# 由工程控制學觀點分析農業生態系統之動態行為

# An Analysis of the Dynamics of Agroecosystems Via the Viewpoint of Engineering Cybernetics

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**摘　　要**

本文乃從工程控制學觀點以邏輯一數學方法分析一描述農業生態系統動態行爲之假設模式。模式結構之關鍵爲一輸出係數矩陣，〔B(k)〕。由〔B(K)〕矩陣之熱力學特性及主特徵值的數學性質可知此一封閉農業生態系統具有一零成長率，且當廢棄物產生時將會迫使該系統做一適切的矯正。若此農業生態系統之控制程序可由受控之技術變化來定義，則該農業生態系統之行爲便可由線性反饋控制系統來描述。該控制系統之狀態變數爲資源數量，而控制變數則爲廢棄物數量。控制輸出則可由系統之廢棄物函數來表示。由模式反饋機構可顯示出如果農業生態系統具可控制性，則所有資源會受系統的影響，若農業生態系統具不可觀測性，則間接以廢棄物爲其控制量測值對受控程序並不能確定其影響效力。

關鍵詞：農業生態系統，工程控制學，技術，反饋控制系統，可觀測性，可控制性。

**ABSTRACT**

A logical-mathematical analysis of an axiomatic model describing the dynamic behavior of an agroecosystem is presented from the standpoint of engineering cybernetics. The key point of the model structure is an output coefficient matrix [B(k)]. From characterisics of thermodynamics and mathematical properties of dominant eigenvalue of matrix [B(k)], the closed agroecosystem has a zero growth rate and will force an adjustment on itself when wastes are generated. A control process is defined as the controlled technological change. Thus the linear feedback control system can be used to decscribe the behavior of the agroecosystem. The state variables of the control system are resource quantities, and the control variables are residual quantities. Control outputs are yielded by the residuals of the system. The model feedback mechanisms show that if the agroecosystem is controllable, then the every resources are affected by the system. If the agroecosystem is unobservable, the application of residuals as control measures will have indeterminate effects on the controlled processes.

**Keywords:** Agroecosystem, engineering cybernetics, technology, feedback control system, observability, controllability.

## NOTATIONS

[A(k)] a gross input coefficient matrix
[$\Delta A(k)$] a gross input coefficient change matrix
[B(k)] a net output coefficient matrix
[$\Delta B(k)$] a net output coefficient change matrix
{e} the unit vector
[I] the identity matrix
{j(k)} a row vector of control variables
[J(k)] a controllability matrix
[K(k)] an observability matrix
[M(k)] a feedback matrix
{q(k)} a row vector of resource quantities
{qr(k)} a row vector of residual quantities
[X(k)] a gross input matrix
[Z(k)] a net output matrix

## INTRODUCTION

A cybernetic system is a system with feedback (Wiener, 1965; Glorioso, 1988; Mesarovis and Takahara, 1989). A typical feedback system consists of two subsystems. The behavior of this system is governed only by past causes. Its feedback structure may passively or actively make its behavior stable, regulate, and may enable it to damp disturbance. Another type of feedback system, its feedback subsystem is a controller through which information about desired output can be introduced. Actual output information is fed back to the controller, and the actual derivation from desired becomes the basic signals for corrective actions. According to Patten and Odum's work (1981), there is a feedback in the movement of energy-matter in the biosphere, and there are informational processes with which to modulate this environment. The closed-loop structure appears to dominate ecosystem level coevolution, which offers potential for the introduction of system perspectives into the evolutionary process. Thus, they claim that ecosystems are cybernetic systems.

The early evolution of agricultural technologies show that the transformation from hunting and gathering to agriculture has been explanned by archeologists in terms of a controlled feedback process (Bender, 1975). Analogously, the processes of some ecological dynamics also have been analyzed in very similar terms (Bentsman and Hannon, 1988; Hannon, 1986; Mulholland and Sims, 1976).

The rotation of crops, the control of weeds and pets, manipulation of drainage, manuring and fertilization are acknowledgements that interactions occur between organism and their environments in farmed land. Agriculture can make energy and mineral resources out of one area into another and in addition it generates shortages in other self-maintaining system.

If agricultural systems are to maintain productivity, they require a continual supply of the nutrients removed in cropping through shortage in the ecosystems. However, organic or inorganic fertilizers are often applied, not simply at a level sufficient to make up losses but at rates designed to maximize crop production. Such use of fertilizer may create changes in neighbouring ecosystem such as lakes and draingeways into which excess nutrient becomes leached. This change can be severe because ecosystems are not entirely closed system. The effect of residual pollutant within ecosystem must be an important part of any sciencs of agricultural ecosystem or referred to as the agroecosystem (Harper, 1974; Cox and Atkins, 1979; Paul, 1990).

