專 論

洪水頻率之統計參數

Statistical Parameters For Flood Frequency

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摘 要

作爲洪水頻率分析之應用，本文提供一個估計平均值和標準偏差之方法。這兩個參數的估計方法是使得相同頻率的實測洪水和預估洪水的相差的平方和爲最小。本文以常態、對數常態，極值和對數極值作爲洪水的或然率分佈，和以威伯（Weibull）的公式來闡釋本方法之應用，並以臺灣的六條河流的洪水資料作實際上的比較。它的結果，正爲理論所顯示，本文所提供統計參數的估計方法比現在一般通用的平均值和標準偏差在洪水頻率分析的效果更好。

Abstract

A method of estimating mean and standard deviation is developed for flood frequency analysis which will make the summation of deviation squares between the observed floods and estimated floods with a certain plotting formula minimum. The normal, log-normal, extreme value, and log-extreme value distribution are assumed for the probability distribution of flood, and the Weibull plotting formula is used for illustration the method. The results of flood frequency analysis for several Taiwan watersheds reveal that the summations of deviation squares for floods estimated by the mean and the standard deviation now commonly used are larger than those for floods estimated by the proposed statistical parameters in this study.

I. Introduction

A flood magnitude-frequency relationship is essential in the design of spillways, highway bridges, culverts, and other flood control structures, or in the planning of flood plain zoning. The method of determining the magnitude-frequency relationship usually used, is to assume a theoretical or an empirical probability distribution for the population of the events and to estimate the statistical parameters thereof from historical data.

The Annual maximum values and the annual exceedence values are often the two types of data selected from the complete-duration series for flood frequency analysis.

As proposed by Chow (1951), a general equation for flood frequency analysis is often used:

$$Q_T = \bar{Q} + KS_Q \quad \cdots\cdots (1)$$

where: Q_T is the magnitude of the flood at a return period T,
K is the frequency factor, depending on the return period and the distribution characteristics,
$\bar{Q}$ is a sample mean of the flood,
S_Q is a sample standard deviation of the flood.

The two statistical parameters, mean and standard deviation, are estimated from historical data by the following two equations:

$$\bar{Q} = \frac{\Sigma Q_i}{N} \quad \cdots\cdots (2)$$

$$S_Q = \left\{ \frac{\Sigma (Q_i - \bar{Q})^2}{N-1} \right\}^{\frac{1}{2}} \quad \cdots\cdots (3)$$

where: Q_i are the observed floods in the annual maximum or the annual exceedence series, and N is the number of years in the record.

For any chosen probability distribution, a relationship can be derived between the return period and the corresponding frequency factor. This relationship is sometimes referred to as the K-T relationship. The return period, or recurrence interval, T, of a given flood magnitude is the average interval of time within which the magnitude of the flood will be equaled or exceeded once. A flood magnitude Q for a given return period T can be determined from Eq. 1 with K found by K-T relationship and the computed statistical parameters from Eqs. 2and 3.

In general, either the method of moments or the method of maximum likelihood is used to estimate the statistical parameters. For a normal distribution, the parameters estimated by the method of moments and by the maximum likelihood methood can be shown to be identical. For some other types of distribution, however, the estimations of parameters by the maximum likelihood method are generally more complex than by the method of moments. In the maximum likelihood method, the parameters of some distributions can not be determined analytically and have to be solved for by the iteration method. Therefore, in practice, it is quite often that the method of moments rather than the maximum likelihood method is used to estimate the statistical parameters for flood frequency analysis.

The objective of this study is to propose the least squares method of estimating statistical parameters for flood frequency analysis. The estimation of statistical parameters is to make the summation of squares of deviations between the observed floods and the estimated floods with a cartain plotting formula minimum. In this study, the normal, log-normal, extreme value, and log-extreme value distribution are assumed for the probability distribution of floods, and the Weibull plotting formula is used for illustrating the method. A comparison of the results of flood frequency analysis for several Taiwan watersheds is made between the statistical parameters estimated by the method of moments and by the proposed least squares method.

II. Plotting Formula

In a series of N annual exceedence floods, the largest flood in the record occurred only once. But in the other series of equal length, N, the same magnitude of flood might occur several times or not at all. In other words, the maximum flood observed in a 50-year record may have a true return period of 10 years, 50 years,100 years or any other number of years, each associated with a different probabilities.

