漸變段水理設計之研究

A Hydraulic Design of Transitions

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摘　要

在水利構造物的設計時，工程師常常遇到漸變段的長度如何決定的問題。本文筆者考慮到任何一水深必有另一共軛水深 (Conjugate depth)， 兩者之間有一變化長度，即所謂之跳水長度。此乃筆者之構想出發點。漸變段的任何水深各具有共軛水深及跳水長度，選其較長跳水長度，將另一水深當做該共軛水深變化中的一水深，則該接的長度所佔跳水長度就可以求出，如果假定跳水深的變化爲直線的話。因目前跳水長度之求解尚未成功，故筆者求其較正確之實驗公式以解決上述問題。又爲流線形漸變段以簡便法求出以供實用上之方便。筆者鑑於求算上述問題之簡捷，附上小型可做程序計算機 (HP-67) 之程式以便應用。最後附四則計算例子以示應方法。

I. Introduction

Transition is a section of canal structure to be used to connect two different adjacent canal sections which have different hydraulic properties. Because of the the difficulty of the theoretical solution for the length of unsteadily varied flow motion, as for the hydraulic jump length, this paper attempts simply to illustrate a manner of hydraulic design for transition based on laboratory experimental data on hydraulic jumps.

2 Hydraulic Jump Length

Laboratory tests on hydraulic jumps were widely done by the U. S. Bureau of Reclamation and one of the results for natural hydraulic jumps on horizontal rectangular flumes is shown by the following manner as in Fig. 2.1.

The writer found the simplicity of the relationship between F_1 and L/D_1 values in Fig. 21 and created an approximate but practical empirical equation for it with symbols defined in Fig. 2.2 as follows:

$$\frac{L}{d_1}_{F1=1\rightarrow 18} = 10\ (F_1-1)^{1-0.0038\ (F_1-2)} \quad \cdots\cdots (2.1)$$

where,

L = jump length

d_1 = jet flow depth (at point 1 in Fig. 2.2)

F_1 = Froude number (at point 1 in Fig. 2.2)

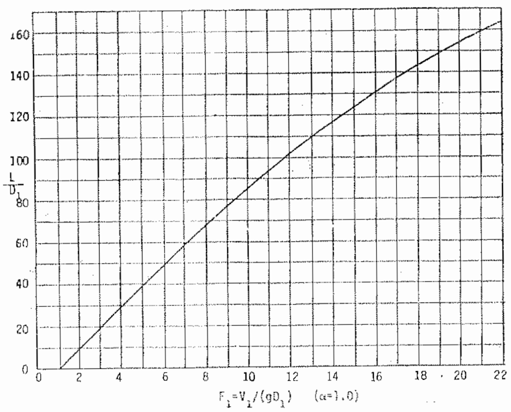


Fig. 2.1 Natural hydralic iump length (USBR)

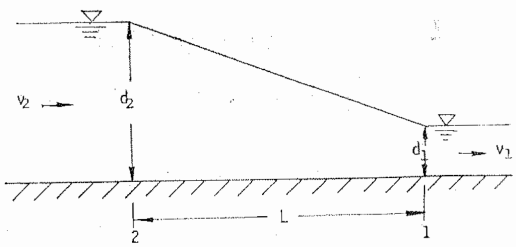


Fig. 2.2 Definition of symbols

Considering flow conditions of a transition usually ranging around $F_1=\pm1.0$, therefore, the Writer's attention was paid on the lower ranges of F_1 values, as well as the L/d_1 values, in Fig. 2.1. Among the range from $F_1=0$ up to about 4.5, the relationship between L/d_1 and F_1 can more slmply be expressed by the following equation.

$$\frac{L}{d_1}\Big|_{F1=0\rightarrow 4.5} \doteqdot 10\ (F_1-1) \qquad (2.2)$$

Here, L may further be redefined as the natural jump transiition length required for connecting the conjugate depths from which d_1 changes to d_2, and the converse.

3. Study and Considerations

Because of the above mentioned relationship between the conjugate depths and the travelling (or transition) length, this concept and the same relationship will also be true for the reversed phenomenon of hydraulic jump, which namely means the drawdown flow condition as shown in Fig. 2.3 below.

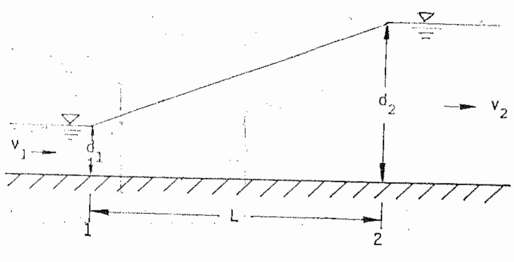


Fig. 2.3 Drawdown flow condition

Because the conjugate depths are mutually reversible, therefore, any water depth can be considered to have another counter-depth (i. e. conjugate depth), which may be imaginary, by taking the critical depth as a boundary, as each pair of conjugate depths has its constant travelling length for a given mass of a flowing body.

