers of these questions, a series of sediment infiltration experiments have to be performed for physically measuring the sediment distribution along the depth of the porous matrix at different times.

CONCLUSION

The stochastic model, nonhomogeneous Poisson process model, is adopted to describe the physical process of sediment particles infiltration into the porous media because of this model's suitability for the unsteady and nonuniform nature of the sediment infiltration process as well as the random behavior of the sediment particles within the porous matrix. The theoretical bases for determining the two parameters of this stochastic model are developed in this phase I study. In order to evaluate the two parameters λ_1 and λ_2 , a series of sediment infiltration experiments were performed to physically measure the time and spatial variations of sediment distribution within the void space of the porous matrix. The description of the experiments and the results from the experimental study and further analysis will be presented in the paper on phase II study.

REFERENCES

Alonso, C. V., Tabios, G. Q. III, and Mendoza, C., 1985. Sediment Intrusion (SEDINT) Model. Report, Project Tucannon River Pilot Study funded by SCS and ARS-USDA. Fort Collins, Colorado.

Berndtsson, R., 1990. Transport and Sedimentation of Pollutants in a River Reach: A Chemical Mass Balance Approach. Water Resour. Res., 26: 1549-1558.

Cerling, T. E., Morrison, S. J., and Sobocinski, R. W., 1990. Sediment-Water Interaction in a Small Stream: Adsorption of ^{137}Cs by Bed Load Sediments. Water Resour. Res., 26:1165-1176.

Diplas, P. and Parker, G., 1985. Pollution of Gravel Spawning Grounds due to Fine Sediment. St. Anthony Falls Hydraulic Laboratory, Proj. Report 240, University of Minnesota, Minneapolis, Minnesota.

Einstein, H. A., 1937. Bed Load Transport as a Probability Problem. Dr. Sc. Thesis, Federal Institute of Tech., Zurich, Switzerland. Translated into English by W. W. Sayre in Sedimentation (Einstein), edited by H. W. Shen, Water Resources Publication, 1972.

Federal Highway Administration, 1989. Design of Riprap Revetment. Hydraulic Engineering Circular No. 11. Publication No. FHWA-IP-89-016. U.S. Department of Transportation.

Hung, C. S. and Shen, H. W., 1972. Research in Stochastic Models for Bed-Load Transport. Appendix B, River Mechanics, Vol. II, edited by H. W. Shen, Fort Collins, Colorado.

Joy, D. M., Lennox, W. C. and Kouwen, N., 1993. Stochastic Model of Particulate Transport in Porous Medium. J. Hyd. Eng., ASCE, 119(7): 846-861.
Lisle, T. E., 1989. Sediment Transport and Resulting Deposition in Spawning Gravels, North Coastal California. Water Resour. Res., 25: 1303-1319.
McDowell-Boyer, L. M., Hunt, J. R. and Sitar, N., 1986. Particle Transport Through Porous Media. Water Resour. Res., 22: 1901-1922.
Sakthivadivel, R., 1966. Theory and Mechanism of Filtration of Non-Colloidal Fines Through a Porous Medium. HEL 15-5, Hydraulic Engineering Laboratory, University of California, Berkeley, California.
Sakthivadivel, R. and Einstein, H. A., 1970. Clogging of Porous Column of Spheres by Sediment. J. Hydraulics Div., ASCE, 96(HY2): 461-472.
Shen, H. W. and Todorovic, P., 1971. A General Stochastic Model for the Transport of Sediment Bed Material. Stochastic Hydraulics, edited by C.-L. Chiu, Proc. of the First International Symposium on Stochastic Hydraulics, University of Pittsburgh, May 31-June 2.
Sherard, J. L., Dunnigan, L. P. and Talbot, J. R., 1984. Basic Properties of Sand and Gravel Filters. J. Geotech. Eng., ASCE, 110(6): 684-700.
Sherard, J. L., Woodward, R. J., Gizienski, S. F., and Clevenger, W. A., 1963. Earth and Earth-Rock Dams. John Wiley and Sons, Inc. pp. 81-91.
Yim, C. S. and Sternberg, Y. M., 1987. Development and Testing of Granular Filter Design Criteria for Stormwater Management Infiltration Structures (SWMIS), Final Report. Maryland Department of Transportation, State Highway Administration Research Report FHWA/MD-87/03.
Wu, F.-C., 1993. Stochastic Modeling of Sediment Intrusion into Gravel Beds. A dissertation submitted in partial satisfaction of the requirements for the degree of Doctor of Philosophy in Civil Engineering, University of California at Berkeley. pp. 25-28.

收稿日期：民國83年2月20日
修正日期：民國83年3月1日
接受日期：民國83年3月11日