



# 恆率乾燥期間非壓實多孔物質之水份分佈 ——擴散理論之不適用\*

## Moisture Distribution During the Constant Rate Drying Period for Unconsolidated Porous Media

### ——Failure of the Diffusion Theory

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#### 摘 要

在進行非壓實多孔物質之乾燥時，一般都會經歷恆率乾燥期。在此期間，乾燥率是受外在質量傳遞所控制，而在多孔物質內水份之移動則由毛細作用力所支配。因此針對本份之內在移動有兩個有趣的問題：(1)是否恆率乾燥過程為準穩態？(2)可否以擴散方程式來做理論模擬？其答案如何？本文將以傳動方程式及邊界條件之理論探討與數值解來做充份的說明。

#### Abstracts

When drying unconsolidated porous media, one usually encounters a constant rate period. During this period the rate of drying is controlled by external mass transfer and moisture transport within the porous media is dominated by capillary forces. Two questions are posed concerning the internal moisture transport: (1) Is the process quasi-steady? (2) Can the process be modelled with a diffusion equation? Answers are provided in terms of theoretical and numerical studies of the governing transport equation and boundary conditions.

#### INTRODUCTION

In the analysis of drying various materials, one sometimes encounters the

simple system illustrated in Fig. 1. It consists of a rigid, inert solid phase, a pure liquid phase (water), and a vapor phase consisting of the evaporating species

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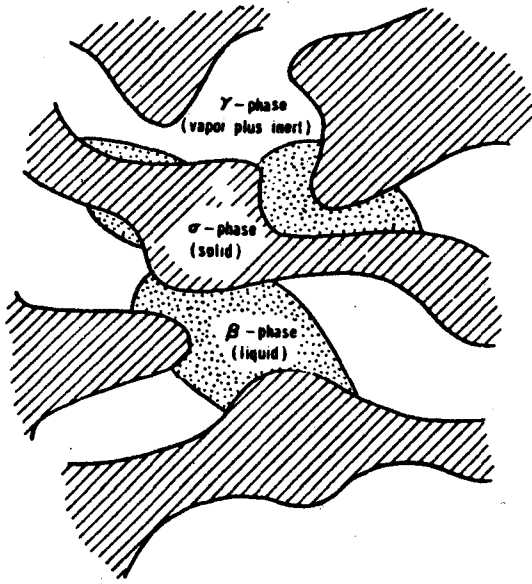


Figure 1. Three-phase system.

(water) and an inert gas (air). For such systems the transport of moisture can be rigorously described (Whitaker, 1977 and 1980) in terms of a saturation transport equation that takes the form

$$\frac{\partial S}{\partial t} = \nabla \cdot \left( \underset{\text{capillary action}}{\underset{\sim}{K}}_S \cdot \nabla S + \underset{\text{Kelvin effect}}{\underset{\sim}{H}} \left( \frac{\partial \langle \omega_1 \rangle^\gamma}{\partial S} \right) \cdot \nabla S + \underset{\text{Marangoni effect}}{\underset{\sim}{K}}_T \cdot \nabla \langle T \rangle \right) + \underset{\text{Clausius-Clapeyron effect}}{\underset{\sim}{H}} \left( \frac{\partial \langle \omega_1 \rangle^\gamma}{\partial \langle T \rangle} \right) \cdot \nabla \langle T \rangle - \underset{\text{gravity effect}}{\underset{\sim}{K}}_B \cdot \underline{\underline{g}} \quad (1)$$

Here  $S$  is the saturation defined by

$$S = \frac{\epsilon_\beta \rho_\beta + \epsilon_\gamma \langle \rho_1 \rangle^\gamma}{\rho_\beta (1 - \epsilon_\sigma)} \quad (2)$$

and  $\langle \omega_1 \rangle^\gamma$  is a mass fraction given by

$$\langle \omega_1 \rangle^\gamma = \langle \rho_1 \rangle^\gamma / \langle \rho_\gamma \rangle^\gamma \quad (3)$$

In both Eqs. 2 and 3 we have used  $\langle \rho_1 \rangle^\gamma$  to represent the intrinsic phase average

density of the evaporating species.