Based on the above assumptions, Eq. 2.2 can be rewritten as follows:

$$\frac{L}{d_1}_{F_1=0\rightarrow 4.5} = ABS\ (10\ (F_1-1)) \quad (2.2a)$$

$$\frac{L}{d_2}_{F_2=0\rightarrow 4.5} = ABS\ (10\ (F_2-1)) \quad (2.2b)$$

The conjugate depth ratio can also be rewritten as follows:

$$K_1 = d_2/d_1 = \tfrac{1}{2}\left((1+8F_1)^{\frac{1}{2}} - 1 \right) \quad (2.3a)$$

$$K_2 = d_1/d_2 = \tfrac{1}{2}\left((1+8F_2)^{\frac{1}{2}} - 1 \right) \quad (2.3b)$$

The Froude number is defined to be,

$$F_1 = v_1/\ (g\ d_{h1})^{\frac{1}{2}} \quad (2.4a)$$

$$F_2 = v_2/\ (g\ d_{h2})^{\frac{1}{2}} \quad (2.4b)$$

where,

- F = Froude number
- v = mean velocity (m/sec)
- g = gravitational acceleration (m/sec^2)
- d_h = hydraulic depth (m) = (A/T)
- A = cross-sectional water area (m^2)
- T = top width of water surface of canal cross section (m)

* Subscripts 1 and 2 represent the condition at point 1 and 2 as shown in Fig. 2.2 and 2.3 respectively.

From above equation it can be concluded by saying that no matter computation begins from any side of conjugate depths, the hydraulic proprties of the other side will be obtainable with the same equations.

4. Transition Length

Assume a hydraulic jump in a rectangular canal and on intermediate depth, d_m, which is higher than the lower of the conjugate depths but is lower of the conjugate depths, as shown in Fig. 4.1 below.

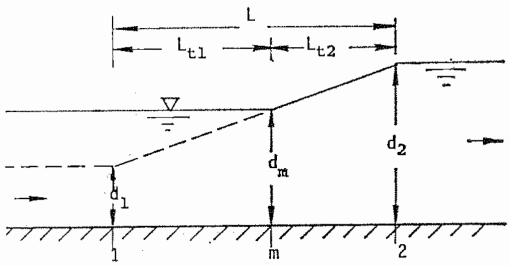


Fig. 4.1 Definition of symbols

The length, L_{t1} or L_{t2} in Fig. 4.1, may be considered to be the natural transition length which is necessary for connecting, two different canal sections for depths d_1 and d_m, or d_m, and d_2. therefore, in practical design, if the imaginary conjugate depth for a given depth and its natural jump transition length between these two conjugate depths have been computed, the transition length connection from any given intermediate depth to either of the conjugate depths can be obtained.

Referring to the symbols used in Fig. 4.1, the proportional relationship of the transition length to the jump transition length, L_{t2}/L or L_{t1}/L, can be expressed as follows by assuming that the variation of the depths is is in a straight line.

$$\frac{d_2-d_1}{L}=\frac{d_m-d_1}{L-L_{t2}} \qquad (4.1)$$

therefore,

$$L-L_{t2}=L_{t1}=\frac{d_m-d_1}{d_2-d_1}\cdot L \qquad (4.2)$$

the dimensionless form of Eq. (4.2) will be,

$$\frac{L_{t2}}{L}=\frac{(d_m/d_1)-1}{(d_2/d_1)-1} \qquad (4.2a)$$

Assuming that $m_1=d_m/d_1$ and $K_1=d_2/d_1$,

$$\frac{L_{t2}}{L}=\frac{m_1-1}{K_1-1} \qquad (4.2b)$$

The transition length from point m to 2 in Fig. 4.1, i.e., L_{t2}, can also be derived from Eq. 4.1 or 4.2 as follows:

From Eq. 4.2, by subtractracting L from both sides,

$$L_{t2}=L-\frac{d_m-d_1}{d_2-d_1}\cdot L \qquad (4.3)$$

the dimensionless form will be,

$$\frac{L_{t2}}{L}=1-\frac{(d_m/d_1)-1}{(d_2/d_1)-1} \qquad (4.3a)$$

or, using the same notations defined above,

$$\frac{L_{t2}}{L}=1-\frac{m_1-1}{K_1-1} \qquad (4.3b)$$

In applying the above equations to canal sections other than rectangular sections, care must be taken as to the depth. Because the conjugate deptes defined so far were based on a regular rectangular canal, of which the hydraulic depth is equal to the depth measured from the canal bottom. In order to avoid confusion in application, the above equations, 4.2b and 4.3b may be rewritten as follows:

$$\frac{L_{t1}}{L}=\frac{m_{h1}-1}{K_1-1} \qquad (4.4)$$

$$\frac{L_{t2}}{L}=1-\frac{m_{h1}-1}{K_1-1} \qquad (4.5)$$

where,

A_m = cross sectional water area at section m in Fig. 4.1

T_m = top width of water surface of the canal cross section at m in Fig. 4.1

$m_{h1}=d_{hm}/d_{h1}$

$K_1=d_{h2}/d_{h1}$

d_{h1} = hydraulic depth at section 1 in Fig. 4.1

d_{h2} = hydraulic depth at section 2 in Fig. 4.1

d_{hm} = hydraulic depth at section m in Fig. 4.1

5. Stream-lined Transition

The traditional ways of designing stream-lined transitions usually proceed as follows:

1) Decide the transition length, L_t.
2) Compute the energy loss for transition, $h_L = h_i + h_f + \cdots\cdots$
3) Decide the variation points of the energy curve, which is to be used as a mid-point of an S-curve.
4) Compute the radius of the S-curve for the energy curve computation, R_{e1} and R_{e2}.
5) Compute the velocity head, h_v, for every sub-station at a specific distance interval within the transition length.
6) Compute the velocity, v, from the already known velocity head, h_v, for for every sub-station.
7) Compute the water area, A, and decide the bottom width, b, or water depth, d, for every station.