Therefore, one of the main concerns in the connection with material inputs in the agroecosystem today is the question about fate of residuals in the environment. To predict more accurate concentration in the various subystems of the ecosystem is important, whether an agent is likely to degrade or to accumulate or a transfer to groundwater, surfacewater or the iair can occur. Traditional answers to these question can be obtained via simulation of the dynamic behavior o a substance in a given agroecosystem. But if the research topics are emphasized directly on the agroecosystem itself, the meanings of ecosystem in terms of agricultural activities will become more articulate. Relatively few studies have utilized engineering cybernetics theory in the analysis of the dynamics of the agroecosystem.

This paper focuses on how to apply (1) the concepts of cybernetic system (Glorioso, 1988; Mesarovic and Takahara, 1989), and (2) linear systems theory (Chen, 1984) to describe the dynamic behavior of an agroecosystem. The methodology chosen here is in a logical-methamatical sense. That is, it comprises a theorem or body of theorem logically deducible from a set of mutually consistent axioms as opposed to a systematic description of the essential interrelations between the variables of reality.

Hopefully, throughout the investigation of the dynamics of an agroecosystem via the standpoint of engineering cybernetics can be illustrated. It is also hoped that this work will stimulate the applications of an research on the engineering cybernetics to the environmental control of the agroecosystem.

## OBJECTIVES

In specific terms, the purposes of this paper are:

1. Toward a construction theory of an axiomatic structure of the agroecosystem via the viewpoint of engineering cybernetics.

2. Explore the time behavior of the indecomposable agroecosystem bounded by the laws of thermodynamics.

3. Develop a formal model to explore the dynamics of the agroecosystem based on the linear systems theory.

## MODEL ASSUMPTIONS

The essential characteristics of an agroecosystem may be summarized in the following six assumptions.

1. The first and most fundamental assumption is that the agroecosystem is thermodynamically closed. This implies that is exchange energy with its environment, but it materially self contained. No matter can pass into or out of the system. Since matter can be neither created or destroyed, it means that all production in the agroecosystem involves the transformation of a fixed mass according to a set of well-defined physical laws.

2. The agroecosystem is not decomposable. It means all subsystems within the agroecosystem are opened with respect to its environment. Closed subsystems within the system can not be identified. Both energy and matter pass between the processes of any subystem, including the agroecosystem and its nearby environment.

3. The material transformation of the agroecosystem is in terms of a finite number of processes, each of which uses and generates a finite number of resources. It is assumed that it is possible to identify the same number of processes and resources; that each is produced by at least one process; and that each process uses at least one resource.

4. The agroecosystem of material flows is described by a system of energy flows. The material transformations of the system are driven by energy, and are therefore limited by the law of thermodynamics. The value of productive resources is universally a function of entropy change – the useful work that they perform. Although energy is neither created nor destroyed – by the First Law of Thernodymanics. Its availability for useful work is limited by the irreversability of entropic processes – the Second Law of Thermodynamics.

5. The existence of high and low entropy states of matter is consistent. Not all resources degrade at the same rate. The outputs of a particular process will include both new products and partially degraded instruments of productions. Hence, wastes is equal to the difference between the mass of all inputs and the mass of these valued outputs.

6. The set of material transformations undertaken in each period of the system is characterized by fixed coefficients of production. It is intended to show that at any given moment the system is operating with an inherited technology embodied in a set of resourcess.

## MODEL STRUCTURE

### *Model prior knowledge: Technology*

In this paper technology means the prior knowledge that bounds all material transformations of the agroecosystem. It represents the sum of all acquired chemicophysics and genetics, theory of communication and cybernetics, modern theories of algebra and informatics, computers and their languages, etc., (Lyotard, 1984) or recorded knowledge of material transformations.

The technology of the agroecosystem for the kth period can be described by a pair of nonnegative matrices, [A(k)] and [B(k)]. From Assumptions (2) to (6), [A(k)] and [B(k)] are both n-square for all $k \geqslant 0$:

$$[A(k)] = \begin{bmatrix} a_{11} & a_{12} \dots & a_{1n} \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ a_{n1} & a_{n2} \dots & b_{nn} \end{bmatrix} (k), \quad (1\text{-}1)$$

$$[B(k)] = \begin{bmatrix} b_{11} & b_{12} \dots & b_{1n} \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ b_{n1} & b_{n2} \dots & b_{nn} \end{bmatrix} (k), \quad (1\text{-}2)$$

where:

[A(k)] = an indecomposable matrix of gross input coefficients,

[B(k)] = a partially decomposable matrix of net output coefficients,

$\{a_{ij}(k)\}$ = vector of gross input coefficient of the jth resource in the ith process per unit mass of the ith resource available to the system in the kth period.