The question then arises as to the true return period of each of the set of annual floods. This true return period for each event is, however never known, but can be estimated in several ways. For example, the sample frequency in its population may be considered to correspond directly to their observed frequency. That is the maximum event in a series of N independent annual maxima or annual exceedence would have a return period of N and a probability of 1/N. This is known as the California method and is given by

$$p=m/N, \text{ or } T=N/m \quad \cdots\cdots (4)$$

where m is the order of the floods, m being 1 for the largest and N for the smallest event in N years of record.

As given by Kite (1977), if the observed frequency of the maximum event in a series of N independent events is considered to be the mean of the population of frequencies for the maximum event, then

$$p=\int_0^1 (1-Z^{1/N})=1/(N+1)$$

where Z is the probability of occurrence of the event in the N-year record.

Use of the mean frequency thus leads to a general equation given by

$$p=m/(N+1), \text{ or } T=(N+1)/m \quad \cdots\cdots (5)$$

This is commonly known as thè Weibull formula for plotting position of floods.

Some other consideration of the observed frequency being the mode or the median of all possible frequencies will give other kind of plotting formula.

For this reason‘ the frequency factor, K, which depending on the return period and the distribution characteristics, is considered in this study to be known for each event in N independent events series.

III. Estimation of Statistical Parameters

For a given plotting formula used and a probability distribution of floods assumed, the frequency factor of each observed flood in series can be determined.

For the purpose of deriving a new relation for estimating the mean and the standard deviation of flood, the general frequency equation given in Eq. 1 is expressed as

$$Q_i=\alpha+K_i\beta \quad \cdots\cdots (6)$$

where α and β are the statistical parameters corresponding to the mean and the standard deviation of a flood series, Q_i is the ith order of flood, Q_1 for the largest flood, and Q_N for the smallest flood, K_i is the frequency factor of ith order of flood, K_1 is the frequency factor of the maximum flood, K_N is the frequency factor of the smallest flood in N-year record.

The determination of α and β is to make that the summation of squares of deviations between recorded floods and estimated floods by Eq 6 for a given plotting formula and a probability distribution of floods to be minimum. In other words, α and β in Eq. 6 are obtained to give a best fit of obseved floods to a giver plotting positions. Based on this consideration, α and β should be obtained by solving the following two normal equations:

$$\frac{\partial}{\partial\alpha}\Sigma\delta_i^2=\frac{\partial}{\partial\alpha}\Sigma\left\{Q_i-\alpha-K_i\beta\right\}^2=0,$$

or

$$\Sigma Q_i-N\alpha-\beta\Sigma K_i=0 \quad \cdots\cdots (7)$$

and

$$\frac{\partial}{\partial\beta}\Sigma\delta_i^2=\frac{\partial}{\partial\beta}\Sigma\left\{Q_i-\alpha-K_i\beta\right\}^2=0,$$

or,

$$\Sigma Q_iK_i-\alpha\Sigma K_i-\beta\Sigma K_i^2=0 \quad \cdots\cdots (8)$$

By solving Eqs. 7 and 8 for α and β, it yields

$$a=\overline{Q}-\overline{K}\{Cov(Q\,K)/Var(K)\} \quad \cdots\cdots (9)$$

and

$$\beta=Cov(Q,K)/Var(K) \quad \cdots\cdots (10)$$

where:

$$\overline{Q}=\frac{1}{N}\Sigma Q_i,$$

$$\overline{K}=\frac{1}{N}\Sigma K_i,$$

$$Var(K)=\frac{1}{N}\Sigma K_i^2-(\overline{K})^2,$$

$$Cov(Q,\,K)=\frac{1}{N}\Sigma Q_iK_i-(\overline{Q})(\overline{K}).$$

Eq. 9 for α may be written as

$$\alpha=\overline{Q}-\overline{K}\beta \quad \cdots\cdots (11)$$

For a symmetrical probability distribution, $\overline{K}=0$ Eq. 11 shows that $\alpha=\overline{Q}$. It seems that the mean traditionally defined as in Eq. 2 is a special case of a defined in this study.

