推進型風力機之設計與分析

Analysis and Design of a Propeller-type Wind Machine

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摘　要

本文係根據剝削原理 (strip theory) 設計風力推進器葉片，基於葉片局部最大能量計算公式，所產生之葉片形狀良好，亦極實用。其估計風能亦甚為合理。

Summary

A design has been proposed for the blades of propeller-type wind machine. It was based on the strip theory by maximizing the local power contribution along the blade. The shape of the blade obtained was found to be good and practical. The power output calculated was also reasonable.

INTRODUCTION

Wind machines have been used for many years by people around the world to convert wind energy into mechanical work. The Dutch used large windmills to pump water that leaked through the dikes back into the sea. The best known "Early American" windmills (many are still working today) were used to pump water and to power various tools.

As energy costs rising, people are once again looking into wind machines for clean and quiet energy supply. Modern wind machines are usually used to generate electricity which can be used to operate other type of device. The use of wind power to pump water for irrigation has also gained popularity in recent years.

Unfortunately, most wind machines in use are not as efficient as they could be(1). This is due largely to the fact that until recently very little was known about the design and optimization of the blades. With few exceptions most wind machines in use today were design through trial and error (using wind tunnels and years of experience with design). It is expensive and time consuming; therefore, the design for specific purpose (specific conditions and location) has been impractical.

Since the blades extract the energy from the wind, they are the most important part of the wind machines. It is very important that they be designed to extract the greatest possible energy for their size. The amount of energy that can be extracted depends upon the blade size and efficiency. The efficiency is determined by the shape of the blades. The analysis discussed and presented here will allow you to determine the blade shape that yields the greatest efficiency for your configuration.

The primary purpose of this paper is to demonstrate the design and analysis of propeeller-type wind machines using portions of mathematical models developed by Walker, Lissaman, Wilson, Patton and others[2]. In order to make a difficult task as easy as possible I will use design examples to illustrate the processes.

MOMENTUM THEORY

Applying the momentum theory to the control Volume in Figure 1, the thrust can be expressed as

$$T = \frac{1}{2}\rho A (V_\infty^2 - V_2^2) \tag{1}$$

and the velocity through the blade is

$$U = \frac{1}{2}(V_\infty + V_2) \tag{2}$$

Now defining the axial induction factor, *a*, by

$$U \equiv V_\infty (1-a) \tag{3}$$

and

$$a = 1 - \frac{V_\infty + V_2}{2V_\infty} \tag{4}$$

V_∞ = Free stream velocity
U = Velocity through blade

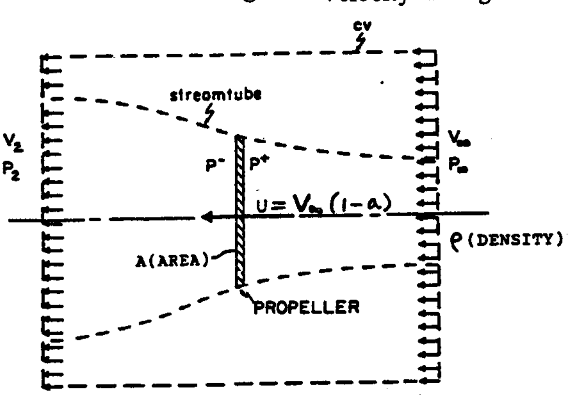

Fig. 1. Control Volume of a Wind Turbine.

Because power is given by mass flow rate times the change in kinetic energy, the power, P, is

$$P = m\,\Delta KE = \rho A U\left(\frac{V_\infty^2}{2} - \frac{V_2^2}{2}\right)$$

$$= \frac{1}{2}\rho A V_\infty^3\, 4a(1-a)^2$$

or

$$P = 2\rho A V_\infty^3\, a(1-a)^2 \tag{5}$$

maximum power occurs when $\frac{dp}{da} = 0$; therefore

$a = \frac{1}{3}$ and

$$P_{max} = \frac{16}{27}\left(\frac{1}{2}\rho A V_\infty^3\right) \tag{6}$$

The coefficient of power, C_p $\left(= \frac{P}{\frac{1}{2}\rho A V_\infty^3}\right)$ is approximately 0.593.

Dutch windmills of very large diameter are said to approach 20 per cent, and modern propeller-type wind machines are more efficient.

