由年齡分佈函數分析通風農業設施中氣體污染質之散佈

Dispersal Analysis of Gaseous Pollutants in a Ventilated Agricultural Structure via Age Distribution Functions

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摘　　要

本文乃針對由年齡分佈函數觀點所建立之可描述通風農業設施中氣體污染質之散佈之方法論做一數學分析。此方法乃對空氣與氣體污染質之濃度對時間曲線的動差做評估，由濃度歷程之動差觀念確可瞭解通風系統中供給空氣與氣體污染質之散佈特性。通風空間中氣流的平均年齡停留時間可由一反系統狀態矩陣〔A〕$^{-1}$，計算而得。〔A〕$^{-1}$ 矩陣中任意一行 i 之元素和即等於通風空間 i 之氣流平均年齡，在通風空間 i 中氣體污染質的平均年齡則爲濃度曲線下之面積除上由氣體污染源所導致的平衡濃度值。〔A〕$^{-1}$ 矩陣則可以反區域流量率矩陣（〔W〕$^{-1}$），傳輸機率矩陣（〔P〕）及空氣體積（〔V〕）表示，即〔A〕$^{-1}$ ＝〔W〕$^{-1}$〔P〕〔V〕。最後，通風空間中氣流的平均年齡便可由〔W〕及〔P〕兩矩陣解釋。爲舉例說明，文中並以一典型畜舍做爲理論應用過程之介紹。

關鍵詞：氣體污染質散佈，通風農業設施，動差，年齡分佈函數，平均年齡，停留時間。

ABSTRACT

Mathematical analysis of a methodology for describing the gaseous pollutant dispersal in a ventilated agricultural structure is presented from the viewpoint of age distribution functions. The method is based on evaluating the moments of concentration vs. time curves of both air and gaseous pollutants. The concept of moments of concentration histories is applicable to characterize the dispersal of the supplied air or gaseous pollutant in a ventilated system. The mean age or residence time of airflow in a ventilated airspace can be calculated from an inverse system state matrix $[A]^{-1}$. The sum elements in an arbitrary row i in matrix $[A]^{-1}$ is equal to the mean age of airflow in airspace i. The mean age of gaseous pollutant in airspace i can be obtained from the area under the concentration profile divided by the equilibrium concentration reading in that space caused by gaseous pollutant sources. Matrix $[A]^{-1}$ can be represented in terms of the inverse local airflow rate matrix ($[W]^{-1}$), transition probability matrix ($[P]$), and air volume matrix ($[V]$), i.e., $[A]^{-1}$=

$[W]^{-1}[P][V]$. Finally the mean age of airflow in a ventilated airspace can be interpreted by the physical characteristics of matrices [W] and [P]. The practical use of the theory is also applied in a typical animal housing.

Keywords: gaseous pollutant, dispersal, ventilated agricultural structures, moments, age distribution functions, mean age, residence time.

INTRODUCTION

Polluted air can be a very serious problem for the producer of greenhouse flower crops. Many different gases, present in relatively low concentrations, affect the growth, quality, and life of flowers and plants. Included in the list of phytotoxic gases are ozone, peroxyacyl nitrates, and other oxidants, ethylene, sulfur dioxide, fluorides, mercury, phenolic compounds herbicides insecticidal fumigants, and chemicals used for pasteurizing soils. Damage to flower crops varied with the toxic gas, the concentrations and dilution of exposure, the sensitivity of a particular crop, and climatic and environmental conditions (Masterlerz, 1977; Langhans, 1990).

Crops grow in greenhouses located in industrialized or heavily populated urban areas are subjected to ozone and other oxidents. Nearby industry is often a source of sulfur dioxide, but this toxic gas also can be caused by the boilers that heat the greenhouses range. Plants injuried by herbicides, plenolic wood preservatives, insecticidal fumigants, soil pasturization chemicals, or mercury vapors may result from the decisions and mistakes of people working in the greenhouses.

Faced with serious community air pollution, greenhouse owners may find it is necessary to move their greenhouses to where phytotoxic gases are not present, or to grow less sensitive species of flower crops.

On the other hand, the increasing use of confinement buildings has resulted an economic need for higher productivity. Enclosing and concentrating animal, however, meant concentrating their waste products and pollutants. Potential health hazards to the workers and animal occur when exposed to these agents. For example, swine producers are concerned about the gases produced in liquid manure storage pits in buildings with partially or totally slatted floors. The main gaseous pollutants contained in a swine confinement building with known physiological effects include ammonia, carbon dioxide, hydrogen sulfide, and methane (Hellickson and Walker, 1983; Albright, 1990).