The first term on the right hand side of Eq. 1 describes the liquid phase transport owing to capillary action. The coefficient  $\underset{\sim}{K}_S$  is given by

$$\underset{\sim}{K}_S = - \frac{K_T K_\beta^0 (\partial p_c / \partial S)}{\mu_\beta (1 - \epsilon_\sigma)} \quad (4)$$

in which  $\underset{\sim}{K}_\beta^0$  represents the single phase Darcy's law permeability tensor. The relative permeability,  $K_T$ , is illustrated in Fig. 2 and the capillary pressure,  $p_c$ , for a

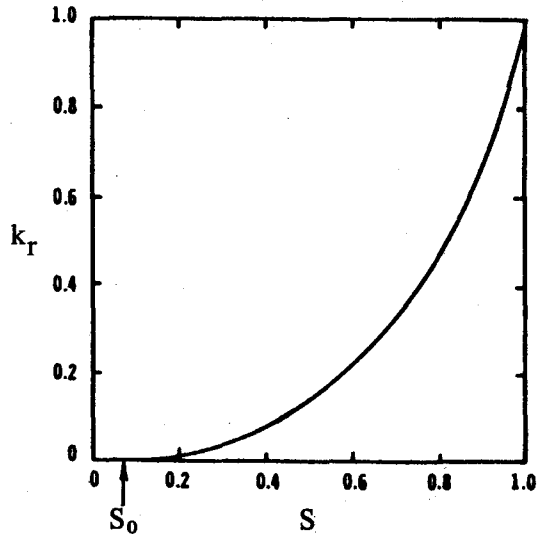


Figure 2. Relative permeability for the wetting phase

drainage process is shown in Fig. 3. While recent studies (Whitaker, 1984) suggest that the traditional representations used for  $k_T$  and  $p_c$  may not be appropriate when the saturation is in the neighborhood of  $S_0$ , they will suffice in this study since we will be confined to values of the saturation greater than  $S_0$ .

The second term on the right hand side of Eq. 1 represents the gas-phase diffusive transport caused by the Kelvin effect, while the third term represents a liquid phase flow caused by surface tension

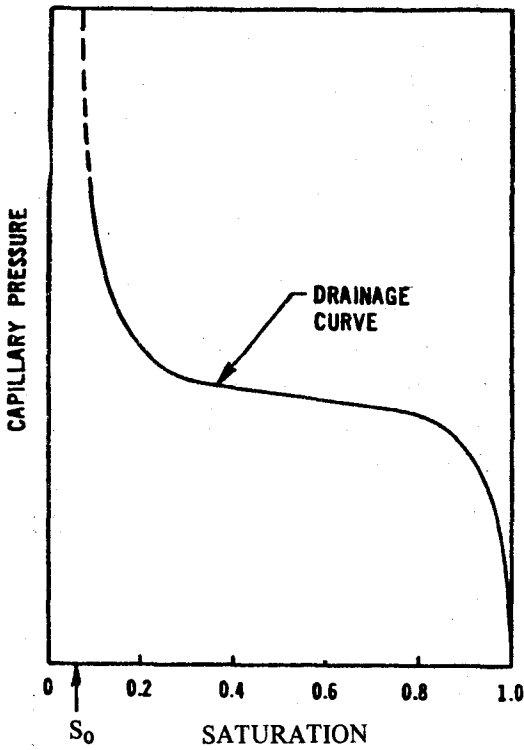


Figure 3. Capillary pressure – saturation curve

gradients (the Marangoni effect). Because surface tension depends on the temperature, this contribution to the liquid phase flow is proportional to the temperature gradient. The fourth term on the right hand side of Eq. 1 represents a vapor-phase diffusive flux caused by gradients of the partial pressure of the evaporating species. For a system in a state of local thermal equilibrium (Whitaker, 1986a), the partial pressure depends on the temperature, and the gradient of the partial pressure is therefore related to the temperature gradient giving rise to the Clausius-Clapeyron effect. The last term in Eq. 1 represents the gravitational effect. Since the flow in the liquid phase is described by Darcy's law (Whitaker, 1986b) the effect of gravity on the moisture transport naturally appears in

the saturation transport equation. It is of some interest to note that Darcy's law for the gas phase has not been used in the derivation of Eq. 1. Arguments supporting this simplification were originally given by Whitaker (Sec. 5, 1980) and have been recently placed on a sound theoretical basis (Whitaker, 1984).