The relations for determining α and β developed in this study, based on the least squares method, are to be used for mean and standard deviation, respectively, for flood frequency analysis. The value of β is defined as the ratio of covariance of floods and its corresponding frequecy factors to the variance of frequency fact ors. The value of α is defined as the difference in the mean of floods and the product of mean of frequency factors and β.

IV. Practical Application and Comparison of Results

In order to show its applications and to compare the difference in the results in flood frequency analysis by the different methods of estimating statistical parameters, the data of annual maximum and annual exceedence flood series with various length of record collected at several Taiwan watersheds are used. Four different types of probability distributions are assumed for the distribution of flood. The Weibull plotting formula is used to determine the frequency factor of each event.

A. Normal Probability Distribution

If Q_i are considered to follow the normal probability distribution, its probability density function, f(Q), is

$$f(Q)=\frac{1}{S_Q\sqrt{2\pi}}\left\{\text{Exp }\{-(Q-\bar{Q})^2/2\,S_Q^2\}\right\} \quad \cdots\cdots (12)$$

According to the Weibull plotting formula, the probability that Q will be exceeding Q_i, the ith order of magnitude of flood in N years record, is

$$P_r\{Q\leq Q_i\}=i/(N+1)=\int_{Q_i}^{\infty} f(Q)\,dQ$$

$$=1-\Phi\left\{\frac{Q_i-\bar{Q}}{S_Q}\right\}=1-\Phi\{K_i\} \quad \cdots\cdots (13)$$

where $\Phi(x)=\int_{-\infty}^{x}\frac{1}{\sqrt{2\pi}}e^{-y^2/2}\,dy$, an integration of the standard normal density function.

Equation 13 can be written as

$$K_i=\Phi^{-1}\left\{\frac{N+1-i}{N+1}\right\}.$$

It may also be expressed as

$$\int_{-\infty}^{K_i}\frac{1}{\sqrt{2\pi}}e^{-x^2/2}\,dx=\frac{N+1-i}{N+1} \quad \cdots\cdots (14)$$

The values of K_i can thus be found from a standard normal table which is available in many statistical textbooks. The values of K_i and variance of K for some values of N are given in Table 1.

With a set of values of K_i for a given value of N, the number of years of record, β can be determined by Eq. 10. It is known that $a=\bar{Q}$ for the normal distribution.

The summation of deviation squares is used to compare the goodness of fitting between two different methods of estimating statistical parameters. The summation of deviation squares is computed by

$$\Sigma\delta_j^2=\Sigma\{Q_i-(\bar{Q}+K_iS_Q)\}^2,$$

and

$$\Sigma\delta_j^2=\Sigma\{Q_i-(\alpha+K_i\beta)\}^2.$$

Table 1 - Values of σ_k^2 and K_i for Various Length of Records When the Distribution of Flood is Normal

j \ N	5	10	15	16	18	20	22	24
1	0.9672	1.3353	1.5342	1.5650	1.6200	1.6690	1.7100	1.7510
2	0.4308	0.9085	1.1505	1.1865	1.2517	1.3094	1.3600	1.4053
3	0	0.6047	0.8870	0.9288	1.0032	1.0674	1.1245	1.1750
4	− 0.4308	0.3489	0.6745	0.7216	0.8050	0.8759	0.9388	0.9946
5	− 0.9672	0.1143	0.4889	0.5415	0.6334	0.7126	0.7810	0.8418
6		− 0.1143	0.3187	0.3776	0.4794	0.5659	0.6406	0.7067
7		− 0.3489	0.1573	0.2228	0.3361	0.4308	0.5120	0.5829
8		− 0.6047	0. 0	0.0736	0.1990	0.3029	0.3914	0.4678
9		− 0.9085	− 0.1573	− 0.0736	0.0606	0.1800	0.2759	0.3584
10		− 1.3353	− 0.3187	− 0.2228	− 0.0606	0.0598	0.1641	0.2533
11			− 0.4889	− 0.3776	− 0.1990	− 0.0598	0.0545	0.1510
12			− 0.6745	− 0.5415	− 0.3361	− 0.1800	− 0.0545	0.0500
13			− 0.8870	− 0.7216	− 0.4794	− 0.3029	− 0.1641	− 0.0500
14			− 1.1505	− 0.9288	− 0.6334	− 0.4308	− 0.2759	− 0.1510
15			− 1.5342	− 1.1865	− 0.8050	− 0.5659	− 0.3914	− 0.2533
16				− 1.5650	− 1.0032	− 0.7126	− 0.5120	− 0.3584
17					− 1.2517	− 0.8759	− 0.6406	− 0.4678
18					− 1.6200	− 1.0674	− 0.7810	− 0.5829
19						− 1.3094	− 0.9388	− 0.7067
20						− 1.6690	− 1.1245	− 0.8418
21							− 1.3600	− 0.9946
22							− 1.7100	− 1.1750
23								− 1.4053
24								− 1.7510
σ_K^2	0.4484	0.6218	0.7046	0.7164	0.7370	0.7548	0.7692	0.7827