The initial assumptions of momentum theory considered no rotation was imparted to the flow. It is possible to develop simple relationships between the angular velocity, ω, imparted to the slip-stream flow and the angular velocity, Ω, of the rotor.

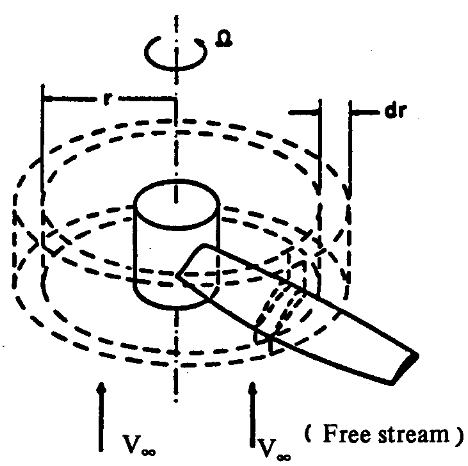

Fig. 2. Rotor Blade Element.

By defining the angular induction factor

$$a' = \frac{\omega}{2\Omega} \tag{7}$$

the thrust can be obtained as:

$$dT = \rho V 2\pi r_L dr_L \;(V_\infty - V_2)$$

$$= 4\pi r_L \,\rho V_\infty^2 \; a(1-a)\, dr_L \qquad (8)$$

where

$r_L = r\,\text{Cos}\psi$

$dr_L = dr\,\text{Cos}\psi$

4 = coning angle

The torque is

$$dQ = 4\pi\, r_L^3\, \rho V_\infty (1-a) a' \Omega\, dr_L \qquad (9)$$

BLADE ELEMENT THEORY

The program uses a model similar to that used for propellers. It is commonly called the blade element theory. It divides the blade into several spanwise elements, by calculating the forces acting on these elements, then integrating over the entire blade we can estimate the performance of the rotor. The theory makes some important assumptions:

1. The blade does not interfere with one another.
2. Lift and drag forces are considered in analysis.
3. Flow through the rotor is steady.

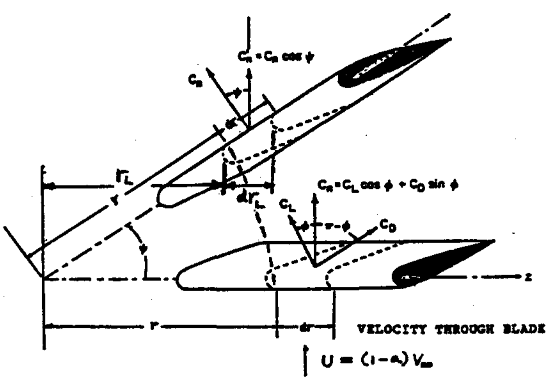

Fig. 3. Blade Coning.

Figure 3 shows the forces acting on element "dr" at a coning angle, ψ. C_n is the force coefficient ($= \dfrac{\text{Force}}{\frac{1}{2}\rho A V_\infty^2}$) normal to the blade element. Only C_n is affected by the coning of the blade. C_t (tangent to the blade element) is not. The force coefficients are:

$$C_n' = C_L\, \text{Cos}\phi \text{Cos}\psi + C_D\, \text{Sin}\phi \text{Cos}\psi \qquad (10)$$

$$C_t = C_L\, \text{Sin}\phi\, C_D\, \text{Cos}\phi \qquad (11)$$

The radical distances are:

$r_L = r\,\text{Cos}\,\psi$

$dr_L = dr\,\text{Cos}\,\psi$

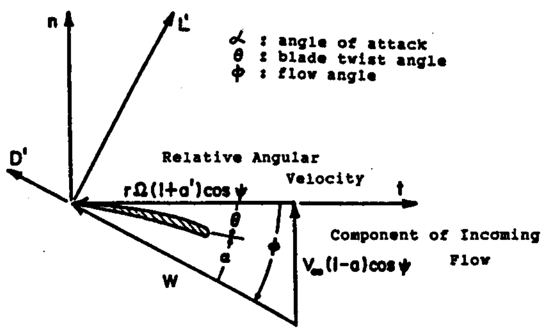

Fig. 4. Velocity Diagram At Blade.
(Including Coning Angle Correction)