There is ample evidence that several of the gaseous pollutants released during the decomposition of excreta, if concentrated, can cause injury or even death to swine. Prolonged exposure to low levels of those gaseous pollutants may be of considerable importance, although health effects are largely unknown.

From the viewpoint of air quality, the main objective of a ventilation system in agricultural structures is to limit animal, plants, and crops, etc. exposure to polluted air. This implies at least for a steady air pollutant source, that its spread and residence time within the agricultural structures shall be minimized. The spread of a pollutant and pollutant itself and the distribution of the supplied air stated above must be quantified. That is to say, the flow field occurring must somewhat be characterized.

Within biology (Weinstein and Dudukovic, 1975) and chemical reaction engineering (Levenspiel, 1981; Nauman, 1981; Nauman and Buffham, 1983) it has for a long time been a well established technique to represent flow by measuring how long the fluid stays, for example, in a section of the human circulation system or a distillation column. This characterization of flow patterns in terms of ages or residence times permits unified and elegant greatment of continuous flow systems which is independent of specific mixing mechanisms. This type of approach seems suitable for use in characterizing flows in ventilated agricultural structures and is the main topic of this paper. When applying this approach, the moments of the obtained concentration histories are calculated. When using this technique in ventilation engineering, there is much to be learnt from the field mentioned above.

In general, a ventilated agricultural structure is a multiport system having several supply and exhaust terminals. It is of vital importance to know where a pollutant goes within a ventilated agricultural structure. Measurements of the residence time distribution of the air extracted in a mechanically ventilated test-house with several exhaust air dusts are reported by Sandberg (1983). In hospital wards, Lidwell (1960) studied the spread of pollutants between rooms. The dispersion of pollutants in a house, simulated by tracer gas, is reported by Freeman et al. (1982). Measurements of equilibrium concentrations of simulated pollutants in an office room are reported by Skarret and Mathisen (1981). Thereasse and Sine (1974) in their study of ventilation for livestock buildings interpreted the results of tracer gas experiments in terms of a concept related to residence time.

Therefore, the aim of this paper is to give a comprehensive presentation of both the method and the theory of the residence time or age concept to problems occurring within ventilation engineering of agriculture. The method is based on evaluating the moments of concentration vs. time curves of both air and gaseous pollutant.

With the development of this methodology, studies of an interdisciplinary nature involving both occupational health and the life of animal and plants will be able to define an acceptable enironment for the animal, plants and operators.

DYNAMIC MATRIX EQUATION

A mathematical model describing the dynamic behavior of carbon dioxide concentration in a ventilated airspace has been presented and can be expressed as a linear dynamic matrix equation (Liao et al., 1991):

$$\{\dot{C}(t)\} = -[A]\{C(t)\} + [V]^{-1}\{m(t)\} + [D]\{C_s(t)\}, \quad \{C(0)\} = \{C_0\} \qquad (1)$$

where:

$\{C(t)\}$ = carbon dioxide concentration vector, ppm,
$\{m(t)\}$ = source generation rate vector, g/hr,
$\{C_s(t)\}$ = supplied air concentration vector, ppm,
$[V]^{-1}$ = inverse of air volume matrix, $1/m^3$,
[A] and [D] = constant matrices.

Matrices [A] and [D] can be expressed as:

$$[A] = [V]^{-1}[Q] \qquad (2\text{-}1)$$

$$[D] = [V]^{-1}[Q_s] \qquad (2\text{-}2)$$

Matrix [Q] is referred to as the flow matrix, consisting of the total flow rates of air between airspaces. The sum of the elements in a row (say No. i) of [Q] is equal to the flow rate of outdoor air entering airspace i. All elements in a column (say No. j) of [Q] is equal to the total flow rate of air transferred directly from airspace j to outdoor. Matrix [A] sometimes referred to as the system state matrix, contains the essential dynamic characters of the system being studied.

In the case of neglecting the supplied air concentrations, Equation (1) becomes:

$$\{\dot{C}(t)\} = -[A]\{C(t)\} + [V]^{-1}\{m(t)\} \qquad (3)$$

In an analogous manner regards to the tracer gas techniques, the vector $\{m(t)\}$ can be seen as a vector consisting of injected flow rate of tracer gas in the ventilated airspaces.

Exhaust concentration can be written as (Liao, 1988):

$$C_e(t) = \{Q_e\}^T\{C(t)\}/Q \qquad (4)$$

where:

$\{Q_e\}$ = extract airflow vector, m^3/hr,

Q = total volumetric flow rate, m^3/hr.

