專 論

Simulation of Unsteady Flow in Rivers

河川變量流之模擬

By

Dr. Chin-lien Yen[1] and Dr. Tsan-wen Wang[2]

臺灣大學土木系客座教授 顏清連

臺灣大學土木系教授兼臺大水工試驗所主任 王燦汶

Abstract

Because of the development and improvement of digital computers, the procedures for computation of unsteady flow in rivers have progressed from much simplified routing methods to more sophisticated simulation models using numerical methods. Unsteady flow simulation in floodplain channels is still a relatively difficult problem. Proposed herein is a "THREE-TUBE MODEL" in which the main channel and the two floodplains are treated separately so that the exchanges of momentum and of mass between main channel and floodplains can be taken into consideration. The implicit scheme of numerical method was employed in constructing the MODEL. Calibration and verification of the MODEL has been carried out for South Yamhill River in U. S A, Cho-Shui River (Meopu-Chichi) and Potzu River (Neuchaochi Bridge-Potzu) in Taiwan.

摘　要

因爲電子計算機的發展與改進，河川變量流的計算已經從很簡化的方法進步到相當複雜而利用數值分析法的模擬模式。在有洪水平原的河川變量流的模擬仍然是一個困難問題。本文所提出的「三管模式」是將河道劃成主河槽及左右兩洪水平原分別處理，使彼此間的質量和動量交換得以加入考慮。模式的建造是以穩式差分法作成，並以美國的 South Yamhill River 與臺灣的濁水溪（苗圃→集集）和朴子溪（牛稠溪橋→朴子）等河段作檢定與校驗。

1. Visiting Professor of Civil Engrg., Natiotnal Taiwan University (on sabbatical leave from Howard Univ., Washington, D. C., U. S. A.)

2. Professor of Civil Engrg., and Director of Hydraulic Research Laboratory, National Taiwan University.

I. INTRODUCTION

The unsteady flow problem in rivers is generally the problem of determining the shape and timing of a flood wave as it propagates downstream. The solution of this problem is of great importance to engineers in planning, design and operetion of engineering projects such as regulation of stream flows and flood control.

The procedures for computation of unsteady flow in rivers have been developed mainly to study flood wave propagation through channels with or without various types of man-made structures. The need for knowledge concerning flood propagation is highlightened by a great number of structures being built along river channels. These structures may modify the movement of flood wave to such an extent that the nature of the flood propagation is totally different from that in its natural state.

Unsteady flow in rivers is a relatively difficult problem to treat mathematically. Basically, the difficulty arises from the fact that many variables enter into the relationship and that the differential equations can not be integrated in closed forms except under greatly simplified conditions. However, the resulting solutions for these simplified cases have limited pratical applications. On the other hand, even if the solutions are based on full unsteady flow equations, one should recognize that the basic assumptions and simplitications made in their derivations so that the reliability of the resulting solution can be assessed in light of these basic assumptions and simplifications. Distinction is usually made between flood routing by simplified approaches and by the theory of wave movement in channels, generally called free-surface unsteady flow. The latter case is referred to as unsteady flow simulation. However, they utilize basically the same physical principles and differ only in degree of approximation. In the past decade, a great deal of advances have been made in the area of unsteady flow simulation because digital computers with larger and larger storage capacity have become available. The large computer capacity has made it possible to handle problems with greater complexity.

Unsteady flow simulation in channel-floodplain complex is one of the many difficult problems in hydraulics. The difficulties arise from the fact that there exist large differences in roughness and flow depth between the main channel and floodplains. There are two types of approaches to this problem: (a) the entire cross-section is considered as a whole unit, and (b) the cross-section is divided into conveyance and storage portions. In the first approach, usually an equivalent roughness is used to account for the differences in roughness and depth between main channel and floodplains [2]. The inadequacy of using equivalent roughness has been pointed out [11]. Furthermore the exchanges of mass and of momentum between the main channel and floodplain, which have long been recognized to exist [5], can not be computed by taking this approach. In the second approach, the floodplain is treated as a storage reservoir without longitudinal velocity [1]. This assumption of no longitudinal velocity on the floodplain portion can result in significant errors in flow computations when the floodplains have relatively large wetted cross-sectional areas.

In this paper, an overview of methods for unsteady flow computations is given first. It is then followed by a discussion of implicit method and its solution technique for general unsteady flow problems. A simulation model (THREE-TUBE MODEL) for unsteady flow in floodplain channels using the implicit scheme is presented. The model is calibrated and verified for three river reaches - two in Taiwan and one in Oregon, U. S. A.