It is important to keep in mind that Eq. 1 represents moisture transport phenomena in the region where liquid and vapor co-exist. In the dry region of a porous medium, the saturation transport equation is greatly simplified and the interested reader is referred to the work of Whitaker and Chou (1983). In order to solve Eq. 1, one must also solve the volume averaged thermal energy equation (Whitaker, Sec. 3, 1980) in order to determine the temperature gradient. However, during the constant rate period, liquid phase transport dominates and the vapor phase transport can be neglected. In addition, temperature gradients within the porous medium are negligible and the Marangoni effect can be ignored. For the system shown in Fig. 4, the constant rate drying period can be described by



Figure 4. One-dimensional drying

$$\frac{\partial S}{\partial t} = \frac{\partial}{\partial x} \left( K_s \frac{\partial S}{\partial x} - K_g g_x \right) \quad (6)$$

$$\text{B.C.1} \quad K_s \frac{\partial S}{\partial x} - K_g g_x = 0, \quad x = 0 \quad (7)$$

$$\text{B.C.2} \quad - \left( K_s \frac{\partial S}{\partial x} - K_g g_x \right) = \frac{K_G}{\rho_\beta (1 - \epsilon_\sigma)} \left( \langle \rho_1 \rangle^\gamma - (\rho_1)_\infty \right), \quad x = L \quad (8)$$

The first boundary condition requires that the moisture flux is zero at  $x = 0$ , while B.C.2 represents the flux at  $x=L$  in terms of a mass transfer coefficient,  $K_G$ , and the driving force,  $\langle \rho_1 \rangle^\gamma - (\rho_1)_\infty$ . In the absence of a solution to the energy equation, one is forced to assume that the temperature at  $x=L$  is the wet-bulb temperature so that the Clausius-Clapeyron equation can be used to determine  $\langle \rho_1 \rangle^\gamma$  at  $x=L$ .

The simplification that we have achieved in arriving at Eqs. 6 through 8 is enormous, and it is reassuring to know that in a recent, exhaustive study of the complete problem (Pilitsis, 1986), it has been demonstrated that Eqs. 6 through 8 represent a reasonable model of the moisture transport process during the constant rate period. However, one must keep in mind that Eqs. 6 through 8 apply *only* to coarse, inert materials such as sand for which the problems of adsorbed or bound moisture can be ignored.

One of the interesting special cases that *might be* associated with Eqs. 6 through 8 is the case of *dominant capillary forces*. This situation can be described by the inequality

$$K_s \frac{\partial S}{\partial x} \gg K_g g_x \quad (9)$$

and it is this inequality that was used by Whitaker (1977b) to suggest a diffusion theory of drying. When Eq. 9 is imposed on Eqs. 6 through 8, we obtain

$$\frac{\partial S}{\partial t} = \frac{\partial}{\partial x} \left( K_s \frac{\partial S}{\partial x} \right) \quad (10)$$

$$\text{B.C.1} \quad \frac{\partial S}{\partial x} = 0, \quad x = 0 \quad (11)$$

$$\text{B.C.2} \quad -K_s \frac{\partial S}{\partial x} = \frac{K_G}{(1 - \epsilon_\sigma)} (\langle \rho_1 \rangle^\gamma - (\rho_1)_\infty), \quad x = L \quad (12)$$

This boundary value problem represents an ancient model of drying first proposed by Lewis (1921). The crucial question to ask before embracing a diffusion theory of drying for unconsolidated material is: Do circumstances exist for which Eq. 9 is valid? Our response is given in the next section and is limited to systems which can be described by Eq. 1.

## THEORY

If one imposes the inequality suggested by Eq. 9, the moisture transport process during the constant rate period is clearly *unsteady* as indicated by Eqs. 10 through 12. The *antithesis* of Eq. 9 is the equality given by

$$K_s \frac{\partial S}{\partial x} = K_g g_x, \quad 0 \leq x \leq L \quad (13)$$

Under these circumstances the saturation distribution,  $S(x,t)$ , is *quasi-steady* and can be determined directly by Eq. 13 for any specified average saturation. This approach was utilized by Ceaglske and Hougen (1937) to determine saturation profiles that were in excellent agreement with experimental values.