As indicated in Table 2, for all flood series analyzed the summations of deviation squares for floods estimated by $\overline{Q}$ and S_Q are larger that the summations of deviation squares for floods estimated by α and β. Their differences range from 0.16% to 44.74% for annual exceedence series and from 0.08% to 99.37% for annual maximum series.

B. Log-Normal Probability Distribution

If logarithmic of flood, In Q, is considered to follow normal distribution, then Q is said to follow the log-normal distribution. In this case,

Table 2 - Statistical Parameters and Comparison of Results for Taiwan Streams When the Distribution of Flood is Considered to be Normal

	STREAMS	CHIN SHUI	WU	KAO PING	KAO PING	CHO SHUI	PEI KANG
	STATIONS	TUNG TOU	KAN TZU LIN	LAO NUNG	CHIU CHU TAN	CHI CHI	PEI KANG
	Watershed Area (Km^2)	259.2	954.24	812.03	3075.66	2304.20	597.46
	Length of Record (year)	16	16	18	20	24	29
Annual Exceedence Series	$\overline{Q}=\alpha$	2760	3087	1889	7765	5321	1655
	S_Q	966	2112	1228	3218	2093	414
	β	1118	2204	1201	3484	2130	441
	$\Sigma\delta^2(\overline{Q},S_Q)$	859176	15745223	8026148	19047654	19906259	442541
	$\Sigma\delta^2(\alpha,\beta)$	593612	15648175	8013252	17490276	19262632	425897
	Differences (%)	44.74	0.62	0.16	8.90	3.34	3.91
Annual Maximum Series	$\overline{Q}=\alpha$	2401	2716	1481	6507	4695	1385
	S_Q	1304	2362	1369	3873	2582	618
	β	1524	2539	1341	4224	2789	676
	$\Sigma\delta^2(Q,S_Q)$	1114671	15749794	9904642	32649462	14860074	446232
	$\Sigma\delta^2(\alpha,\beta)$	559089	15396032	9896863	30811279	13904250	369752
	Differences (%)	99.37	2.30	0.08	5.97	6.87	20.70

Units for $\overline{Q}$, S_Q and β are CMS, for $\Sigma\delta^2$ are $(CMS)^2$.

$$\text{Differences }(\%)=\frac{\Sigma\delta^2(\overline{Q},S_Q)-\Sigma\delta^2(\alpha,\beta)}{\Sigma\delta^2(\alpha,\beta)}$$

$$\alpha=\overline{\ln Q}=\frac{1}{N}\Sigma\ln Q_i \qquad (15)$$

$$\beta=\frac{1}{Var(K)}\left\{Cov(\ln Q,K)\right\} \qquad (16)$$

The estimated flood magnitude Q_i for the log-normal distribution is

$$Q_i=e^{\alpha+\beta K_i} \qquad (17)$$

It is found, however, that in the analysis of some series of floods the summations of deviation squares for floods estimated by α and β are larger than those for floods estimated by $\overline{Q}$ and S_Q. Therefore, α and β in Eq. 17 should be determined from the following two equations:

$$\frac{\partial}{\sigma\alpha}\Sigma\left\{Q_i-e^{\alpha+\beta K_i}\right\}^2=0,$$

or.