The decay from a given initial concentration distribution $\{C_0\}$, Equation (3) is governed by:

$$\{C(t)\} = \exp(-[A]t)\{C_0\} \qquad (5)$$

MOMENTS

From the statistical point of view, the moments about the origin of concentration profiles are (Feller, 1968):

$$M_c^{(n)} = \int_0^\infty t^n C(t)dt, \quad n = 1,\ 2,\ 3,\ \ldots,\ n \qquad (6)$$

where $M_c^{(0)}$ is the area under the concentration profile. After use of the technique of Laplace transformation, the application of Equation (6) on the relation of Equation (5) can be obtained as:

$$\{M\}_c^{(n)} = n!([A])^{-(n+1)}\{C_0\} \qquad (7)$$

After inserting Equation (5) into the Equation (4) of the exhaust concentration becomes:

$$C_e(t) = 1/Q\{Q_e\}^T(\exp(-[A]t)\{C_0\}) \qquad (8)$$

Because

$$\{Q_e\}^T = \{1\}^T[Q] \qquad (9)$$

By inserting Equation (9) into Equation (8) and calculating the moments in accordance with Equation (6), obtains:

$$\begin{aligned} M_{C_e}^{(n)} &= n!/Q(\{1\}^T[Q][A]^{-(n+1)})\{C_0\} \\ &= n!/Q(\{1\}^T[V][V]^{-1}[Q][A]^{-(n+1)})\{C_0\} \\ &= n!/Q(\{1\}^T[V][A][A]^{-(n+1)})\{C_0\} \\ &= n!/Q(\{1\}^T[V][A]^{-n})\{C_0\} \end{aligned} \qquad (10)$$

where $[A]^0 = [I]$

From Equation (7), it is shown that Equation (10) can be expressed as:

$$M_{C_e}^{(n)} = n/Q(\{1\}^T[V]\{M\}_c^{(n-1)}) \qquad (11)$$

By dividing each side in Equation (11) by total volume, V, and rearranging the terms, obtains:

$$1/V(\{1\}^T[V]\{M\}_c^{(n-1)}) = Q/V\,1/n M_{C_e}^{(n)} \qquad (12)$$

The matrix multiplication in Equation (12) yields a summation of the moment, $M_{C_i}^{(n-1)}$, in each airspace weighted by the corresponding airspace's fraction of the total volume, i.e.,

$$\sum_{i=1}^{N}(V_i/V)M_{C_i}^{(n-1)} = (Q/V)(1/n)M_{C_e}^{(n)} \qquad (13)$$

The left-hand side in Equation (13) is the system's average (n–1)th moment, $\bar{M}_C^{(n-1)}$, and therefore Equation (13) can be rewritten as:

$$\bar{M}_c^{(n-1)} = (Q/V)(1/n)M_{C_e}^{(n)} \qquad (14)$$

RESIDENCE TIME DISTRIBUTION

Definition of age distribution function

Consider a ventilated agricultural structure in Figure 1, the gaseous pollutants are show in their path through the structure from outdoor until their departure. Each air and gaseous pollutant entering the ventilated airspace will spend some time in the airspace before leaving. It is obvious that the exit time of one gaseous pollutant is different from that of another not only because of the circulation of air flow in a ventilated airspace but because of the internal mixing (due to the molecular diffusion and turbulent movement, etc.) in each airspace. Therefore, there is an exit age distribution in the leaving airflow. This exit age distribution function will be denoted by E(t). Intimately related to the exit age distribution function is internal age distribution function, I(t), which accounts

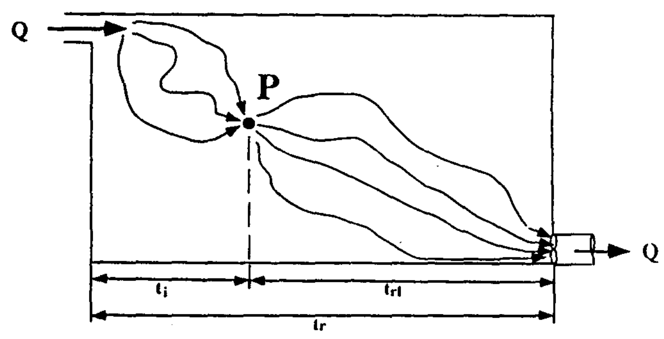

Fig. 1. Diagram shows different ages of a gaseous pollutant from outdoor to a specific point P within a ventilated agricultural structure.

for the distribution of the ages (the length of time has elapsed since the gaseous pollutant goes into the structure) of air and gaseous pollutants at any moment in a ventilated airspace.