$\{b_{ij}(k)\}$ = vector of net output coefficient of the jth resource in the ith process per unit mass of the ith resource available to the system in the kth period.

To see the construction of these coefficients, two further matrices, [X(k)] and [Z(k)] are defined. The elements of [X(k)] and [Z(k)], $x_{ij}(k)$ and $z_{ij}(k)$ are denoted as the gross input and the net output of the jth resource in the ith process in the kth period, respectively. A nonnegative, time-dependent, n-dimensional row vector, $\{q(k)\}$, is then defined as follows:

$$\{q(k)\} = \{q_1(k)\ q_2(k)\ \dots\ q_n(k)\}, \quad (2)$$

where

$q_i(k)$ = the mass of the ith resource available to the system at the beginning of the kth period.

The coefficients $a_{ij}(k)$ and $b_{ij}(k)$ are thus defined by,

$$a_{ij}(k) = q_i(k)^{-1} x_{ij}(k),$$
$$b_{ij}(k) = q_i(k)^{-1} z_{ij}(k). \quad (3)$$

In a technologically stationary system, [A(k)] = [A], and [B(k)] = [B]. In a time-varying system:

$$[A(k)] = [A(k-1)] + [\Delta A(k-1)], \quad (4\text{-}1)$$

$$[B(k)] = [B(k-1)] + [\Delta B(k-1)]. \quad (4\text{-}2)$$

where $[\Delta A(k\text{-}1)]$ and $[\Delta B(k-1)]$ are the changes in the coefficients of [A(k−1)] and [B(k−1)], respectively, as the results of the fluctuation of the system.

The elements of $\Delta a_{ij}(k)$ and $\Delta b_{ij}(k)$ may be greater than, equal to, or less than zero depending on whether the input or output coefficients of the jth resource in the ith process is argumented, unchanged, or diminished in the (k+1)th priod.

### *Time behavior of the model*

The mass of the ith resource available to the agroecosystem at the beginning of the kth period, $q_i(k)$, may be greater than, equal to, or less than $q_i(k+1)$ for all k. This implies that output of the ith resource may contract, unchanged, or expand from one period to the next. However, the conservation of mass condition (Assumption 1) means that the combined mass of all the $q_i(k)$, which using the unit vector, $\{e\}$, such as $\{q(k)\}\{e\}$, must be constant for all k. That is,

$$\{q(k)\}\{e\} = \{q(k+1)\}\{e\}. \quad (5)$$

For the n resources, the mass of the output of any process at a given period must be equal to the mass of the inputs of all periods. That is,

$$q_i(k)\{a_{ij}(k)\}\{e\} = q_i(k)\{b_{ij}(k)\}\{e\},$$
$$j \in [1, \dots, n]. \quad (6)$$

To see the systematic implications of Equations (5) and (6), consider the time behavior of an agroecosystem defined by a given technology at which all resources are fully applied in all periods and all resources are produced in positive quantities. This means that the system is technologically stationary.

In all cases the outputs of an agroecosystem in the kth period are given by the first-order difference equation:

$$\{q(k+1)\} = \{q(k)\}\ [B(k)]. \qquad (7)$$

Equation (7) clearly shows that the time path of $\{q(k)\}$ without explicit reference to the input matrix $[A(k)]$. In the special case of a technologically stationary system, the general solution of Equation (7) is (Hosteller, 1988):

$$\{q(k)\} = \{q(0)\}\ [B]^k, \qquad (8)$$

where $\{q(0)\}$ is the vector of initial value of resource mass at k=0.

Equation (8) is referred to as an automatous behavior, i.e., the system automatically repeated activities of the previous periods. The time path $\{q(k)\}$ in such a way is depended on the values of the components of $\{q(0)\}$, and on the structure of [B].

***Physical meanings of matrix [B]***

(i) Suppose matrix [B] is indecomposable. This implies reducing [B] to a block diagonal or block triangular form is impossible. This means that it is impossible to identify any subsystem producing a discrete set of outputs.

As can be seen, when k approaches infinity, $\{q(k)\}$ is convergent. Precisely, $\{q(k)\}$ approaches a left eigenvector of [B] corresponding to the dominant eigenvalue of [B], $\lambda_{max}(B)$ (Gantmacher, 1959). Since [B] is a square nonnegative matrix, it has a dominant eigenvalue: an eigenvalue that is real, positive, and greater in absolute value than all other eigenvalues (Seneta, 1981). The set of all the eigenvalues of [B] can be written as the components of the vector $\{\lambda\} = \{\lambda_1, \lambda_2 \ldots \lambda_n\}$, and let these components be ordered such that $\lambda_{max} = \lambda_1$.