This method appears reasonable, but is a time consuming and labour-intensive job. The writer suggests that since the reasonable transition length, the energy loss and the mid-point of the transition length have been decided, it can be simplified by directly deciding the plan shape of the transition, which precludes the necessity of computing the minor details.

The simplified method of transition design may be illustrated as follows:

1) Decide the transition length, L_t.
2) Compute the energy loss for transition, $h_L = h_i + h_f + \cdots\cdots$
3) Decide the mid-point of the energy curve variation, as well as the distances from both ends of a transition, L_a, L_b.
4) Compute the radius of reversal curvatures (i. e. S-curves) for top widths and bottom widths in plan designs, and for bottom elevation variations in profile designs if desired.

Comparing the above two methods, the steps from 1) to 3) are the same. The transition length, L_t, in Step 1) can be computed by the ways mentioned in previous sections. The energy loss for the stream-lined transition in Step 2) can be computed by the following equations.

$$h_i = f_i \cdot h_v \quad \cdots\cdots (5.1)$$

$$h_o = f_o \cdot h_v \quad \cdots\cdots (5.2)$$

$$h_f = s_m \cdot L_t \quad \cdots\cdots (5.3)$$

where,

h_i = inlet loss of transition due to eddy (m)

h_o = outlet loss of transition due to eddy (m)

h_f = friction loss of transition (m)

f_i = coefficient of inlet loss = 0.10 (for stream-lined transition)

f_o = coefficient of outlet loss = 0.20 (for stream-lined transition)

h_v = velocity head (m) = v/ (2g)

v = velocity (m/sec)

s_m = mean energy slope = $\frac{1}{2}(s_1 + s_2)$

s_1 = energy slope on inlet side
s_2 = energy slope on outlet side
L_t = transition length (m)

Taking an example of flume transitions for both the inlet and outlet, the head loss computation is shown in Fig. 5.1.

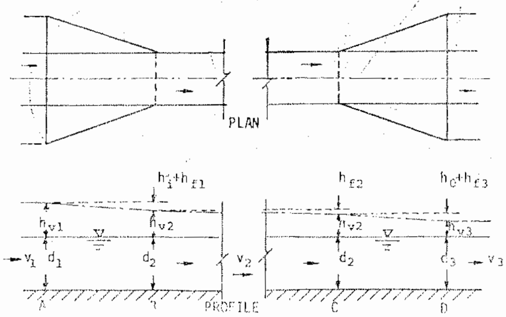


Fig. 5.1 Head losses of a flume

The mid-point for the energy curve variation, as well as the distances from both ends of a transition, L_a, L_b can be computed by the transition length divided by the proportional ratio of Froude numbers for the inlet and outlet of the canal sections. The computation method is explained as follows:

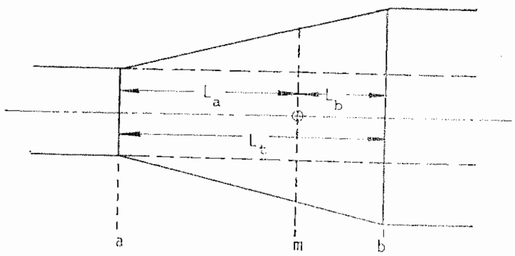


Fig 5.2 Definition of symbols

$$\frac{L_a}{L_t} = \frac{F_a}{F_a + F_b} \quad \cdots\cdots (5.4)$$

$$\frac{L_b}{L_t} = \frac{F_b}{F_a + F_b} \quad \cdots\cdots (5.5)$$

where,

F = Froude number = $v/(g\,dh)^{\frac{1}{2}}$
v = velocity (m/sec)
g = gravitational accelaration = 9.8 m/sec^2
d_h = hydraulic depth = A/T
A = cross sectional water area (m^2)
T = top width of water surface at canal section (m)

* The subscripts, a and b respectively represent the point at the inlet and outlet of a transition.

The radii of curvatures for the known distances, x, and y, which are defined in Fig. 5.3 can be simply computed by a geometrical theorem (i. e. the theorem of a homologous triangle) which may be illustrated as follows.

$\because \quad x : y = y : (2R-x)$

$\therefore \quad R = \frac{1}{2}\left(\frac{y^2}{x}+x\right)$ .... (5.6)

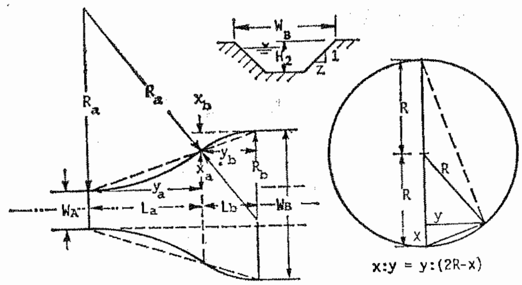


Fig. 5.3 Definition of symbols for an enlarging transition

Based on Eq. 5.6, the radii for curvatures for any condition in a transition can be computed by following branch equations shown as follows:

$K_a = \frac{F_a}{F_a+F_b}$ .... (5.7)

$K_b = \frac{F_b}{F_a+F_b}$ .... (5.8)

$X = ABS\left(\frac{1}{2}(W_A-W_B)\right)$ .... (5.9)

$y_a = L_a = k_a \cdot L_t$ .... (5.10)

$y_b = L_b = k_b \cdot L_t = L_t - y_a$ .... (5.11)

$x_a = k_a \cdot X$ .... (5.12)

$x_b = k_b \cdot X = X - x_a$ .... (5.13)

$R_a = \frac{1}{2}\left(\frac{y_a^2}{x_a} + x_a\right)$ .... (5.14)

$R_b = \frac{1}{2}\left(\frac{y_b^2}{x_b} + x_b\right)$ .... (5.15)

where,

W_A = top width of canal at point 1 (m)

$= b_1+2\ Z_1H_1 = T_A$ .... (Fig. 5.3)

W_B = top width of canal at point 2 (m)

$= b_2+2\ Z_2H_2$

6. Program for Pocket Programmable Calculator (HP-67)

If a packet programmable calculator, as Model HP-67 produced by Hewlett Packard, is available, the computation can be much facilitated. The flow chart of the programmed calculation is shown in Fig. 6.1 and the program is shown in Fig. 6.2.