II. OVERVIEW OF FLOOD ROUTING METHODS

There have been a large number of flood routing mehtods developed since the beginning of this century. This large number reflects the desire for routing flood flow accurately from one location of a stream to another. These methods can be grouped into three categories: (a) Storage Methods based on storage-discharge relationhip; (b) Diffusion Methods based on convective-diffusion eqnation; and (c) Finite Difference Methods based on numerical solution of the full equations of motion and continuity. Each of these methods has its own assumptions, advantages and disadvantages, which are briefly discussed in the following:

1. *Storage Methods*: The methods in this category are generally the simplest of all. They are called storage routing because they are based on the concept of storage of flood water in the channel reach or reservoirs. The effects of flow resistance and accelerations are not taken into account. So, the storage routing of flood flow in a river reach utilizes the continuity equation which can be written as

$$\frac{dS}{dt} = Q_i - Q_o \qquad (1)$$

where $\frac{dS}{dt}$ = rate of change of water in storage; Q_i = rate of inflow; Q_o = rate of outflow; S = volume of water in storage; t = time. Normally $Q_i(t)$ is given, but $Q_o(t)$ and S(t) are the unknown variables to be solved for. Obviously, a second relationship between storage S and the rate of outflow Q_o and/or inflow Q_i is required in order to solve for the two unknowns. One of the most frequently used method of this type is the well known Muskingum Method [7] which uses the relationship

$$S = K\left[XQ_i + (1-X)Q_o\right] \qquad (2)$$

where K = average wave propagation time; and X = a weighting factor, Other symbols are the same as those in Eq. (1). The actual values of K and X have to be determined from past flood records or channel characteristics.

Equation (2) automatically assumes that the relationship between discharge and storage is unique. However, the fact is that for a particular storage (i. e., given stage) the discharge is greater when the flood is rising than when it is falling. Furthermore, the Muskingum method does not predict attenuation of flood peak in prismatic channels. Although Muskingum method ignores dynamic effects on the flood wave, Cunge [3] has shown that it is possible to improve the method so that its solution is a good approximation to the Diffusion Methods.

2. *Diffusion Methods*: The Diffusion Method was first proposed by Hayami [4] based on a linear convective-diffusion equation as the following:

$$\frac{\partial y}{\partial t} + V_W \frac{\partial y}{\partial x} = D\frac{\partial^2 y}{\partial x^2} \qquad (3)$$

where y = depth of water; V_W = propagation speed of flood wave; t = time; x = distance along the direction of flow; D = diffusion coefficient. In deriving Eq. (3), Hayami's reasoning was that the flood propagations in rivers are affected by the irregularities of channel boundaries. The effect of these irregularities is the diffusion of flood wave as it moves downstream. Because there is no direct way to calculate the diffusion coefficient, Hayami suggested that D should be determined by the trial-and-error method from previous records of flood in the river under study. Once the diffusion coefficient is known and the propagation speed defined, Hayami's method of routing will yield reasonably good agreement with actual floods. However, the uncertainty in diffusion coefficient remains as a major shortcoming of the Diffusion Methods.

By treating a flood wave as a kinematic wave and by including modification to this due to the diffusive action of the water surface slope, Lighthill and Whitham [6] were able to arrive at the same Equation (3). However, the Lighthll and Whitham's diffusion coefficient takes into consideration only the effect of water surface slope while Hayami's diffusion coefficient include the effects of both channel irregularities and water surface slope. The irregularities in channel boundary is viewed in effect as a series of small reserviors which increase the storage capacity of the river through which the flood wave propagates. This effect is accentuated particularly when the flood flows out over the floodplains. Recently Price [9] has shown that the effects of channel irregularities in natural rivers can be quantified. So, the diffusion coefficient can be determined without the trial-and-error procedure suggested by Hayami.

One major problem in using the Diffusion Methods is the inclusion of discharges from major tributaries. The problem is how to prevent the discrete lateral inflow from upsetting the stability of the governing equation. The simplest solution to the problem is to route a flood from tributary to tributary and then sum up the hydrographs from both the main river and the tributary. This procedure assumes naturally that the flow from the tributary has no backwater effect on the main river.

3. *Finite Difference Methods*: Because of the limitations in various flood routing methods proposed in the past and the availability of high-speed digital computer, increasing attention is being paid to numerical solutions of the full equations of motion and continuity. There have been a number of finite difference methods of flood routing developed since 1960. These methods differ from each other primarily in the techniques used in solving the differential equations. There are basically two types of methods for the solution of the governing equations: The methods of characteristics and fixed-grid methods.