$$\Sigma Q_i e^{\beta K_i}-e^{\alpha}\Sigma e^{2\beta K_i}=0 \qquad (18)$$

and

$$\frac{\sigma}{\sigma\beta}\Sigma\left\{Q_i-e^{\alpha+\beta K_i}\right\}^2=0,$$

or,

$$\Sigma K_i Q_i e^{\beta K_i} - e^{\alpha} \Sigma K_i e^{2\beta K_i} = 0 \qquad \cdots\cdots (19)$$

Dividing Eq. 18 by Eq. 19, it yields

$$\frac{\Sigma Q_i e^{\beta K_i}}{\Sigma K_i Q_i e^{\beta K_i}} = \frac{\Sigma e^{2\beta K_i}}{\Sigma K_i e^{2\beta K_i}},$$

or,

$$\{\Sigma Q_i e^{\beta K_i}\}\{\Sigma K_i e^{2\beta K_i}\} - \{\Sigma e^{2\beta K_i}\}\{\Sigma K_i D_i e^{\beta K_i}\} = 0 \qquad \cdots\cdots (20)$$

The value of β can therefore be determined from Eq. 20 by the iteration method. After finding β, value of α can be obtained from Eq. 18 as

$$\alpha = \ln\left\{\frac{\Sigma Q_i e^{\beta K_i}}{\Sigma e^{2\beta K_i}}\right\} \qquad \cdots\cdots (21)$$

The values of β and α obtained from Eqs. 20 and 21 are denoted by β' and α', respectively, so that it would not be confused with the values of α and β obtained by Eqs. 15 and 16. The summations of deviation squares for floods estimated by α' and β' are much smaller than the summations of deviation squares for floods

Table 3 - Statistical Parameters and Comparison of Results for Taiwan Streams When the Distribution of Flood is Considered to be Log-normal

	STREAMS	CHIN SHUI	WU	KAO PING	KAO PING	CHO SHUI	PEI KANG
	STATIONS	TUNG TOU	KAN TZU LIN	LAO NUNG	CHIU CHU TAN	CHI CHI	PEI KANG
Annual Exceedence Series	$\alpha = \overline{\ln Q}$	7.8587	7.8422	7.3926	8.8731	8.5153	7.3814
	$S_{\ln Q}$	0.3647	0.5940	0.5221	0.4122	0.3427	0.2426
	β	0.4228	0.6755	0.5900	0.4627	0.3687	0.2606
	α'	7.8678	7.8221	7.3136	8.8756	8.4751	7.3810
	β'	0.3995	0.7760	0.7601	0.4677	0.4462	0.2715
	$\Sigma\delta^2$ $(\overline{\ln Q}, S_{\ln Q})$	690206	12275684	6269756	10795878	13755566	343498
	$\Sigma\delta^2$ (α, β)	538495	7856367	4658179	6445611	10976161	268794
	$\Sigma\delta^2$ (α', β')	490418	6009869	3118140	6349681	8202864	259774
	Difference (%)	40.74	104.26	101.07	70.02	67.69	32.23
Annual Maximum Series	$\alpha = \overline{\ln Q}$	7.5613	7.5176	6.9809	8.6062	8.2835	7.1131
	$S_{\ln Q}$	0.7572	0.9154	0.7774	0.5938	0.6353	0.5212
	β	0.8448	1.0692	0.8908	0.6729	0.6840	0.5593
	α'	7.6874	7.6396	6.8426	8.6250	8.3257	7.1608
	β'	0.5636	0.9197	1.1042	0.6570	0.5879	0.4482
	$\Sigma\delta^2$ $(\overline{\ln Q}, S_{\ln Q})$	4914520	9844027	6228129	16081280	7435905	1038796
	$\Sigma\delta^2$ (α, β)	9365312	8944027	3214304	9168593	11488005	1474264
	$\Sigma\delta^2$ (α', β')	2358044	7019739	1758217	8411351	6574315	772162
	Difference (%)	108.42	40.23	254.23	91.19	13.11	34.53

$$\text{Differences } (\%) = \frac{\Sigma\delta^2(\overline{\ln Q}, S\ln Q) - \Sigma\delta^2(a', \beta')}{\Sigma\delta^2(a', \beta')}$$

estimated by $\bar{Q}$ and S_Q. The comparison of results in the summations of deviation squares are shown in Table 3. It shows that their differences range from 32.23% to 104.26% for annual exceedecne series and from 13.11% to 254.23% for annual maximum series.