Certainly it is of interest to ask if Eq. 9 is possible and a *diffusion theory* of drying results as indicated by Eqs. 10 through 12, or whether Eq. 13 is possible and a quasi-steady *hydrostatic theory* of drying results. The crux of the matter is that Eq. 9, along with the boundary condition given by Eq. 8, implies that

$$K_{g\beta x} \ll \frac{K_g}{(1-\epsilon_\sigma)} (\langle \rho_1 \rangle \gamma - (\rho_1)_\infty), x=L \quad (14)$$

It is important to note that Eq. 9 indicates that "capillary forces are large compared to gravitational forces" while Eq. 14 represents an entirely independent constraint imposed by a *boundary condition*. In order to determine whether both Eq. 9 and Eq. 14 can be satisfied, it will be convenient to express the general problem in dimensionless form. To do so, we express the relative permeability as

$$K_r = \left( \frac{S - S_0}{1 - S_0} \right)^3 \quad (15)$$

and make Eqs. 6 through 8 dimensionless to obtain

$$\frac{\partial S}{\partial \theta} = \frac{\partial}{\partial X} \left\{ \left( \frac{S - S_0}{1 - S_0} \right)^3 \left[ - \left( \frac{\partial H}{\partial S} \right) \left( \frac{\partial S}{\partial X} \right) + 1 \right] \right\} \quad (16)$$

$$\text{B.C.1} \quad \left( \frac{\partial S}{\partial X} \right) = \left( \frac{\partial H}{\partial S} \right)^{-1}, X=0 \quad (17)$$

B.C.2

$$\left( \frac{\partial S}{\partial X} \right) = \left( \frac{\partial H}{\partial S} \right)^{-1} \left[ 1 + \Omega \left( \frac{1 - S_0}{S - S_0} \right)^3 \right], X=1 \quad (18)$$

Here  $H$  represents the dimensionless capillary height given by

$$H = p_c / \rho_\beta g L = h_c / L \quad (19)$$

while  $\Omega$  represents the dimensionless mass flux defined by

$$\Omega = \frac{K_G (\langle \rho_1 \rangle \gamma - (\rho_1)_\infty) \mu_\beta}{\rho_\beta^2 g K_\beta^0} \quad (20)$$

Clearly the dimensionless length is given

by  $X = x/L$  and in order to complete the problem statement we need an initial condition which was taken to be

$$\left. \begin{aligned} S=1, X=0 \\ \left( \frac{\partial S}{\partial X} \right) = \left( \frac{\partial H}{\partial S} \right)^{-1}, 0 \leq X \leq 1 \end{aligned} \right\} \theta=0 \quad (21)$$

This condition requires that the saturation be unity at the bottom of the layer illustrated in Fig. 4 and that the dimensionless flux given by

$$\left( \frac{\text{dimensionless}}{\text{flux}} \right) = \left( \frac{\partial H}{\partial S} \right) \left( \frac{\partial S}{\partial X} \right) + 1 \quad (22)$$

be zero at  $\theta = 0$ . In order to determine how closely the solution of Eqs. 16 through 20 resembles the quasi-steady profiles, we compared our general results with those given by the dimensionless form of Eq. 13 which is

$$\left( \frac{\partial S}{\partial X} \right) = \left( \frac{\partial H}{\partial S} \right)^{-1}, 0 \leq X \leq 1 \quad (23)$$

Before presenting some of our numerical results, we should point out that Eqs. 16 through 18, along with Eqs. 21, clearly indicate that the departure of the complete solution from the quasi-steady solution can only be significant when

$$\Omega \left( \frac{1 - S_0}{S - S_0} \right)^3 \geq 1, \text{ non-quasi-steady drying} \quad (24)$$

While this situation can occur when  $S$  approaches  $S_0$ , only a small portion of the entire drying process occurs under this condition.

## CALCULATIONS

The numerical solution of Eq. 16 is described in detail by Chen (1982) and here we will only present some of the results. In order to compare theory with experiment, we have used a single phase permeability given by (Bear, 1972)

$$K_{\beta}^g = 6.17 \times 10^{-4} d_p^2 \quad (25)$$

and expressed the capillary height as

$$h_c = p_c / \rho_{\beta} g = P \left\{ 1 - \exp [-Q(1-S)] \right\} + R(1-S)^Z + U(S-S_0)^{-F} \quad (26)$$

When the parameters are chosen according to

$$P = 4.9 \text{ cm}, Q = 40.0, R = 1.8 \text{ cm}, \\ U = 0.08 \text{ cm}, F = 1.15, Z = 1.0, S_0 = 0.09$$

one obtains good agreement with the capillary pressure measured by Ceaglske and Hougen (1937). From Ceaglske (1936) we obtain the following parameters associated with the experimental saturation profiles illustrated in Fig. 5.