There exists a nonnegative matrix [S], and so a matrix $[T] = [S]^{-1}$, such that $[B] = [S]\ [D_\lambda]\ [T]$, in which the first row of [T], $\{t_1\}$, and the first column of [S], $\{s_1\}$, are the left and right eigenvectors of [B], respectively, corresponding to $\lambda_1$, in which $[D_\lambda]$ is the diagonal matrix formed from the vector $\{\lambda\}$.

By the Perron-Frobenius Theorem (Seneta, 1981), the components of $\{t_1\}$ and $\{s_1\}$ are strictly positive; and matrix [B] is an indecomposable, nonnegative, square matrix. Then Equation (8) can be rewritten as,

(9

$$\{q(k)\} = \{q(0)\}\ [S]\ [D_\lambda]^k\ [T]. \qquad (9)$$

Multiply both sides of Equation (9) by $\lambda_1^{-k}$,

$$\{q(k)\}\ \lambda_1^{-k} = \{q(0)\}\ [S]([D_\lambda]\ \lambda_1^{-1})^k\ [T]. \qquad (10)$$

Since $\lambda_1$ is dominant eigenvalue of [B], as k approaches infinity, $([D_\lambda]\lambda_1^{-1})^k$ tends to

$$[D_1] = \begin{bmatrix} 1 & 0 & \ldots & 0 \\ 0 & 0 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & 0 \end{bmatrix} \qquad (11)$$

Consequently,

$$\lim_{k \to \infty} \{q(k)\}\ \lambda_1^{-k} = \{q(0)\}\ [S]\ [D_1]\ [T]. \qquad (12)$$

Since $\{q(0)\}[S]\ [D_1]$ is a row vector that is positive in its first component only, while the first row of [T] is a strictly positive left eigenvector of [B], for k very large $\{q(k)\}$ becomes very close to a left eigenvector of [B] corresponding to $\lambda_{max}(B)$.

(ii) if matrix [B] is not indecomposable, it may be permuted to the form,

$$[B] = \left[\begin{array}{c|c} [B_{11}] & [B_{12}] \\ \hline [0] & [B_{22}] \end{array}\right] \qquad (13)$$

in which the submatrices $[B_{11}]$ and $[B_{22}]$ are mxn and (n-m)xn matrices, respectively. If $\{q(k)\}$ is partitioned conformably, that is,

$$\{q(k)\} = \{\{q_1(k)\} | \{q_2(k)\}\}, \quad (14)$$

such that $\{q_1(k)\}$ and $\{q_2(k)\}$ are m- and (n-m)-dimensional vectors, respectively. As can be seen that the time path of the first m resources in the system is entirely independent of the time path of the last (n-m) resources. That is,

$$\{q_1(k)\} = \{q_1(0)\} [B_{11}]^k, \quad (5)$$

On the other hand, the time path of the last (n-m) resources depends on all the processes in the system. That is, $\{q_2(k)\}$ is found in the solution to,

$$\{\{q_1(k)\} | \{q_2(k)\}\} = \{\{q_1(0)\} | \{q_2(0)\}\} [B]^k. \quad (16)$$

Therefore, whether the first m processes of the system are significant in the limit depends on the relative potential growth rates of the two subsystems.

(iii) if the potential growth rate of the subsystem described by $[B_{11}]$ is greater than that of the subsystem described by $[B_{22}]$, then the dominant eigenvalue of [B] will be the dominant eigenvalue of the submatrix $[B_{11}]$, and vice versa. If the dominant eigenvalue of [B] is the dominant eigenvalue of $[B_{11}]$, then by a theorem of Gantmacher (1959) on reducible matrices, the left eigenvector of [B] corresponding to $\lambda_{max}(B)$ will be positive. But if the dominant eigenvector of [B] is the same as the dominant eigenvalue of $[B_{22}]$, the left eigenvector of [B] corresponding to $\lambda_{max}(B)$ will be semi-positive and zero in its first m components. This means that, in the limit, the first m processes will be significant in determining the magnitudes of the last (n-m) resources only if the potential growth rate of the first m resources is greater than that of the last (n-m) resources.