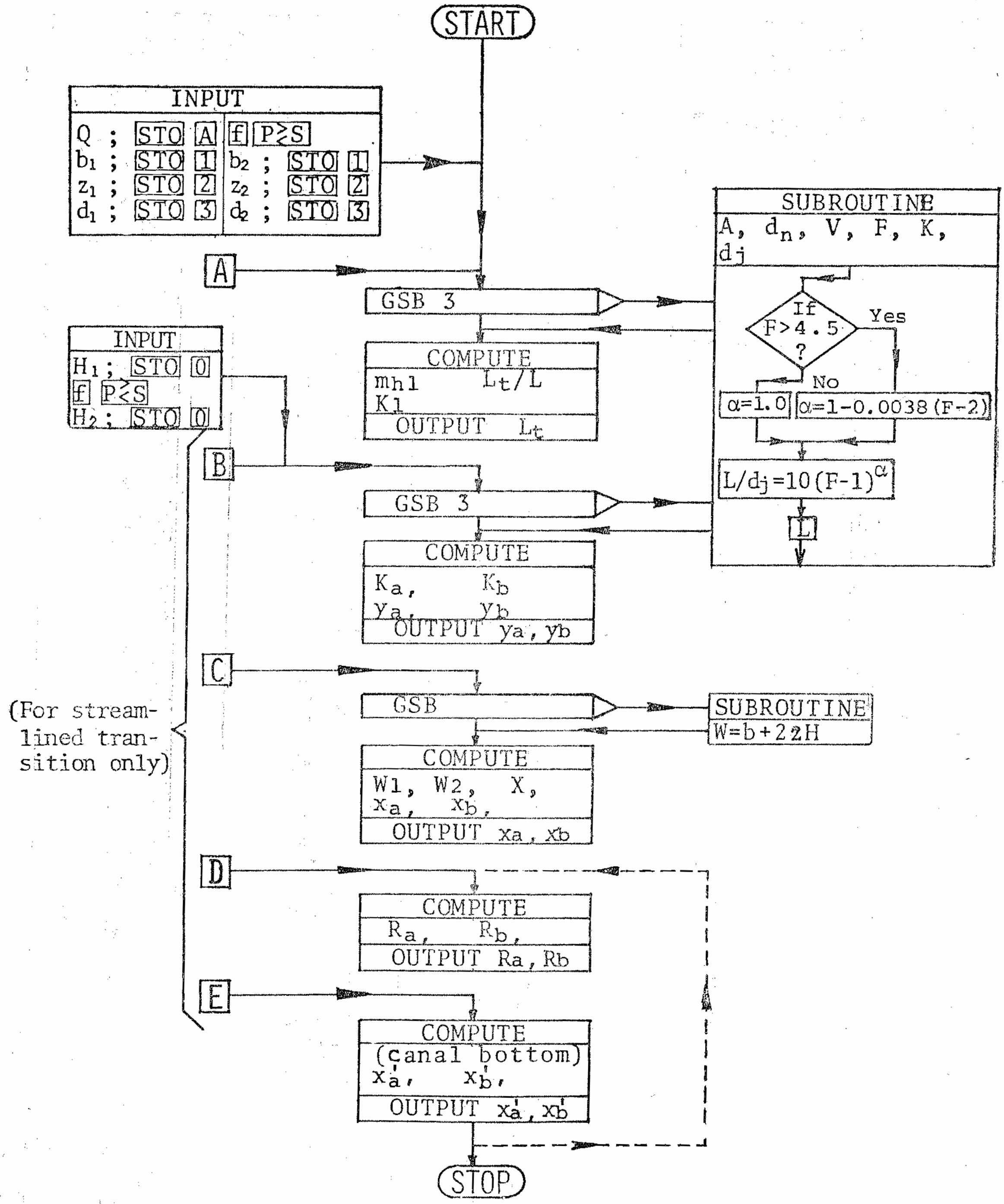


Fig. 6.1 Flow Chart of Programmed Computation (HP-67)

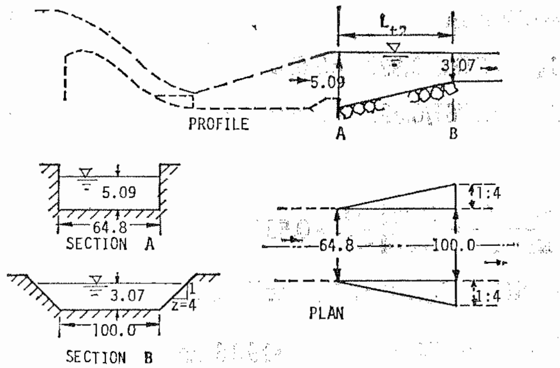
Fig. 6.2 A HYDRAULIC DESIGN OF TRANSITIONS (HP-67)