The method of characteristics is based on the characteristic form of the governing equations and has been known for many years. The basic principle is to fill the x-t plane with characteristic lines so that the dependent variables are defined at intersections of the characteristic lines. In fact, this method has been used for graphical integration and for overland flow studies. However, there remain difficulties preventing wide use of this method for unsteady flow simulation in natural rivers. For example, a small change in continuity to account for off-stream storage will result in changes in characteristic formulation and solution to a large extent. Also the stability criterion requires the time-step of the solution scheme be limited to a relatively small size. Particularly when the spacings between river sections are very uneven, the size of time-step is almost always controlled by the smallest spacing. As the size of time-step decreases the total computational time increases. This is another major reason that the method of characteristics is not widely employed in dealing with practical problems.

In contrast to the method of characteristics, the fixed-grid methods are based on the original differential equations from which the dependent variables are solved for at a finite number of prescribed rectangular grid points on the x-t plane. There are two different schemes; the explicit and the implicit schemes. In the explicit scheme, the equations are arranged in such a way that the solution is advanced to a new time level one point at a time. In other words, at the new time level the solution for each section is found independently of any other points on the same time line. This is to say that the solution for the new time level is only dependent on the conditions at the old time level, without referring to other dependent variables which are also to be solved for. This is the reason for calling this scheme as explicit scheme.

On the other hands, the implicit scheme solves for a group of points on the advanced time level by using simultaneous equations which contain the unknown variables for all the points. Naturally the simultaneous equations also include the conditions at the old time level which had been solved previously. The explicit scheme, like the characteristic methods, requires that the time interval be small enough so that the stability criterion can be satisfied. The implicit scheme has practically no limitations on the size of time-step in its solution. This scheme is always stable if the weighting factor in the time-direction is greater than 0.5. However, it should be realized that as the size of time-step increases the accuracy of the resulting solution decreases. So, there must be some sort of trade-off between the computational time and accuracy.

As the storage capacity of computer increases with the advances in computer technology, the Finite Difference Methods can be extended to include as much details of geometrical properties of the main channel and floodplains as may be required. However, the problems of great disparities in boundary roughness and in flow depth between the main channel and floodplains, as well as problems of mass and momentum exchanges as flood rises and falls, are some of the many major difficulties that need to be overcomed in order to make physical and mathematical treatments of unsteady flow in rivers comparable.

III. IMPLICIT METHOD AND SOLUTION TECHNIOUE

1. *Governing Equations*

The equations governing unsteady flow in natural rivers are basically the same as those governing any open channel flow. Because the geometrical properties in natural rivers vary greatly from section to section, these equations are usually written in terms of total discharge and cross-sectional area. They are as the follows:

$$\frac{\partial Q}{\partial t}+\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right)+gA\frac{\partial E}{\partial x}=-gAS-qu \quad \cdots\cdots (4)$$

and

$$\frac{\partial A}{\partial t}+\frac{\partial Q}{\partial x}=q \quad \cdots\cdots (5)$$

where Q = total flow rate at a section;
A = flow cross-sectional area;
E = water surface elevation;
x = distance along flow direction;
g = gravitational acceleration;
S = frictional slope;
q = lateral inflow per unit length of channel; and
u = x-component of lateral flow velocity.

The second term of Eq. (4) should carry a momentum correction factor which is considered to be unity in the treatment in this section. The product $q \cdot u$ in the last term on the right handside of the same equation is the longitudinal momentum due to lateral flow. The meaning of all the other terms in Eqs. (4) and (5) can be found elsewhere [2]

2. *Finite Diference Scheme*

Of many implicit schemes developed for solution of unsteady flow in open channels, the "four-point" schemes are more advantageous since they do not require equal space-intervals nor equal time-intervals. Many investigators [8] have shown that this type of schemes is quite satisfactory. A description of the implicit four-point finite difference scheme follows.

The x-t plane, on which solution of water surface elevation E and flow rate Q are sought, is divided into rectangular grid of diserete points as shown in Fig. 1. The grid points are defined by the intersection of straightlines drawn pararell to x-axis and the ones to t-axis. Those pararell to x-axis are time lines having spacing Δt and those pararell to t-axis represent different locations along the river having spacing of Δx.