(iv) if [B] is indecomposable and $\lambda_{max}(B) > 1$, as k gets very large, the system will get very close to an expansion path at which it will be growing exponentially, with all resources growing at the same rate. The same result will occur if [B] is decomposable and the dominant eigenvalue of $[B_{11}]$ and $[B_{22}]$ have an absolute value greater than one. If [B] is indecomposable and $\lambda_{max}(B) < 1$, the system will collapse completely. The components of $\{q(k)\}$ will converge to zero. If [B] is decomposable, then it will be possible for one subsystem to collapse while the other expands exponentially. The eventual growth rate of an automatous system operating with a given technology is therefore $\lambda_{max}(B) - 1$. The essential point here is that it is only if $\lambda_{max}(B) = 1$ in indecomposable and decomposable, that the components of $\{q(k)\}$ converge in the limit to stable absolute values. In other words, only if growth rate is equal to zero will an agroecosystem not contradict the conservation of mass condition.

(v) The agroecosystem described here is not protected by the assumption of resources required in production are freely available in limitless quantities, i.e., the free gift assumption (Georgescu-Roegen, 1971). The conservation of mass condition requires that the dominant eigenvalue of the output matrix [B] be equal to unity. If the free gifts assumption fails, as it must under the conservation of mass condition, then physical growth rate of the system must be zero. The growth rate of the agroecosystem, G, can be defined as follows:

$$G = (\{q(k-1)\}\{e\} / \{q(k)\}\{e\}) - 1, \quad (17)$$

In view of Equations (5) and (17), the growth rate of the system, G, must be equal to zero.

***Physical meanings of matrix [A(k)]***

(i) The conservation of mass has one very clear implication for the matrix [A] describing the allocation of resources. In a closed agroecosystem, [A(k)] will fully account for all resources in the system in the kth period. It implies that,

$$\{q(k)\} = \{q(k)\} [A(k)], \quad (18)$$

This follows from the fact that in a closed agroecosystem there is no free disposal of resources. Waste material can not be ejected from the system.

To see the difference between this condition and the free disposal case, if the system is protected by the assumption of free disposal, then the

Equation (18) is replaced by the condition:

$$\{q(k)\} \geqslant \{q(k)\} [A(k)], \tag{19}$$

This condition means the possibility that there will be resources generated within the system that will be unused by the system. It assumes that resources that are not used may be costlessly dumped either inside or outside the system. If $\{q(k)\}$ $[A(k-1)]$, the inherited technology will obviously satisfy the conservation of mass condition (18). But if $\{q(k)\} \neq \{q(k)\} [A(k-1)]$, then the inherited technology will not satisfy the conservation of mass condition.

(ii) To obtain $[A(k)] = [A(k-1)]$ only if the vector of residuals,

$$\{q_R(k)\} = \{q(k)\} ([I] - [A(k-1)]) = \{0\}, \tag{20}$$

Equation (6) shows that $\{a_{ij}(k)\}\{e\} = \{b_{ij}(k)\}\{e\}$, for $j \in [1, 2, \ldots, n]$. Therefore, whenever $\{a_{ij}(k)\}\{e\} \neq \{a_{ij}(k-1)\}\{e\}$, then $b_{ij}(k) \neq b_{ij}(k-1)$ for at least one $j \in [1, 2, \ldots, n]$. In other words, the conservation of mass condition (5) insists that any change in the mass of all or any of the inputs to the ith process will be matched by an equivalent change in the mass of any or all outputs of the precess.

(iii) Suppose that there exists excess demands for the ith resource in the kth period. This implies that,

$$\{q_t(k)\} < \{q(k)\}\{a_{ij}(k-1)\}, \quad j \in [1, \ldots, n], \tag{21}$$

then, there exists a scalar $\rho < 1$ such that,

$$\{q_t(k)\} = \{q(k)\} \rho \{a_{ij}(k-1)\}, \quad j \in [1, \ldots, n], \tag{22}$$

Equation (22) shows that by operating the process or processes using the ith resource of $\rho$ of capacity, the condition may be satisfied.

Therefore, the importance of the distribution between Equations (18) and (19) is that a closed agroecosystem will always be bounded by Equation (18), while an open system or a subsystem within an indecomposable agroecossystem may be bounded only by Equation (19). Hence, any agroecosystem subjects to change in the relative mass of resources will be time varying.

In the general case, the time path of the resources of the technologically nonstationary system will not be defined by Equation (8), but by (Hosteller, 1988):

$$\{q(k)\} = \{q(0)\} \prod_{t=0}^{k-1} [B(i)], \tag{23}$$

## MODEL FEEDBACK MECHANISMS

### A linear cybernetic system

This section considers the capacity of controlled technological change to affect the limits imposed by the environment on the agroecoystem. The problem is thus the role of controlled technological change in enabling the agricultural agents to manage their environment to extend the life of resources in fixed supply, and to minimize the damaging effects of waste disposal. In this paper, controlled technological change means the deliberate application of residual quantities in new combinations or forms (Georgescu-Goegen, 1971). Therefore, technological change in the general sense is a function of residuals disposal.