INPUT:		Example:		Output:
Q ; STO A	f P≷S	Q=2.0		A Lt=(3.746)=3.800
H_1 ; STO 0	H_2 ; STO 0	H_1=1.50	H_2=1.50	B ya=2.527, yb=1.273
b_1 ; STO 1	b_2 ; STO 1	b_1=1.10	b_2=1.00	C xa=1.463, xb=0.737
z_1 ; STO 2	z_2 ; STO 2	z_1=0	z_2=1.5	D Ra=2.914, Rb=1.467
d_1 ; STO 3	d_2 ; STO 3	d_1=1.20	d_2=1.20	(at canal bottom)
				E xa'=0.033, xb'=0.017
				D Ra'=96.057, Rb'=48.368

L_t	y_a, y_b	x_a, x_b	R_a, R_b	x_a', x_b'
A	B	C	D	E
f LBL A	÷	f √x	h RTN	2
DSP 3	STO E	1	f LBL B	×
f P≷S	h RTN	-	RCL 4	RCL 1
f GSB 3	f LBL 2	2	↑	+
f P≷S	f P≷S	÷	f P≷S	STO 9
f GSB 3	f GSB I	STO 6	RCL 4	h RTN
RCL 8	f P≷S	RCL 5	f P≷S	f LBL D
f P≷S	h RTN	×	+	RCL C
RCL 8	f LBL 3	STO 7	÷	g x²
g x>y	RCL 3	4	STO A	RCL E
GTO 1	RCL 2	•	RCL E	÷
GTO 2	×	5	×	RCL E
f LBL 1	RCL 1	RCL 4	STO C	+
RCL 5	+	g x>y	f -x-	2
RCL 7	RCL 3	GTO 4	1	÷
÷	×	1	RCL A	f -x-
STO B	STO 4	STO 8	-	RCL D
f P≷S	RCL 3	GTO 5	STO B	g x²
RCL 5	RCL 2	f LBL 4	RCL E	h RCI
f P≷S	×	RCL 4	×	÷
RCL 7	2	2	STO D	h RCI
÷	×	-	h RTN	+
STO C	RCL 1	•	f LBL C	2
1	+	0	f GSB 6	÷
-	STO 9	0	f P≷S	h RTN
RCL B	RCL 4	3	f GSB 6	f LBL E
1	h x≷y	8	f P≷S	RCL 1
-	÷	×	-	f P≷S
÷	STO 5	1	2	RCL 1
1	RCL A	h x≷y	÷	f P≷S
h x≷y	RCL 4	-	h ABS	-
-	÷	STO 8	h STI	2
STO D	RCL 5	f LBL 5	RCL A	÷
RCL 8	9	RCL 4	×	h ABS
×	•	1	STO E	h STI
h PAUSE	8	-	f -x-	RCL A
1	×	h ABS	RCL B	×
0	f √x	RCL 8	h RCI	STO E
×	÷	h yˣ	×	f -x-
•	STO 4	1	h STI	RCL B
6	g x²	0	h RTN	h RCI
+	8	×	f LBL 6	×
f INT	×	RCL 5	RCL 0	h STI
1	1	×	RCL 2	h RTN
0	+	STO 8	×	

7. Examples of Application

Ex. 1 Transition (or Protection) Length After Stilling Pool



* Given data:

Q = discharge = 750 cms

Items	Section	
	A	B
Bottom Width	b_A = 64.8 m	b_B = 100.0 m
Side Slope	Z_A = 0	Z_B = 4.0
Water Depth	d_A = 5.09 m	b_B = 3.07 m

* Solution:

Firstly, compute the hydraulic properties of the two sections which are to be connected.

No.	Item/Equation	Section	
		A	B
1	$A_2 = d_2 (d_2 Z_2 + b_2)$	329.832	344.700
2	$T = b_2 + 2 d_2 Z_2$	64.800	124.560
3	$d_{h2} = A_2 / T_2$	5.090	2.767
4	$V_2 = Q / A_2$	2.274	2.176
5	$F_2 = V_2 / \sqrt{g\ d_{h2}}$	0.322	0.418
6	$K_2 = d_{h1} / d_{h2} = \frac{1}{2} \left(\sqrt{1+8\ F_2^2} - 1\right)$	0.176	0.274
7	$d_{h1} = K_2 \cdot d_{h2}$	0.897	0.758
8	$L/d_{h2} = 10\ (F_2 - 1)$	6.797	5.841
9	$L = (L / d_{h2}) \cdot d_{h2}$	34.598	16.164

* The imaginary jump transition length for Section A condition requires longer length (i. e. $L_A = 12.855 > L_B = 2.892$), therefore, take Section A as a basis and take B's depth as the intermediatediate depth to compute the length required for transition.

$d_{hm} = d_{h2B} = 2.767$ m

$d_{h2} = d_{h2A} = 5.090$ m

$d_{h1} = d_{h1A} = 0.897$ m

$k_1 = d_{h2} / d_{h1} = 5.090/0.897 = 5.674$

$m_{h1} = d_{hm} / d_{h1} = 2.767/0.897 = 3.980$

then,

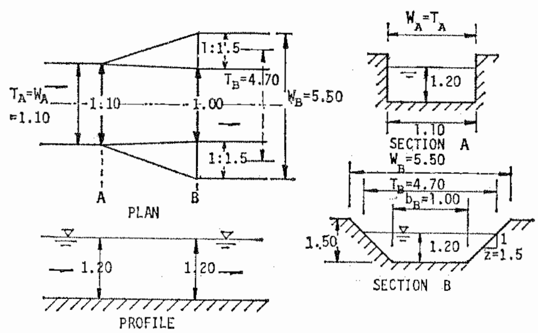
$$\frac{L_{t2}}{L} = 1 - \frac{m_{h1}-1}{k_1-1} = \frac{3.085-1}{5.674-1} = 0.554$$

therefore, the transition length, L_{t2}, will be

$$L_{t2} = \left(\frac{L_{t2}}{L}\right) \cdot L = 0.554 \cdot 34.598 = 19.16 \text{ m}$$

use L_{t2} = 19.20 m or 20.00 m

Ex. 2 Transition Length of Canal and Structure



* Given data:

Q = discharge = 2.0 cms

Items	Section	
	A	B
Bottom Width	b_A = 1.10 m	b_B = 1.00 m
Side Slope	Z_A = 0	Z_B = 1.5
Water Depth	d_A = 1.20 m	d_B = 1.20 m

* Solution:

Because it is hard to judge which section requires a longer natural transition length. hydraulic property computations are to be made for both sections as follows:

No.	Item/Requation	Section	
		A	B
1	$A_2 \quad = d_2\ (d_2 Z_2 + b_2)$	1.320	3.360
2	$T_2 \quad = b_2 + 2d_2 Z_2$	1.100	4.600
3	$d_{h2} \quad = A_2\ /T_2$	1.200	0.730
4	$V_2 \quad = Q\ /A_2$	1.515	0.595
5	$F_2 \quad = V_2\ /\sqrt{g\ d_{h2}}$	0.442	0.222
6	$K_2 \quad = d_{h1}/d_{h2} = \frac{1}{2}\left(\sqrt{1+8\ F_2^2} - 1\right)$	0.300	0.091
7	$d_{h1} \quad = K_2 \cdot d_{h2}$	0.360	0.066
8	$L/d_{h2} = 10\ (F_2 - 1)$	5.601	7.788
9	$L \quad = (L/d_{h2}) \cdot d_{h2}$	6.726	5.689

* From the above results, it is now known that L_A is longer than L_B. Therefore, take Section A as the base for computing the transition length required for connecting Section A and B.

$$d_{hm} = d_{h2B} = 0.730 \text{ m}$$
$$d_{h2} = d_{h2A} = 1.200 \text{ m}$$
$$d_{h1} = d_{h1A} = 0.360 \text{ m}$$
$$k_1 = d_{h2}/d_{h1} = 1.200/0.360 = 3.330$$
$$m_{h1} = d_{hm}/d_{h1} = 0.730/0.360 = 2.027$$

then,

$$\frac{L_{t2}}{L} = 1 - \frac{m_{h1}-1}{k_1 - 1} = 1 - \frac{2.028-1}{3.333-1} = 0.559$$

therefore, the transition length, L_{t2}, will be

$$L_{t2} = \left(\frac{L_{t2}}{L}\right) \cdot L = 0.550 \cdot 6.690 = 3.746 \text{ m}$$

use

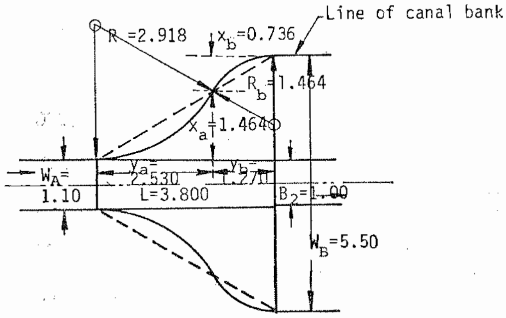
$$L_{t2} = 3.800 \text{ m}$$

* If stream-lined (or warped) transition is to be designed, proceed the computation as follows:

$$k_a = \frac{F_a}{F_a + F_b} = \frac{0.442}{0.442+0.222} = 0.66566$$
$$k_b = \frac{F_b}{F_a + F_b} = \frac{0.222}{0.442+0.222} = 0.33434$$
$$X = \text{ABS}\ (\tfrac{1}{2}\ (W_A - W_B)) = \text{ABS}\ (\tfrac{1}{2}\ (1.10\text{-}5.50) = 2.200 \text{ m}$$
$$y_a = L_a = k_a \cdot L_t = 0.66566 \cdot 3.80 = 2.530 \text{ m}$$
$$y_b = L_b = k_b \cdot L_t = 0.33434 \cdot 3{,}80 = 1.270 \text{ m}$$
$$x_a = k_a \cdot X = 0.66566 \cdot 2.200 = 1.464 \text{ m}$$
$$x_b = k_b \cdot X = 0.33434 \cdot 2.200 = 0.736 \text{ m}$$

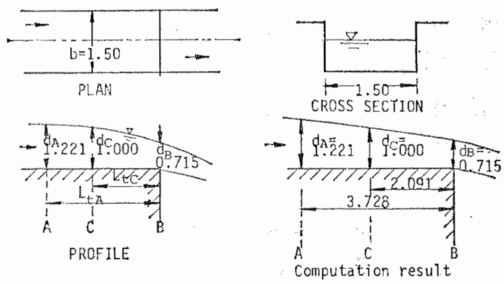
$$R_a = \frac{1}{2}\left(\frac{y_a^2}{x_a} + x_a\right) = \frac{1}{2}\left(\frac{2.530^2}{1.464} + 1.464\right) = 2.918 \text{ m}$$

$$R_b = \frac{1}{2}\left(\frac{y_b^2}{x_b}+x_b\right) = \frac{1}{2}\left(\frac{1.270^2}{0.736}+0.736\right) = 1.464 \text{ m}$$



Ex. 3 Drawdown Length Before a Straight Drop

* Suppose a straight drop is planned to be used as a measuring device, the distance of the critical depth from the brink is figured as follows:



* Given data:

Q = discharge =4.696 cms

Items	Section		
	A	C	B
Bottom Width	1.50	1.50	1.50
Side Slope	0	0	0
Water Depth	1.221	1.000	0.715†

† The depth at brink is assumed to be 0.715 d_c based on H. Rouse's recommendation.