There are three concentrations of gaseous pollutants that are considered in terms of age distribution (Zwietering, 1959):

1. A total internal concentration consisting of all gaseous pollutants within the structure.
2. A local internal concentration at any arbitrary airspace within the structure.
3. An external concentration consisting of gaseous pollutants leaving the structure.

Each concentration of gaseous pollutants may be characterized by their statistical cumulative age distribution function (CDF), and corresponding probability density function (PDF). The CDF is dimensionless and the magnitude of the function at a particular time, t, gives the fraction of the considered concentration with an age less than or equal to t. The magnitude of PDF whose dimension is $1/t$, is the derivative of the CDF at time t. For the exit age distribution function, E(t), the general relationship between a CDF, F(t), and corresponding PDF, E(t), is:

$$dF(t)/dt = E(t), \quad \int_0^t E(t)dt = F(t) \tag{15}$$

The term E(t)dt represents the fraction of the fluid elements in the exit stream having spend the time between t to $t+dt$ in a ventilated airspace, and I(t)dt represents the fraction of fluid elements with internal age between t and $t+dt$. Therefore,

$$\int_0^\infty E(t)dt = \int_0^\infty I(t)dt = 1 \tag{16}$$

Since age cannot be negative, F(t) is defined over (0, ∞). The range of F(t) is 0 to 1 with F(−0)=0 and F(∞)=1 (Nauman, 1981). The moments about the origin regard to exit age distribution function are:

$$M_E^{(n)} = \int_0^\infty t^n E(t)dt = \int_0^1 t^n dF(t) \tag{17}$$

The variance, σ_E^2, can be expressed as:

$$\sigma_E^2 = \int_0^\infty (t - M_E^{(1)})^2 E(t)dt = M_E^{(2)} - (M_E^{(1)})^2 \tag{18}$$

The moment, $M_E^{(n)}$, also can be calculated from the CDF, F(t), by integrating by parts and applying Equation (17), it leads to (Liao, 1988):

$$M_E^{(n)} = n\int_0^\infty t^{(n-1)}(1 - F(t))dt, \quad (n > 0) \tag{19}$$

Furthermore, for gaseous pollutants within a ventilated agricultural structure, the following ages can be defined (Figure 1) (Zwietering, 1959):

1. The internal age, t_i: the travel time for component to move from a point at outdoor to a specific airspace within structures.
2. The residual life time, t_{rl}: the travel time from a specific airspace to the exhaust.
3. The residence time, t_r: the average time of release to time of exhaust.

The CDF and PDF for the total internal gaseous pollutants and for the local at arbitrary airspace or point p at different ages are shown in Table 1. To aid in gaining an appreciation of the several ages, an analogy to human populations may be helpful. The age corresponds to the common usage of the word, while the residual time is the life span. The mean internal age is the average age of the population living at any time. The mean residence time is the average age at the time of death.

Therefore, at any time a mass blance of the second airflow over a ventilated airspace gives, [entered the system]=[still in the system]+[left the system]:

$$Qt_r = V\int_0^{t_r} I(t)dt + Q\int_0^{t_r} F(t)dt \tag{20}$$

Differentiating Equation (20) with respect to t yields:

$$1 - F(t) = V/QI(t) \tag{21}$$

where $V/Q=\tau_n$, is mean-holding time of the ventilation air.

For the air or gaseous pollutants in the ventilated agricultural structures, on leaving the system, the mean-holding time is always equal to τ_n. This can be shown by making use of definition of the mean and Equation (16) and express as follows:

Table 1. The cumulative distribution function probability density function for different age distributions

Age	cumulative distribution function		probability density function	
	total	local	total	local
t_i	Φ	Φ_p	I	I_p
t_{rl}	Ψ	Ψp	Ψ	Ψp
t_r	X		χ	

$$\tau_n = \int_0^\infty tE(t)dt \tag{22}$$

According to equation (19) and applying Equation (21), leads to:

$$\tau_n = \int_0^\infty (1 - F(t))dt = \tau_n \int_0^\infty I(t)dt = \tau_n \tag{23}$$

Experimental measurement of age distribution

There are two perfectly equivalent methods of obtaining experimentally or numerically, the age distribution functions (Himmelblau and Bischoff, 1968; Nauman and Buffham, 1983):

1. Step-up method: At time t=0, a fraction of the supplied air is labeled with gaseous pollutant, and the concentration of gaseous pollutant is measured at the point(s) in the system where the local age distribution is to be obtained.