The application of residuals to the agroecosystem to achieve a particular goal is a process that has all the characteristics of a controlled feedback process: the application of a linear combinations of the resources, or state variables, of the agroecosystem to change it from an initial state to some other state. In other words, controlled technological change seeks to change the combination of resources available to the agroecosystem in future periods by changing the combination of resources adavenced now.

The engineering cybernetic description of technological change as a control pocess is straightforward. Substituting Equation (4-2) into Equation (7) yields,

$$\{q(k+1)\} = \{q(k)\} ([B(k-1)] + [\Delta B(k-1)]), \tag{24}$$

where:

$[B(k-1)]$ = inherited technology from the previous period,

$[\Delta B(k-1)]$ = changes in the elements of $[B(k-1)]$ by the residual disposals.

If all technological change is assumed to be controlled, Equation (24) may be written in the form of the state-space representation of a linear feedback control system:

$$\{q(k+1)\} = \{q(k)\} [B(k-1)] + \{j(k)\} [M(k)], \qquad (25)$$

where:

$\{j(k)\}$ = n-dimensional row vector of control variables applied at the beginning of the kth period,

$[M(k)]$ = n-square feedback matrix describing the changes in the elements of $[B(k-1)]$.

The vector of control variables is the vector of residual quantities generated by the system in the kth period under the technology inherited from the (k–1)th period. In view of Equation (20), vector $\{j(k)\}$ can be written as:

$$\{j(k)\} = \{q(k)\} [[I] - [A(k-1)]], \qquad (26)$$

if $[[I] - [A(k-1)]]$ is nonsingular, then the feedback matrix $[M(k)]$ can be written as:

$$[M(k)] = [[I] - [A(k-1)]]^{-1} [\Delta B(k-1)], \qquad (27)$$

When the control output of the system is defined to be the residuals of the system, a complete description of an agroecosystem in terms of the state-space representation can be written to be a linear cybernetic system as follows:

(i) the system equation:

$$\{q(k+1)\} = \{q(k)\} [B(k-1)] + \{j(k)\} [M(k)], \qquad (28\text{-}1)$$

(ii) the output relationship:

$$\{j(k0\} = \{q(k)\} [[I] - [A(k-1)]]. \qquad (28\text{-}2)$$

A block diagram for describing this agroecosystem model is illustrated in Figure 1.

A nonstationary system of this type is said to be controllable if for any initial state $\{q(0)\}$, and any final state, $\{q(f)\}$, there exists a finite period, k, and a control sequencs, $\{j(t)\}$, t=0, 1, . . . , k–1, such that $\{q(k)\} = \{q(f)\}$ (Hosteller, 1988). Generally, such a system may be said to be controllable if it is possible to transform it into a system in which state variables, $q_i(k)$, are dependent on the control vector (Freeman, 1965; Chen, 1984). The structure of the resources produced in the agroecosystem may be brought to a particular state in a finite period through the application of residuals to the system only if the production of all resources in the system is determined by the control variables.

The controllability of such a system implies the knxn controllibility matrix, $[J(k)]$, is of full rank n (Aoki, 1976). The controllability matrix is formed from the sequence of state and feedback matrices as follows (Hosteller, 1988):

$$[J(k)] = \begin{bmatrix} [M(0)] \\ \hdashline [B(0)]\,[M(1)] \\ \hdashline [B(1)]\,[B(0)]\,[M(2)] \\ \hdashline \cdots\cdots \\ \hdashline \prod_{t=0}^{k-2} [B(t)]\,[M(k-1)] \end{bmatrix} \qquad (29)$$

The matrix decribes the effects of the controls applied to the agroecosystem over the k periods of the control sequence. The final state can be determined from the system transition equation giving the general solution of the controlled nonstationary agroecosystem (Hosteller, 1988):

$$\{q(f)\} = \{q(0)\} \prod_{t=0}^{k-1} [B(t)] + \sum_{t=0}^{k-1} \{j(t)\} \left( \prod_{h=0}^{t-1} [B(h)] \right) [M(t0], \qquad (30)$$

The first term on the right-hand said of Equation (30) describes the contribution of the