* Solution:

Critical depth, d_c, at Section C is

$$d_c = \sqrt[3]{\frac{Q^2}{gb^2}} \cdots\cdots\cdots\cdots\text{(for rectangular)}$$

$$= \sqrt[3]{\frac{4.696^2}{g \cdot 1.5^2}} = 1.000 \text{ m}$$

$$A_c = \frac{Q}{b\ d_c} = \frac{4.696}{1.5 \cdot 1.0} = 3.131 \text{ m/s}$$

$$h_{vc} = V_c^2 / (2g) = 3.131^2/(2g) = 0.500 \text{ m}$$

$$E_c = d_c + h_{vc} = 1.00+0.500=1.500 \text{ m}$$

* Cmpute the hydraulic properties at Points A and B

No.	Item/Equation	Section	
		A	B
1	$A_2 = d_2 (d_2Z_2+b_2)$	1.832	1.073
2	$T_2 = b_2 + 2d_2Z_2$	1.500	1.500
3	$d_{h2} = A_2 / T_2$	1.221	0.715
4	$V_2 = Q / A_2$	2.564	4.379
5	$F_2 = V_2 / \sqrt{g\ d_{h2}}$	0.741	1.654
6	$K_2 = d_{h1} / d_{h2} = \frac{1}{2} \left(\sqrt{1+8\ F_2^2} -1\right)$	0.661	1.892
7	$d_{h1} = K_2 \cdot d_{h2}$	0.808	1.353
8	$L/d_{h2} = 10\ (F_2-1)$	2 588	6.541
9	$L = (L/d_{h2}) \cdot d_{h2}$	3.160	4.677

* The transition length for the imaginary jump for Section B conditions is required to be longer, therefore, take the B conditions as a basis and take Section A's depth as the intermediate depth to compute the length required for transition.

$$d_{hm} = d_{h2A} = 1.221 \text{ m}$$
$$d_{h2} = d_{h2B} = 0.715 \text{ m}$$
$$d_{h1} = d_{h1B} = 1.353 \text{ m}$$
$$k_1 = d_{h2}/d_{h1} = 0.715/1.353 = 0.529$$
$$m_{h1} = d_{hm}/d_{h1} = 1.221/1.353 = 0.903$$
$$\frac{L_{t2}}{L} = 1 - \frac{m_{h1}-1}{K_1-1} = 1 - \frac{0.904-1}{0.528-1} = 0.793$$
$$\therefore\ L_{t2} = \left(\frac{L_{t2}}{L}\right) \cdot L = 0.797 \cdot 4.677 = 3.728 \text{ m} \doteqdot 3.70 \text{ m}$$

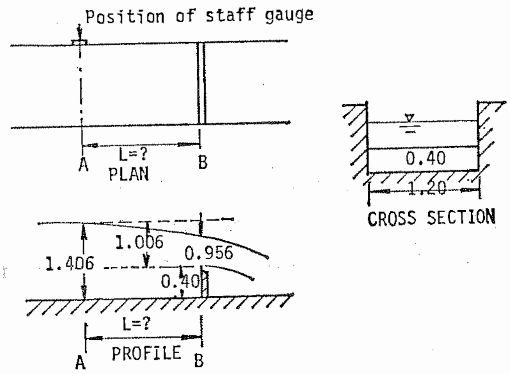
* The length between Points C and B (i.e. the distance from d_c to the brink.)
$Q = 4.696$ cms

No.	Item/Equation	Section	
		A	B
1	$A_2 = d_2 (d_2Z_2+b_2)$	1.500	1.073
2	$T_2 = b_2 + 2d_2Z_2$	1.500	1.500
3	$d_{h2} = A_2 / T_2$	1.000	0.715
4	$V_2 = Q / A_2$	3.131	4.379
5	$F_2 = V_2 / \sqrt{g\ d_{h2}}$	1.000	1.654
6	$K_2 = d_{h1} / d_{h2} = \frac{1}{2} \left(\sqrt{1+8\ F_2^2} -1\right)$	1.000	1.892
7	$d_{h1} = K_2 \cdot d_{h2}$	1.000	1.353
8	$L/d_{h2} = 10\ (F_2-1)$	0.001	6.541
9	$L = (L/d_{h2}) \cdot d_{h2}$	0.001	4.677

* Take Section C as the intermediate condition,

$$d_{hm} = d_{h2A} = 1.000 \text{ m}$$
$$d_{h2} = d_{h2B} = 0.715 \text{ m}$$
$$d_{h1} = d_{h1B} = 1.353 \text{ m}$$
$$k_1 = d_{h2}/d_{h1B} = 0.715/1.353 = 0.528$$
$$m_{h1} = d_{hm}/d_{h1} = 1.000/1.353 = 0.739$$
$$\frac{L_{t2}}{L} = 1 - \frac{m_{h1}-1}{k_1-1} = 1 - \frac{0.739-1}{0.528-1} = 0.447$$
$$\therefore \quad L_{t2} = (L_{t2}/L) \cdot L = 0.447 \cdot 4.677 = 2.091 \text{ m}$$