C. Extreme Value Distribution

Based on the theory of extreme value distribution given by Gumbel (1941) for very large N, the K-T relation is given by Chow (1951) as

$$K = -\left\{0.45 + 0.7797 \ln \{-\ln [1 - \frac{1}{T}]\}\right\}.$$

when the Weibull plotting formula is used, $T_i = (N+1)/i$, and

Table 4 – Values of $\bar{K}$, σ_K^2 and K_i for Various Lenth of Records When the Distribution of Flood is Extreme Value.

i \ N	5	10	15	20	21	23	25
1	0.8770	1.3828	1.6868	1.9049	1.9420	2.0114	2.0751
2	0.2539	0.8023	1.1199	1.3447	1.3828	1.4538	1.5189
2	-0.1642	0.4422	0.7757	1.0079	1.0470	1.1199	1.1864
4	-0.5233	0.1692	0.5214	0.7620	0.8023	0.8770	0.9452
5	-0.9047	-0.0596	0.3154	0.5653	0.6069	0.6837	0.7537
6		-0.2647	0.1387	0.3993	0.4422	0.5214	0.5933
7		-0.4590	-0.0190	0.2539	0.2983	0.3801	0.4540
8		-0.6541	-0.1642	0.1230	0.1692	0.2539	0.3300
9		-0.8659	-0.3016	0.0026	0.0508	0.1387	0.2174
10		-1.1319	-0.4349	-0.1101	-0.0596	0.0319	0.1134
11			-0.5678	-0.2173	-0.1642	-0.0686	0.0161
12			-0.7074	-0.3208	-0.2647	-0.1642	-0.0761
13			-0.8517	-0.4223	-0.3625	-0.2564	-0.1642
14			-1.0208	-0.5233	-0.4590	-0.3463	-0.2494
15			-1.2451	-0.6257	-0.5557	-0.4349	-0.3326
16				-0.7316	-0.6541	-0.5233	-0.4145
17				-0.8443	-0.7565	-0.6128	-0.4961
18				-0.9691	-0.8659	-0.7047	-0.5782
19				-1.1161	-0.9875	-0.8010	-0.6618
20				-1.3181	-1.1319	-0.9047	-0.7484
21					-1.3299	-1.0208	-0.8398
22						-1.1597	-0.9388
23						-1.3515	-1.0503
24							-1.1844
25							-1.3709
$\bar{K}$	-0.0923	-0.0639	-0.0501	-0.0419	-0.0405	-0.0381	-0.0361
σ_K^2	0.3821	0.5482	0.6332	0.6867	0.6952	0.7106	0.7227

$$K_i = -\left\{0.45 + 0.7797 \ln \{-\ln [\frac{N+1-i}{N+1}]\}\right\} \quad \cdots\cdots (22)$$

The mean and the variance of frequency factors for some values of N are computed from Eq. 22 and are given in Table 4. It is shown in Table 5 that the summations of deviation squares are smaller when α and β defined in Eqs. 9 and 10 are used for floods estimation as compared with the summations as compared with the summations of deviation squares when $\bar{Q}$ and S_Q defined in Eqs. 2 and 3 are used. Their differences in the summations of deviation squares range from 4.42% to 115.4% for annual exceedence series and from 5.76% to 72.03 for annual maximum series.

Table 5 - Statistical Parameters and Comparison of Results for Taiwan Streams When the Distribution of Floods is Considered to be Extreme Value.

	STREAMS	CHIN SHUI	WU	KAO PING	KAO PING	CHO SHUI	PEI KANG
	STATIONS	TUNG TOU	KAN TZU LIN	LAO NUNG	CHIU CHU TAN	CHI CHI	PEI KANG
Annual Exceedence Series	α	2809	3255	1919	8132	5502	1666
	β	1165	2562	1281	4270	2562	458
	$\Sigma\delta^2_i\ (\bar{Q}, S_Q)$	978145	10505662	5477794	12479436	10775560	324928
	$\Sigma\delta^2_i\ (\alpha, \beta)$	454100	9130218	5245821	7225089	9429546	260564
	Differences (%)	115.40	15 06	4.42	72 72	14.27	24 70
Annual Maximum Series	α	2486	2745	1587	6559	4904	1415
	β	1601	2562	1601	4270	3203	712
	$\Sigma\delta^2_j\ (\bar{Q}, S_Q)$	2107770	10562292	6291490	18730168	9351408	728555
	$\Sigma\delta^2_i\ (\alpha, \beta)$	1225224	8394134	5948729	11240092	6302353	585496
	Differences (%)	72 03	25.83	5.76	66.64	48.38	24.43

D. Log-extreme Value Distribution

When the logarithmic of flood, ln Q, is considered to follow extreme value distribution, then Q is said to follow the log-extreme value distribution.