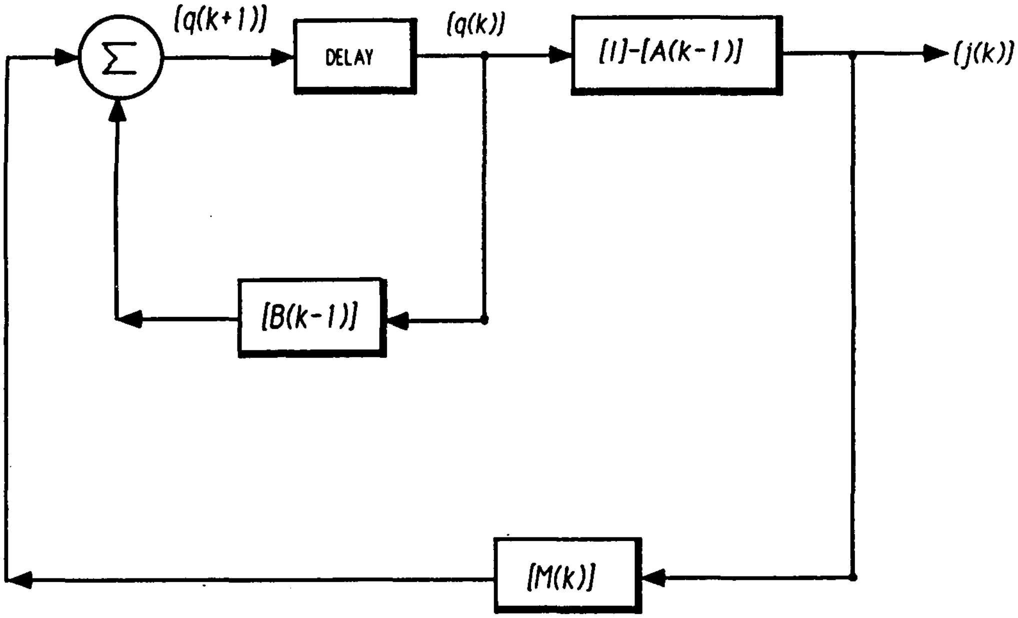


Figure 1. Block diagram showing the relation between signal vectors in a discrete-time state variable agroecosystem model.

initial resource quantities. The second term describes the contribution of the control efforts over the interval (0, k−1). The second term is in facct the product of the knxn controllability matrix [J(k)] and the kn-dimensional vector, {j(0,k)}, formed by combining the control vectors [j(t)] over the same interval.

If the structure of all outputs in the agroecosystem is to be controlled, the vector,

$$\{q(f)\} - \{q(0)\} \prod_{t=0}^{k-1} [B(t)] = \{j\ (0,k)\}\ [J(k)], \qquad (31)$$

will have no zero-valued components and the mtrix [J(k)] must be of full rank (Hosteller, (1988).

Symmetrical to the problem of controllability is the problem of observability. The observability of a system implies that it is possible to determine the state of the system by measuring the signals that are system's outputs in a control sense (Freeman, 1965; Chen, 1984). A system is said to be observable if for any initial state, {q(0)}, and any final state, {q(f)}, there exists a finite period, k, and a control output sequencs, [K(t)], t=0, . . . , k−1, such that a knowledge of [K(t)] and {q(f)} is sufficient to determine {q(0)} (Freeman, 1965). A system will be observable if and only if the observability matrix, [K(k)], formed from the sequance of state and output matrices, is of full rank.

Therefore, in view of Equation (28), the observability matrix, [J(k)], will have the form (Hosteller, 1988):

$$[K(k)] = \begin{bmatrix} [[I]-[A(0)]] \\ \hline [[I]-A(1)]]\,[B(0)] \\ \hline [[I]-[A(2)]]\,[B(1)]\,[B(0)] \\ \hline \cdots\cdots \\ \hline [[I]-[A(k-1)]] \prod_{t=0}^{k-1} [B(t)] \end{bmatrix} \qquad (32)$$

The system will be observable if [[I] −A(h)]] is of full rank n for all h ϵ [0, . . . , k−1]. In other words, the agroecosystem will be observable if the outputs in a control sense are dependent u upon the state variables.

*Control limits*

Considing a subsystem of the agroecosystem described by the submartices, $[A_1(k)]$ and $[B_1(k)]$, are as follows:

$$[A_1(k)] = \left[\begin{array}{c|c} [A_{11}] & [A_{12}] \\ \hline [A_{21}] & [A_{22}] \end{array}\right] (k), \quad (33\text{-}1)$$

$$[B_1(k)] = \left[\begin{array}{c|c} [B_{11}] & [0] \\ \hline [0] & [B_{22}] \end{array}\right] (k), \quad (33\text{-}2)$$

where:

$[A_{11}(k)]$, $[A_{12}(k)]$, and $[B_{11}(k)]$ = the inputs and outputs of the m processes of the system, (mxn) matrix,

$[A_{21}(k)]$, $[A_{22}(k)]$, and $[B_{22}(k)]$ = the inputs and outputs of the last (n-m) processes of the system, 9n-m) xn matrix.