Ex. 4 Check-up Length Before a Measuring Weir



* Suppose there is a non-suppressed rectangular measuring weir installed as shown above. The depth at Point A is obtained from following equation by neglecting the approach velocity

$$Q = C_Q \cdot b \cdot d_A^{\frac{3}{2}}$$

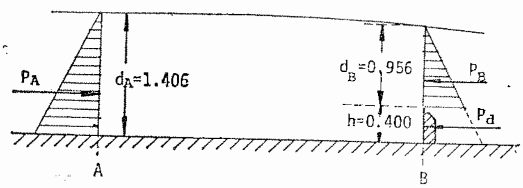
where,

$$C_Q = \text{discharge coefficient} = \tfrac{2}{3}\, C\sqrt{2g}$$
$$\doteqdot 1.85 \text{ (metric system)}$$

Then,

$$d_A = \left(\frac{Q}{C_Q \cdot b}\right)^{\frac{2}{3}} = \left(\frac{2.241}{1.85 \cdot 1.20}\right)^{\frac{2}{3}} = 1.006 \text{ m}$$

* The depth at Point B will obtained by applying momentum theory as follows:



$$\frac{qw}{g}\left(\frac{q}{d_B} - \frac{q}{d_A}\right) = P_A - P_B - P_d$$

Since $P_A = \frac{w}{2} d^2_A$, $P = \frac{w}{2} d^2_B$ and $P_d = \frac{w}{2}\,(2d_B + h)$

therefore,

$$\frac{q^2 w}{g\ d_B} - \frac{q^2 w}{g\ d_A} = \frac{w}{2} d^2_A - \frac{w}{2}\ (2d_B + h)$$

By eliminating w and re-arranging the items, the final form will be

$$d^3_B + 2d^2_d - \left(\frac{2\ q^2}{g\ d_A} + d^2_A - h\right) d_B - \left(\frac{2\ q^2}{g}\right) = 0$$

Since $q = Q/b = 2.241/1.20 = 1.868$, $d = 1.406$ and $h = 0.40$,
then,

$$d^2_B + 2d^2_B - \left(\frac{2 \cdot 1.868^2}{g \cdot 1.406} + 1.406^2 - 0.40\right) d_B - \left(\frac{2 \cdot 1.868^2}{g}\right) = 0$$

$$d_B^3 + 2d_B^2 - 2.083\ d_B - 0.712 = 0$$

$$\therefore\ d_B = 0.956\ \text{m}$$

* Given data:

q=discharge =2.241 cms

Items	Sections	
	A	B
Bottom Width	1.200	1.200
Side Slope	0	0
Water Depth	1.406	0.956

* Solution

No.	Item/Equation	Sections	
		A	B
:	$A_2 = d_2\ (d_2 Z_2 + b_2)$	1.687	1.147
	$T_2 = d_2 + 2d_2 Z_2$	1.200	1.200
3	$d_h{}^2 = A_2 / T_2$	1.406	0 956
4	$V_2 = Q / A_2$	1.328	1.953
5	$F_2 = V_2 / \sqrt{g\ d_{h2}}$	0.358	0 638
6	$K_2 = d_{h1} / d_{h2} = \frac{1}{2}\left(\sqrt{1+8\ F_2^2} - 1\right)$	0.211	0.532
7	$d_{h1} = d_2 \cdot d_{h2}$	0.297	0.508
8	$L/d_{h2} = 10\ (F_2 - 1)$	6.440	3.637
9	$L = (L/d_{h2}) \cdot d_{h2}$	9 054	3.477

* Because $L_A > L_B$, therefore, take the A's imaginary jump condition as basis and take the depth of Section B as the intermidiate depth to compute the length required for transition.

$$d_{hm} = d_{h2B} = 0{,}956\ \text{m}$$

$$d_{h2} = d_{h2A} = 1.406\ \text{m}$$

$$d_{h1} = d_{h1A} = 0.297\ \text{m}$$

$$k_1 = d_{h2}/d_{h1} = 1.406/0.297 = 4.734$$

$$m_{h1} = d_{hm}/d_{h1} = 0.956/0.297 = 3.219$$

then,

$$\frac{L_{t3}}{L} = 1 - \frac{m_{h1}-1}{K_1 -1} = 1 - \frac{3.219-1}{4.734-1} = 0.406$$

therefore, the transition length will be

$$L_{t2} = \left(\frac{L_{t2}}{L}\right) \cdot L = 0.406 \cdot 9.054 = 3.676 \text{ m} \div 3.700 \text{ m}$$

* As a mater of interest, the ratio $L_{t2}/d_A = 3.678$ is a little bit smaller than the ratio, 4.0, recommended by the standard installation of staff gauge for measuring weir.

8. Conclusion

In this paper, USBR' S test results on natural hydraulic jump length and the writer's empirical formula were introduced. Then, based on she assumption that the water surface profile between the conjugate depths is a straight line, the idea of inserting a depth into between the conjugate depths to obtain a ratio which is prortional to the imaginary jump length was introduced. Of which method the Writer thought was a way to obtain the minimum but the reasonable transition length. A way of obtaining a stream-iined (or warped) transition was also introduced by the proportional ratio of Froude numbers. The program of using pocket programable culculator was attached for convenience in computation. A few example computations were shown for application of the introduced method. Although the method is simple and approximate, but it may be applied to almost all kinds of hydraulic transitions.

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