From the velocity diagram (Fig. 4).

$$\tan\phi = \frac{1-a}{1+a'}\;\frac{V_\infty \text{Cos}\psi}{r_L \Omega} \qquad (12)$$

and the angle of attack, α, is related by the blade angle, θ, by

$$\alpha = \phi - \theta \qquad (13)$$

The thrust can also be determined to be:

$$dT = BC\frac{1}{2}\rho w^2 C_n' \;\frac{dr_L}{\text{Cos}\psi} \qquad (14)$$

where B = number of blades
C = chord

The torque acting on the blade element is given by the following expression

$$dQ = r_L BC(\frac{1}{2}\rho w^2) C_t \frac{dr_L}{\text{Cos}\psi} \quad (15)$$

STRIP THEORY

Using both momentum and blade element theories, a series of relationships can be developed. By equating the thrust determined from the momentum and blade element theories, we can obtain the following.

$$\frac{a}{1-a} = \frac{\delta_L C_n \text{Cos}^2 \psi}{8 \text{Sin}^2 \phi} \quad (16)$$

where

$$\delta_L = \frac{BC}{\pi r_L}$$

Equating the torque obrained from both theories, one obtains

$$\frac{a'}{1+a'} = \frac{\delta_L C_t}{8 \text{Sin}\phi \text{Cos}\phi} \quad (17)$$

In the calculations of a and a', the drag terms could be omitted due to the fact that the drag is confined to thin helical sheets in the wake and have little effects on the induced flows. So the equations become

$$\frac{a}{1-a} = \frac{\delta_L C_L \text{Cos}^2\psi \text{Cos}\phi}{8 \text{Sin}^2 \phi} \quad (18)$$

and

$$\frac{a'}{1+a'} = \frac{\delta_L C_L}{8 \text{Cos}\phi} \quad (19)$$

By using the relations developed, a and a' can be determined for a given differential element by the iteration process.

TIP AND HUB LOSSES

The Losses occur as a result of air flowing around the blade tip and root. The air flows around the ends of the blade because the pressure on the upwind side of the blade is higher than on the downstream side. This means that the torque acting upon a blade element near the ends of the blade is reduced. The losses can be significant, especially at the blade tip since the elements farthest out on the blade contribute most to the total torque. While hub loss is less critical, it does decrease the total torque (and power) that can be produced.

There are many different methods of calculating tip and hub losses. Prandtl's method involves the use of a parameter called Prandtl's factor, F_p.

$$F_p = \frac{2}{\pi} \text{arc Cos } e^{-f} \quad (20)$$

where

$$F = \frac{B}{2} \frac{R-r}{R \text{Sin}\phi} \quad (21)$$

R = radius of the rotor

and the total losses, F, are

$$F = F_p(\text{tip}) \times F_p(\text{hub}) \quad (22)$$

Then, the final form of equations can be written as

$$\frac{dT}{dr} = 4\pi r \rho V_\infty^2 (1-a) aF \quad (23)$$

$$\frac{dQ}{dr} = 4\pi r^3 \rho V_\infty \Omega (1-a) a' F \quad (24)$$

and

$$\frac{a}{1-a} = \frac{\delta_L C_L \text{Cos}^2\psi \text{Cos}\phi}{8F \text{Sin}^2 \phi} \quad (25)$$

$$\frac{a'}{1+a'} = \frac{\delta_L C_L}{8F \text{Cos}\phi} \quad (26)$$

OPTIMUM PERFORMANCE

Optimum performance may be determined by maximizing in the power output at each station along the blade. The power developed at a differential element, dr, located at radial position, r, is

$$dp = \Omega dQ = \Omega r BC \frac{\rho}{2} w^2 C_t dr \quad (27)$$

Defining local and overall tip-speed-rations and solidity,

$$x = \frac{r\Omega}{V_\infty}, X = \frac{R\Omega}{V_\infty} \text{ and } \delta_L = \frac{BC}{\pi r}$$

by neglecting the drag, the differential power contibution is

$$\frac{dC_p}{dx} = \delta_L C_L \operatorname{Sin}\phi (\frac{x}{X})^2 (\frac{w}{V_\infty})^2 \qquad (28)$$

where

$$C_p = \frac{P}{\frac{1}{2}\rho V_\infty^3 \pi R^2}$$

$\frac{dC_p}{dx}$ is maximized by varying the axial velocity, U, or its dimensionless equivalent, (1–a), until the power contribution becomes stationary. The equation relating blade torque to the fluid momentum is

$$\frac{\delta_L C_L}{8 \operatorname{Cos}\phi} = \frac{a' F}{1 + a'} \qquad (29)$$

or

$$\frac{B (c\, C_L/R)}{8\pi (r/R) \operatorname{Cos}\phi} = \frac{a' F}{1 + a'} \qquad (30)$$