Equation (33) means the technological matrices represent the subsystem of the environment from that of the agroecosystem. It is assumed that the gross input matrix is indecomposable and that the net output matrix is totally decomposable.

Suppose that the outputs of the system in a control sense are the residuals generated by the agroecosystem, i.e., the quantity of agricultural resources in excess supply under the inherited technology of each period. In the tth period they are given by $\{q_1(t)\}[[I]-[A_{11}(t-1)]]$. Hence, the knxn observability submatrix for a sequence of k periods is of the form:

$$[K(k)] = \left[\begin{array}{c} \left[\begin{array}{c|c} [I]-[A_{11}(0)] & [0] \\ \hline [0] & [0] \end{array}\right] \\ \left[\begin{array}{c|c} [I]-[A_{11}(1)] & [0] \\ \hline [0] & [0] \end{array}\right] [B(0)] \\ \left[\begin{array}{c|c} [I]-[A_{11}(k-1)] & [0] \\ \hline [0] & [0] \end{array}\right] \prod_{t=0}^{k-2} [B(t)] \end{array}\right] \quad (34)$$

This is of rank m at most, since the rank of product matrix can not exceed the rank of each factor matrix (Gantmacher, 1959). Therefore, the system is not observable.

Similary, let the control variables of the system be selected from the same vector of control outputs, but assume that the agricultural agents of the agroecosystem have the option of disposing the residuals as waste. In other words, let $\{j_j(t)\} < \{q(t)\}[a_{ij}(t-1)]$ for all $\{q_i(t)\} - \{q(t)\}[a_{ij}(t-1)] > 0$, $i \in [1, \ldots, m]$, and $j \in [1, \ldots, n]$. The knxn controllability matrix, $[J(k)]$, is also of rank m at most:

$$[J(k)] = \left[\begin{array}{c} \left[\begin{array}{c|c} [M(0)] & [0] \\ \hline [0] & [0] \end{array}\right] \\ [B(0)] \left[\begin{array}{c|c} [M(1)] & [0] \\ \hline [0] & [0] \end{array}\right] \\ \cdots\cdots \\ \prod_{t=0}^{k-2} [B(t)] \left[\begin{array}{c|c} [M(k-1)] & [0] \\ \hline [0] & [0] \end{array}\right] \end{array}\right] \quad (35)$$

The vector $\{j(0,k)\}[J(k)]$ is also at most positive in its first m components. This implies that the last (n-m) resources in the agroecosystem ar are not governed directly by the control variables.

Equations (34) and (35) means that the subsystem within an agroecosystem has a limited set of observations on the state of the agroecosystem. If the subsystem controls the outputs of only a limited set of resources, then it can not determine the performance of the agroecosystem.

## SUMMARY AND CONCLUSIONS

A logical-mathematical analysis of an axiomatic model describing the dynamic behavior of an agroecosystem is presented from the standpoint of engineering cybernetics. The following conclusions can be drawn.

1. The system equation of an agroecosystem can be written in the form of the state-space representationof a linear feedback control system by a discrete-time vector-matrix difference equation as:

$$\{q(k+1)\} = \{q(k)\} [B(k-1)] + \{j(k)\} [M(k)],$$

and control outputs are yielded by the residual of the system as:

$$\{j(k)\} = \{q(k)\} [[I] - [A(k-1)]],$$

in which $\{q(k)\}$ is a row vector of resource quantities (state variables), $[B(k)]$ is a net output coefficient matrix, $\{j(k)\}$ is a row vector of residual quantities (control variables), $[M(k)]$ is a feedback matrix, and $[A(k)]$ is a gross input coefficient matrix.

2. From physical and mathematical properties of the dominant eigenvalue of matrix $[B(k)]$ and constraint of mass conservation, a closed agroecosystem has a zero growth rate. From physical meanings of matrix $[A(k)]$, the agroecosystem will force an adjustment on itself whenever wastes are generated. This implies that an agroecosystem generating residuals does not have the option of standing still.

3. If the agroecosystem is controllable, it must be possible to affect every resource in it. If the feedback matrix describing the technological changes associated with the control are of less than full rank, the controls will not reach all the resources produced in the system. The agroecosystem will be observable if the outputs in a control sense are dependent upon the state variables.

4. If the agroecosystem is unobservable, the application of residuals as control measures will have indeterminate effects on the controlled processes. That is, technological change informed by the system's signals based on the existence of residuals in the agroecosystem can not have determinate effects on the environment.

5. If the subsystem within an agroecosystem has a limited set of observations on the state of the agroecosystem, and if it controls the outputs of a limited set of resources, then it can not determine the performance of the agroecosystem.

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