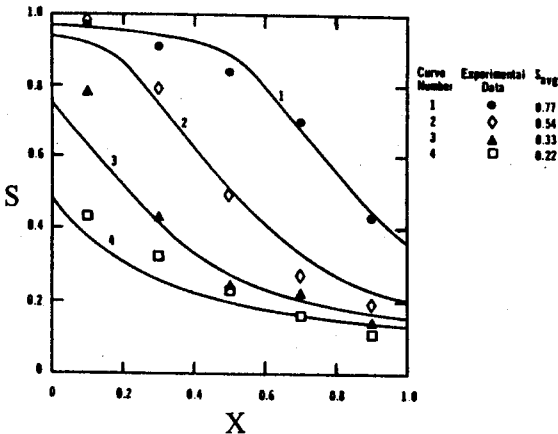


Figure 5. Comparison Between Theory and Experiment for the Data of Ceaglske and Hougen

$$L = 2.54 \text{ cm}, d = 1280 \mu\text{m}, \\ \Omega = 2.07 \times 10^{-4}, 1 - \epsilon_o = 0.41$$

Since Ceaglske and Hougen used sand in their drying studies we should expect good agreement between theory and experiment. In addition, we should also expect the good agreement between the complete theory and the quasi-steady solution given by Eq. 23 both because  $\Omega$  is so small compared to one and because Ceaglske and Hougen had demonstrated this fact in 1937. The agreement is shown in Fig. 6.

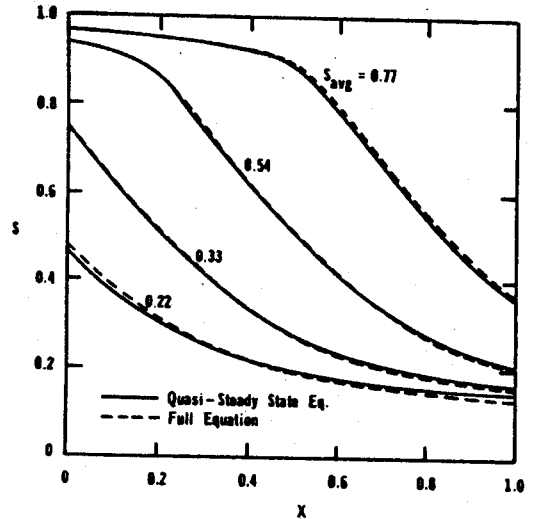


Figure 6. Comparison of the Complete Theory with the Quasi-Steady Solution

In earlier studies (Whitaker, Sec. VII, 1977a; Whitaker 1977b) it was suggested that a diffusion theory of drying would result if gravitational effects were negligible. One might think that this situation could be achieved with very large values of the capillary pressure, and to explore this possibility we used Eq. 26 along with the parameters

$$P = 49 \text{ cm}, R = 18 \text{ cm}, U = 0.8 \text{ cm}$$

while holding the remaining parameters in Eq. 26 constant. This represents a ten-fold increase in the ratio of capillary to gravitational effects and the computed saturation profiles for  $\Omega = 0.05$  are shown in Fig. 7. Obviously the profiles shown in

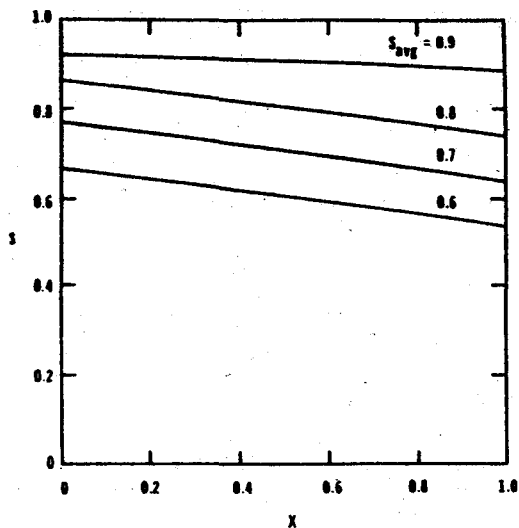


Figure 7. Theoretical Saturation Profiles for a Ten-Fold Increase in the Capillary Height

Fig. 7 do not result from a diffusion equation, and when the capillary height is further increased by the use of

$$P = 490 \text{ cm}, R = 180 \text{ cm}, U = 8 \text{ cm}$$

we obtain the flat profiles illustrated in Fig. 8. Obviously these profiles represent a solution to Eq. 23 with  $\partial H/\partial S$  becoming very large compared to one.