The corresponding equation relating the blade force to the fluid momentum is

$$\frac{\delta_L C_L \operatorname{Cos}\phi}{8 \operatorname{Sin}^2\phi} = \frac{(1\, aF) aF}{(1 - a)^2} \qquad (31)$$

or

$$\frac{B (c\, C_L/R)}{8\pi (r/R) \operatorname{Sin}^2\phi} = \frac{(1 - aF) aF}{(1 - a)^2} \qquad (32)$$

Equation (28) becomes

$$\frac{dC_p}{dx} = 8a'(1 - a) F \frac{x^3}{X^2} \qquad (33)$$

Equations (29) and (31) may be combined to give

$$a(1 - aF) = a' x^2 (1 + a') \qquad (34)$$

At each radial station, the power distribution is obtained by varying the axial interference factor, a, until Eq. (33) is maximized. In the process, Eq. (34) must also be satisfied. Additionally, F is a function of a, a', x and X. Therefore, for each trial value of a, iteration is required to obtain consistent value of a' and F.

The flow angle, ϕ, and the product of cC_L are determined as a function of blade radius. The overall power coefficient is obtained by integrating the power coefficient contributions along the blade.

The blade twist (blade angle θ) can be determined by selecting either the chord distriction or the lift distribution along the blade.

DESIGN PROJECT (B= 3, X= 5)

The project was to design a wind machine of 3 blades and advance tip speed radio of 5 for maximum power extraction. The surrounding condition was assumed to be at standard sealevel condition with wind speed at 15 mph (24 kph). The size of blade (R) was selected to be 17.5 m (57.41 ft). The coning angle (ψ) was decided at 5.

Next was the selection of the airfoil for the blades of the wind machine. The NACA 23012 was selected from the Abbott's book [3] , which has the maximum lift to drag ratio (or $\frac{L}{D}$) at 8° of angle of attack (α) and 0.9 for the lift coefficient (C_L).

Using Basic programming code, let $\frac{r}{R}$ range from 0.05 to 1.00 in 0.05 increments. The axial induction factor, a, was set from 0.100 to 0.500 in increments of 0.010. The tip loss factor, F, was set equal to 1 (no tip loss assumption) for a matter of simplification. The value of a' and $\frac{dC_p}{dx}$ were calculated for each value of a, and the maximum $\frac{dC_p}{dx}$ was found. This process was repeated for each blade station (or $\frac{r}{R}$).

Values of $\frac{cC_L}{R}$, ϕ, θ and C were also calculated for each point. Plots of these values versus $\frac{r}{R}$ were made (Figure 5 to Figure 8).

The derivative of the power coefficient, $\frac{dC_p}{dx}$, was plotted against $\frac{r}{R}(= \frac{x}{X})$ (Fig. 9). The power coefficient (C_p) was obtained from the area under the $\frac{dC_p}{dx}$ curve. The value of chord length (c) were manipulated and plotted against r (Fig. 10).

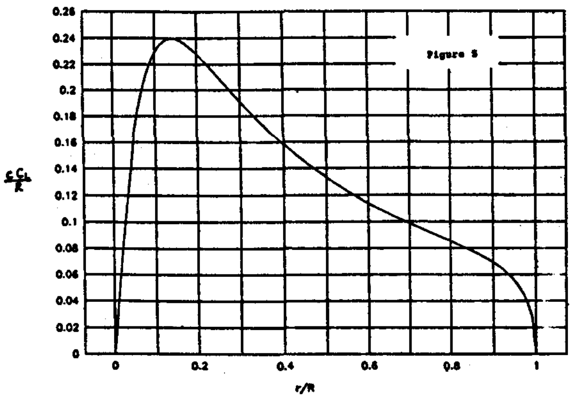

Fig. 5.