2. Step-down method: The system is initially filled with known homogeneous concentration of gaseous pollutant and is then supplied fresh air, the decay of concentration is recorded at point(s) where the local age in a step-down simulation is to be obtained.

The equation for the local age in a step-up simulation is now derived. At time t=0, a fraction, C_0, of all the entering molecules in the system are labeled. Consider an arbitrary point P within the system at time t where the concentration is Cp(t). A fraction $Cp(t)/C_0$ of the molecules at time P thus have an age less than or equal to t. Since $Cp(t)/C_0$ has the characteristics of a cumulative distribution function, $\Phi p(t) = Cp(t)/C_0$ is defined as the cumulative distribution function of the local age at point P (Table 1).

From Equation (19), the local mean age, $M_{Ip}^{(1)}$, can be calculated as:

$$M_{Ip}^{(1)} = \int_0^\infty (1 - \Phi_p(t))dt = \int_0^\infty (1 - C_p(t)/C_0)dt \tag{24}$$

For step-down simulation with $Op(t) = 1 - Cp(t)/C_0$, the local mean then is:

$$M_{Ip}^{(1)} = \int_0^\infty (1 - \Phi_p(t))dt = 1/C_0 \int_0^\infty C_p(t)dt = 1/C_0 M_{Cp}^{(0)} \tag{25}$$

MEAN AGES

Mean age of airflow

From the definition of experimental measurement of age distribution functions by step-down simulation (Equation (25)), it is possible to determine the internal age of airflow in a ventilated agricultural structure by starting from the same initial concentration of air in each airspace as the reference concentration C_0, in which $C_0 = C(0)$; and from definition of the theory of matrices:

$$\{C(0)\} = C(0)\{1\} \tag{26}$$

Therefore, by the definition of Equation (25) and the recorded concentration-time relations of airflow, the mean age of the airflow in a ventilated airspace i can be obtained by dividing the 0th moment, i.e., the area under the recorded concentration-time curve by the initial concentration, C(0), and by using the matrix notation, it becomes:

$$\{M\}_i^{(1)} = 1/C(0)\{M\}_C^{(0)} \tag{27}$$

According to Equation (7):

$$\{M\}_C^{(0)} = [A]^{-1}\{C(0)\} \tag{28}$$

After combining Equations (26) and (28), it leads to:

$$\{M\}_C^{(0)} = [A]^{-1}\{1\}C(0) \quad (29)$$

Inserting Equation (29) into Equation (27) gives the important relations:

$$\{M\}_I^{(1)} = [A]^{-1}\{1\} \quad (30)$$

Equation (30) means the row sum in matrix $[A]^{-1}$, say the ith row, is equal to mean age of airflow in airspace i.

The mean age of air when it leave the ventilated airspace, $M_E^{(1)}$, can be obtained from the concentration reading in the exhaust duct as:

$$M_E^{(1)} = M_{C_e}^{(0)}/C(0) \quad (31)$$

According to Equation (10), the 0th moment is:

$$M_{C_e}^{(0)} = 1/Q(\{1\}^T[V][I]\{C(0)\}) = V/QC(0) \quad (32)$$

and Equation (31) becomes:

$$M_E^{(1)} = V/Q = \tau_n \quad (33)$$

The mean age of air in the whole ventilated sstem, $\overline{M}_I^{(1)}$, can be expressed as:

$$\overline{M}_I^{(1)} = M_{C_e}^{(1)}/M_{C_e}^{(0)} \quad (34)$$

The first moment, $M_{C_e}^{(1)}$, according to Equation (10) is equal to:

$$M_{C_e}^{(1)} = 1/Q(\{1\}^T[V][A]^{-1}\{1\})C(0) \quad (35)$$

Using Equations (32), (35) aand (30) in Equation (33), the following expression for the mean age of air in the whole ventilation systems can be obtained as:

$$\begin{aligned}\overline{M}_I^{(1)} &= (\{1\}^T[V][A]^{-1}\{1\})/V \\ &= (\{1\}^T[V]\{M\}_I^{(1)})/V \quad (36)\end{aligned}$$

The matrix multiplication in the denominator of Equation (36) gives rise to a summation of the mean age, $M_{I_i}^{(1)}$, in each airspace i, weighted by the corresponding volume fraction of the total volume of systems:

$$\overline{M}_I^{(1)} = \sum_{i=1}^{N}(V_i/V)M_{I_i}^{(1)} \quad (37)$$

Mean age of gaseous pollutants

The mean age of gaseous pollutants can be obtained by the same procedure as for the airflow. The only difference is that it starts from an equilibrium concentration, $C(\infty)$, caused by gaseous pollutant sources. The equilibrium concentration attained from a linear dynamic matrix equation is given by (Liao et al., 1991):

$$\{C(\infty)\} = [Q]^{-1}\{m(\infty)\} \quad (38)$$

The equilibrium concentration is not necessarily the same in each airspace.