It should be clear at this time that it is the flux boundary condition given by Eq. 18 that controls the nature of the saturation transport process. One might think that if  $\Omega$  were made sufficiently large, a diffusion-like process would be a possibility. However, our results for various values of  $\Omega$  shown in Fig. 9 indicate only small deviations (up to 5.6% for  $\Omega = 0.05$ )

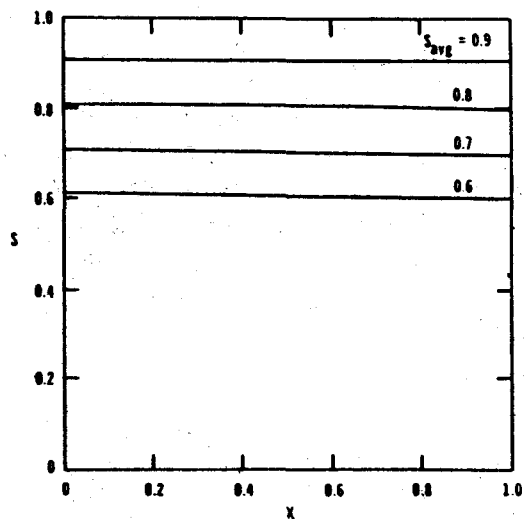


Figure 8. Theoretical Saturation Profiles for a Hundred-Fold Increase in the Capillary Height

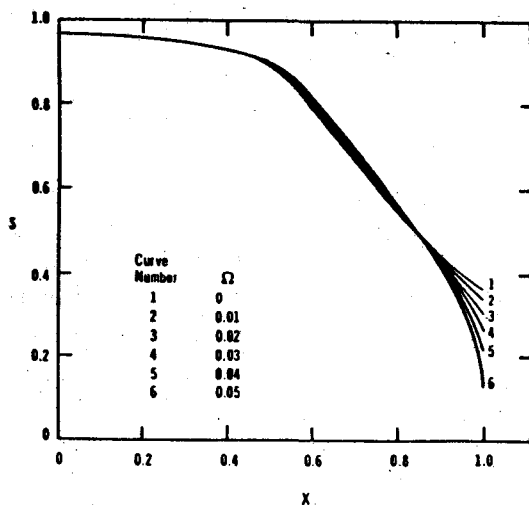


Figure 9. Influence of the Interfacial Flux on the Saturation Profile

from the quasi-steady solution. Obviously one can obtain significant *local* deviations from the quasi-steady solution by making  $\Omega$  sufficiently large, but large values of  $\Omega$  quickly drive the surface saturation to  $S_0$  and the constant rate period is terminated. Vapor phase diffusion within the porous medium takes over as the controlling mechanism, and in regions where

$S$  is greater than  $S_0$  the liquid is redistributed in a quasi-steady manner.

## CONCLUSIONS

The process of drying unconsolidated, inert solids during the constant rate period appears to be a self-controlled operation in which capillary forces and gravitational forces are balanced. This allows one to use a quasi-steady analysis to predict the saturation profiles during the constant rate period.

## ACKNOWLEDGEMENT

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## NOMENCLATURE

$d_p$	effective particle diameter, m
$g$	gravitational constant, $m/s^2$
$h_c$	capillary height, m
$H$	$h_c/L$ , dimensionless capillary height
$K_{\beta}^0$	Darcy's law permeability tensor, $m^2$
$K_r$	relative permeability
$L$	depth of porous medium, m
$p_c$	capillary pressure, $N/m^2$
$S$	saturation
$S_0$	residual saturation
$t$	time, s
$x$	distance, m
$X$	$x/L$ , dimensionless distance
$\epsilon_{\sigma}$	volume fraction of solid phase
$\epsilon_{\beta}$	volume fraction of liquid phase
$\epsilon_{\gamma}$	volume fraction of vapor phase
$\mu_{\beta}$	viscosity of liquid, $Ns/m^2$
$\rho_{\beta}$	density of liquid, $kg/m^3$
$\langle \rho_1 \rangle^{\gamma}$	density of evaporating species in the vapor phase, $kg/m^3$
$(\rho_1)_{\infty}$	density of the evaporating species

	far from the porous medium, $kg/m^3$
$\langle \omega_1 \rangle^{\gamma}$	mass fraction of the evaporating species
$\Omega$	dimensionless interfacial mass flux

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