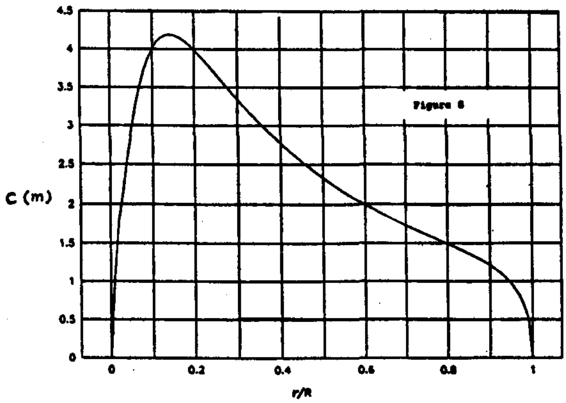

Fig. 8.

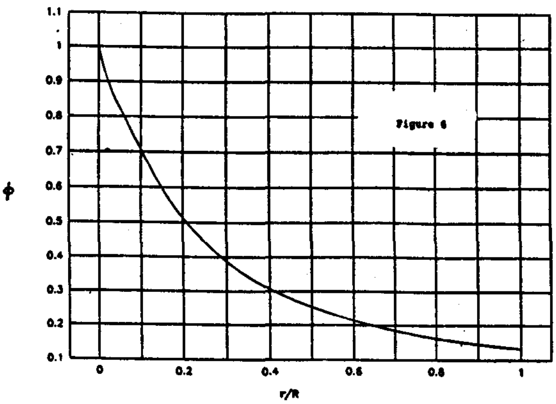

Fig. 6.

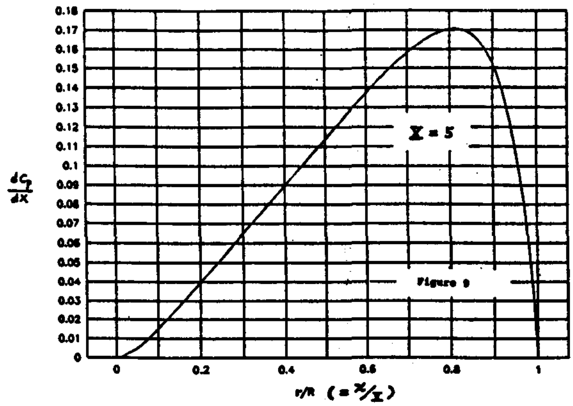

Fig. 9.

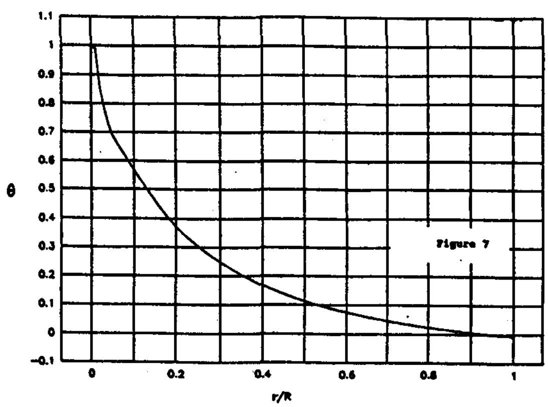

Fig. 7.

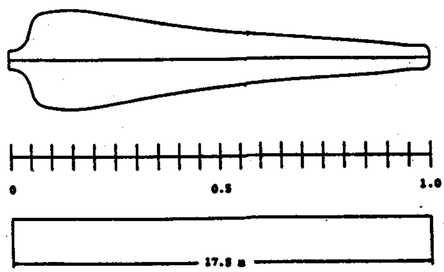

Fig. 10

RESULTS

From the plot of $\frac{dC_p}{dx}$ versus x, the maximum power coefficient can be determined by the area under the curve. The value of C_p is approximately equal to 0.538. The maximum power output is found to be 94.0 kw.

It should be noted that no tip losses were assumed; that is, F, the tip loss factor, was set equal to 1. This gives slightly optimistic results, but it can be considered a reasonable estimate.

The shape of the blade obtained was found to be reasonably good. The power output was also realistic.

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3. I.H. Abbott and A.E. Van Doenhoff; *Theory of Wing Sections*, Dover, N.Y., 1959.