The mean age of the gaseous pollutants in airspace i is therefore calculated from the concentration reading in that airspace as:

$$M_{I_i}^{(1)} = M_{C_i}^{(0)}/C_i(\infty) \quad (39)$$

The mean age of the whole gaseous pollutants presented in the ventilation system, $\overline{M}_I^{(1)}$, can be calculated from the concentration reading in the exhaust duct as:

$$\overline{M}_I^{(1)} = M_{C_e}^{(1)}/M_{C_e}^{(0)} \quad (40)$$

From Equation (10), the 0th moment in the exhaust duct can be obtained as:

$$M_{C_e}^{(0)} = 1/Q(\{1\}^T[V]\{C(\infty)\}) \quad (41)$$

By carrying out the matrix multiplication in Equation (41), it becomes:

$$M_{C_e}^{(0)} = V/Q\overline{C}(\infty) \quad (42)$$

where $\overline{C}(\infty)$ is the average concentration of gaseous pollutant in the systems. By using Equation (11), the 1st moment in the exhaust can be written as:

$$M_{C_e}^{(1)} = 1/Q(\{1\}^T[V]\{M\}_C^{(0)}) \quad (43)$$

By inserting Equations (42) and (43) into Equation (41) to obtain following expression for the mean age of gaseous pollutants presented in a ventilated airspace:

$$\overline{M}_I^{(1)} = (\{1\}^T[V]\{M\}_C^{(0)})/(V\overline{C}(\infty)) \quad (44)$$

By carrying out the matrix multiplication in the denominator of Equation (44) and using Equation (27), the final expression of the mean age of the gaseous pollutants presented in a ventilated greenhouse can be obtained:

$$\overline{M}_{i}^{(1)} = \sum_{i=1}^{N}((V_i C_i(\infty))/(V\overline{C}(\infty)))M_{l_i}^{(1)} \qquad (45)$$

As can be seen that the system's average age is obtained by summing the mean age in each airspace weighted by the fraction of gaseous pollutant presented in the corresponding airspace of the total gaseous pollutant presented in the system.

Physical interpretation of vector $\{M\}_l^{(1)}$

From Equation (2), the inverse system matrix $[A]^{-1}$ can be written as:

$$[A]^{-1} = [Q]^{-1}[V] \qquad (46)$$

Furthermore, the inverse flow matrix, $[Q]^{-1}$, can be expressed in terms of the local (purging) airflow rates and transition probabilities of gas contration as (Liao et al., 1991):

$$[Q]^{-1} = [W]^{-1}[P] \qquad (47)$$

where:

- [W] = diagonal matrix of local purging flow rates, m^3/hr,
- [P] = square transition probability matrix (dimensionless).

The exact mathematical and physical meanings of matrices [W] and [P] have already been introduced by author (Liao and Feddes, 1990; Liao et al., 1991).

Inserting Equation (47) into Equation (46) leads to an important relation as:

$$[A]^{-1} = [W]^{-1}[P][V] \qquad (48)$$

By carrying out the matrix multiplication in definition of matrix $[A]^{-1}$ in Equation (48), becomes:

$$[A]^{-1} = [W]^{-1}[P][V]$$

$$= \begin{bmatrix} V_1/W_1 & V_2/W_1P_{12} & \cdots & V_n/W_1P_{1n} \\ \cdots & \cdots & \cdots & \cdots \\ V_1/W_nP_{n1} & V_2/W_nP_{n2} & \cdots & V_n/W_n \end{bmatrix} \qquad (49)$$

By matrix notation, the mean age vector of airflow finally can be expressed in terms of the matrices of local purging flow rate and transition probability as:

$$\{M\}_l^{(1)} = [W]^{-1}[P][V]\{1\} \qquad (50)$$

In view of Equation (30) that the sum elements in an arbitrary row P in $[A]^{-1}$ is equal to the mean age of airflow in airspace P, $M_{l_p}^{(1)}$, that is:

$$M_{l_p}^{(1)} = 1/W_p \sum_{j=1}^{N} V_j P_{pj}$$

$$= (V_p/W_p) + 1/W_p \sum_{\substack{j=1 \\ (j \neq p)}}^{N} V_j P_{pj} \qquad (51)$$