As in the case of the log-nromal distribution, the summation of deviation squares using the log-extreme value distribution are larger in several flood series analyzed when α and β are used. In the same manner as described previously, α and β should be determined from Eqs. 20 and 21. The results of the summations of deviation squares for three different methods of estimating statistical parameters are tabulated in Table 6. It is clear that among the three different methods of estimating statistical parameters, β' and α' obtained from Eqs. 20 and 21 give the

Table 6 - Statistical Parameters and Comparison of Results for Taiwan Streams When the Distribution of Floods is Considered to be Log-extreme Value.

	STREAMS	CHIN SHUI	WU	KAO PING	KAO PING	CHO SHUI	PEI KANG
	STATIONS	TUNG TOU	KAN TZU LIN	LAO NUNG	CHIU CHU TAN	CHI CHI	PEI KANG
Annual Exceedence Series	α	7.8797	7.8770	7.4206	8.8932	8.5301	7.3900
	β	0.4347	0.7225	0.6249	0 4800	0.3964	0.2716
	α'	7.8984	7.9295	7.4103	8.9132	8.5340	7.3960
	β'	0.3697	0.6594	0.6703	0.4244	0.3995	0.2509
	$\Sigma\delta^2$ ($\overline{\ln Q}$, $S_{\ln Q}$)	1397260	12124244	4704424	13442426	10036406	431984
	$\Sigma\delta^2$ (α, β)	1677701	8315847	2268692	14549086	5942472	435861
	$\Sigma\delta^2$ (α', β')	1165327	7685414	2113750	10238739	5911645	403883
	Difference (%)	19.90	57.76	122.56	31 29	69.77	6 96
Annual Maximum Series	α	7.6015	7.5709	7.0228	8.6354	8.3088	7.1311
	β	0.8326	1 1040	0.9381	0.6974	0.6823	0.5514
	α'	7.7401	7.7800	7.0371	8.6997	8.3875	7.1976
	β'	0.4987	0.7635	0.9026	0.5648	0.5035	0.3850
	$\Sigma\delta^2$ (lnQ, $S_{\ln Q}$)	10922242	15570154	4064086	25165924	25129100	3186494
	$\Sigma\delta^2$ (α,β)	19297724	36078952	1262147	41535760	42308008	4249056
	$\Sigma\delta^2$ (α', β')	4574951	11399058	1141764	19524919	13743541	1804746
	Difference (%)	138.74	36.59	255.95	28.89	82.84	76.56

smallest summation of deviation squares between obseved and estimated floods. For an annual maximum flood series, a difference in the summation of deviation squares is as high as 256%.

V. Conclusions

Based on the theory provided in this study and the results of analysis of flood series, several conclusions can be reached.

1. The statistical parameters of α and β defined in Eqs. 9 and 10, depending on plotting formula and probability distribution function, provide a better fitting to a chosen plotting positions than the mean and the standard deviation that now commonly used.
2. It may be shown that the mean and the standard deviation that now in use are the special case of the statistical parameters defined in this study.
3. The statistical parameters α and β do not possess invariance property. The statistical parameters β' and α' obtained from Eqs. 20 and 21 should be used when the logarithm of flood magnitude, instead of its original series, are analyzed. In several flood series analyzed, the summations of deviation squares

between the observed and estimated floods by α' and β' are remarkably reduced. The use of α' and β' may be more critical when the logarithm of the flood magnitude is to be analyzed.

4. The computation of statistical parameters α and β or α' and β' are comparatively tedious. It has to be re-computed each time when new hydrologic information is added. However, with the abvent of digital computer, the extra work of computation involved seems trivial.

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