It is true that $P_{pj} \leqslant 1$ and therefore the following upper bound of the local mean age of the airflow in airspace P can be obtained as:

$$M_{l_p}^{(1)} \leq 1/W_p \sum_{j=1}^{N} V_j = V/W_p \qquad (52)$$

or,

$$W_p M_{l_p}^{(1)} \leq V \qquad (53)$$

Relation (53) connects two important quantities, the local purging flow rate and local mean age of air. Equation (53) can be rewritten as:

$$W_p \leq (\tau_n / M_{l_p}^{(1)})Q \qquad (54)$$

When $M_{l_p}^{(1)} < \tau_n$, i.e., when local mean age of airflow is less than mean-holding time of the system at airspace P; relation (54) gives rise to the following restriction to the local purging flow rate:

$$W_p > Q \quad (when \;\; M_{l_p}^{(1)} < \tau_n) \qquad (55)$$

A MODEL EXAMPLE

An example of a simple model will be studied in some details to give insight into the meaning of the concepts introduced. The system is a typical agricultural structure unit with a negative pressure ventilation system (Figure 2). The geometric and system parameters used in the model example are listed in Table 2 (Liao and Feddes, 1990). In this model a four-airspace model, in which equal air volume was assumed. Airflow patterns and the four-airspace model for this system are shown in Figure 2.

The computational procedures are stated in detail as follows.

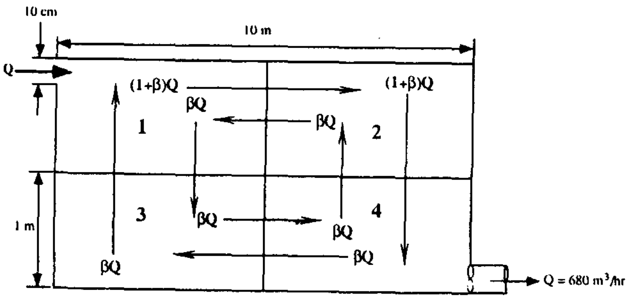

Fig. 2. A four-airspace model of an animal housing unit.

Airflow matrix [Q]

With the assumption of the transfer flow rates are assumed to be entirly generated by entrainment air into the jet stream from the air duct, the airflow matrix [Q] can be expressed as (Liao et al. 1991):

$$[Q] = [\beta]Q \qquad (56)$$

where $[\beta]$ is a square entrainment ratio function matrix with positive diagonal elements and negative off-diagonal elements. Therefore, [Q] can be expressed as (Liao and Feddes, 1990):

$$[Q] = \begin{bmatrix} Q_{11} & -Q_{12} & -Q_{13} & -Q_{14} \\ -Q_{21} & Q_{22} & -Q_{23} & -Q_{24} \\ -Q_{31} & -Q_{32} & Q_{33} & -Q_{34} \\ -Q_{41} & -Q_{42} & -Q_{43} & Q_{44} \end{bmatrix} = Q\,[\beta]$$

$$= Q \begin{bmatrix} 2\beta+1 & -\beta & -\beta & 0 \\ -(\beta+1) & 2\beta+1 & 0 & -\beta \\ -\beta & 0 & 2\beta & -\beta \\ 0 & -(\beta+1) & -\beta & 2\beta+1 \end{bmatrix}$$

$$= 11.33 \begin{bmatrix} 11 & -5 & -5 & 0 \\ -6 & 11 & 0 & -5 \\ -5 & 0 & 10 & -5 \\ 0 & -6 & -5 & 11 \end{bmatrix} \; (m^3/min)$$

The inverse flow matrix $[Q]^{-1}$ can be calculated as:

$$[Q]^{-1} = \begin{bmatrix} 8.02 & 7.21 & 7.27 & 6.56 \\ 7.96 & 8.33 & 7.56 & 7.21 \\ 7.96 & 7.568 & 8.48 & 7.27 \\ 7.94 & 7.96 & 7.96 & 8.00 \end{bmatrix} \times 0.01 \; (min/m^3)$$

Table 2. The input parameters used in the model example

Geometric parameters:	Colume, V = 120 m^3 (10x6x2 m) Height, H = 2 m Slot Width = 10 cm
System parameters:	Ventilation airflow rate, Q = 680 m^3/hr Mean-holding time of air, τ_n = 10.59 min Entrainment ratio, β = 5.0

The matrices [A]$^{-1}$, [W], and [P]

From Equation (46), $[A]^{-1}$ can be calculated as:

$$[A]^{-1} = [Q]^{-1}[V] = \begin{bmatrix} 2.406 & 2.163 & 2.181 & 1.969 \\ 2.388 & 2.399 & 2.268 & 2.163 \\ 2.388 & 2.268 & 2.544 & 2.181 \\ 2.382 & 2.388 & 2.388 & 2.400 \end{bmatrix} \text{(min)}$$

Again, from Equation (49), it can be shown that the diagonal elements of matrix $[A]^{-1}$ are the inverse values of local purging flow rate, and from the off-diagonal elements of matrix $[A]^{-1}$ the transition probability matrix can be calculated. The results are as follows:

$W_1 = 748.13 \quad W_2 = 720.29$

$W_3 = 707.55 \quad W_4 = 750.00\ (m^3/hr)$

and,

$$[P] = \begin{bmatrix} 1.00 & 0.89 & 0.91 & 0.82 \\ 0.96 & 1.00 & 0.91 & 0.87 \\ 0.94 & 0.89 & 1.00 & 0.86 \\ 0.99 & 0.99 & 0.99 & 1.00 \end{bmatrix}$$

Mean age of air, $M_{C_i}^{(1)}$

In view of Equation (30) that the sum elements in an arbitrary row i in $[A]^{-1}$ is equal to the mean age of airflow in airspace i. Therefore, the mean age of airflow in four-airspace model can be calculated as:

$M_{C_1}^{(1)} = 8.72$

$M_{C_2}^{(1)} = 9.32$

$M_{C_3}^{(1)} = 9.38$

$M_{C_4}^{(1)} = 9.56$ (min

The overall results of model example are summarized in Table 3.

SUMMARY AND CONCLUSIONS

The dynamic behavior of gaseous pollutants in ventilated agricultural structures is mainly caused by molecular diffusion, turbulent movement, and circulation of airflow. Also the problem of determination of space air distribution for ventilation systems with multiple inlets and outlets are more difficult, and there does not exist any definition of a local flow rate which would describe the local flow situation. Velocity profiles can answer the problem, but these are usually hard to obtain. Therefore, at any airspace or at any point, air and gaseous pollutants can be presented by age distribution. From the mathematical analysis of age distribution functions and model example for a typical agricultural structure unit, the following conclusions can be drawn.

1. The concept of moements concentration histories of gaseous pollutants (i.e., multiplying concentration reading by time of reading, then integrating with regard to time) is applicable to characterize either the dispersal of the supplied air or gaseous pollutant in the ventilated systems.

2. The mean age of airflow in a ventilated airspace, $\{M\}_I^{(1)}$, can be calculated from the following relation:

$$\{M\}_I^{(1)} = [A]^{-1}\{1\},$$

where $[A]^{-1}$ is an inverse system state matrix. This means the sum elements in an arbitrary row i in matrix $[A]^{-1}$ is equal to the mean age of airflow in airspace i.

3. The mean age of gaseous pollutants in airspace i, $M_{I_i}^{(1)}$, can be obtained from the equilibrium concentration reading in that airspace, $C_i(\infty)$, caused by gaseous pollutant sources as:

$$M_{I_i}^{(1)} = M_{C_i}^{(o)}/C_i(\infty),$$

where $M_{C_i}^{(o)}$ is the area under the concentration profile.

4. Based on the physical and methematical interpretations, the mean age vector of airflow, $\{M\}_I^{(1)}$, can be represented in term of the inverse local airflow matrix, $[W]^{-1}$, transition probability matrix, $[P]$, and air volume matrix, $[V]$, i.e., $\{M\}_I^{(1)} = [W]^{-1}[P][V](1)$. From matrix $[A]^{-1}$, the localairflow rate and transition probabilities of air between airspaces can be calculated.

Table 3. The overall results obtained from model example

Airspace	Purging flow rate (m^3/hr)	Mean age of air (min)	Transition probability	
1	748.13	8.72	**Space 3**	
2	720.29	9.32	0.94 ↑ ↓ 0.91	**Space 2**
3	707.55	9.38	**Space 1**	0.87 ↑ ↓ 0.99
4	750.00	9.56	0.89 ↑ ↓ 0.96	**Space 4**
			Space 2	0.99 ↑ ↓ 0.82
			0.91 ↑ ↓ 0.89	**Space 1**
			Space 3	
			0.86 ↑ ↓ 0.99	
			Space 4	

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