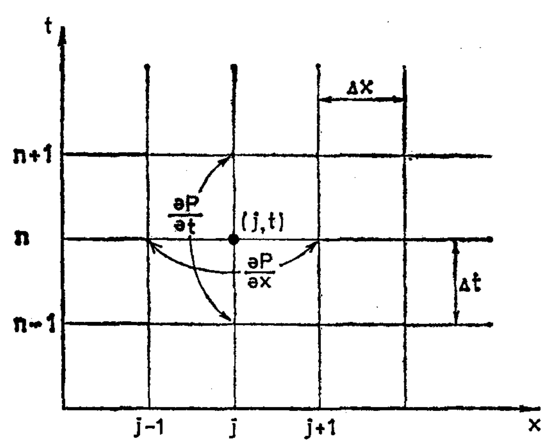


Fig.1 Finite Difference Grid

The time derivative of any variable is approximated by a forward difference as

$$\frac{\partial P}{\partial t} = \frac{1}{2\Delta t}(P_{j+1}^{n+1} + P_{j}^{n+1} - P_{j+1}^{n} - P_{j}^{n}) \quad \cdots\cdots (6)$$

where P represents any variable; j designates x-position; and n designates t-position.

The spatial derivative is approximated by a forward difference operator located between two adjacent time lines according to weighting factors θ and $(1-\theta)$:

$$\frac{\partial P}{\partial x}=\frac{\theta}{\Delta x}(P^{n+1}_{j+1}-P^{n+1}_{j})+\frac{(1-\theta)}{\Delta x}(P^{n}_{j+1}-P^{n}_{j}) \qquad (7)$$

And any variable or function other than the derivatives mentioned above is approximated by its value evaluated at the time level defined by the weighting factors θ and $(1-\theta)$, i. e., that

$$P=\frac{\theta}{2}(P^{n+1}_{j+1}+P^{n+1}_{j})+\frac{(1-\theta)}{2}(P^{n}_{j+1}+P^{n}_{j}) \qquad (8)$$

Since the computation always proceeds from the old time line to the new one, the superscripts n and (n+1) will be dropped from now on. Instead, a variable with superscript "0" represents its value at the old time line and the one without stands for the new time level.

Application of Eqs. (6), (7) and (8) to Eqs. (4) and (5) yields:

$$\begin{aligned}
&\frac{1}{2\Delta t}(Q_{j+1}+Q_{j}-Q^{0}_{j+1}-Q^{0}_{j})\\
&+\frac{1}{\Delta x}\{\theta(\frac{Q^{2}_{j+1}}{A_{j+1}}-\frac{Q^{2}_{j}}{A_{j}})+(1-\theta)[\frac{(Q^{0}_{1+j})^{2}}{A^{0}_{j+1}}-\frac{(Q^{0}_{j})^{2}}{A^{0}_{j}}]\}\\
&+\frac{1}{2\Delta x}g[\theta(E_{j+1}-E_{j})-(1-\theta)(E^{0}_{j+1}-E^{0}_{j})]\cdot\\
&\quad\cdot[\theta(A_{j+1}+A_{j})+(1-\theta)(A^{0}_{j+1}+A^{0}_{j})]\\
&+\frac{1}{4}g[\theta(S_{j+1}+S_{j})+(1-\theta)(A^{0}_{j+1}+A^{0}_{j})]\cdot\\
&\quad+[\theta(A_{j+1}+A_{j})+(1-\theta)(A^{0}_{j+1}+A^{0}_{j})]\\
&+\frac{1}{4}[\theta(q_{j+1}+q_{j})+(1-\theta)(q^{0}_{j+1}-q^{0}_{j})]\cdot\\
&\quad\cdot[\theta(u_{j+1}+u_{j})+(1-\theta)(u^{0}_{j+1}-u^{0}_{j})]=0 \qquad (9)
\end{aligned}$$

and

$$\begin{aligned}
&\frac{1}{2\Delta t}(A_{j+1}+A_{j}-A^{0}_{j+1}-A^{0}_{j})\\
&+\frac{1}{\Delta x}[\theta(Q_{j+1}-Q_{j})+(1-\theta)(Q^{0}_{j+1}-Q^{0}_{j})]\\
&-\frac{1}{2}[\theta(q_{j+1}+q_{j})+(1-\theta)(q^{0}_{j+1}-q^{0}_{j+1})]=0 \qquad (10)
\end{aligned}$$

Eqs. (9) and (10) can be expressed in general form as

$$\left.\begin{aligned}F_{j}(Q_{j+1}, E_{j+1}, Q_{j}, E_{j})=0\\ G_{j}(Q_{j+1}, E_{j+1}, Q_{j}, E_{j})=0\end{aligned}\right\} \qquad (11)$$

In the above equations, the cross-sectional area A is a function of water surface elevation E, and the frictional slope S is a function of E and flow rate Q.

If there are N locations along x-axis, then there are (N-1) number of rectangular grids. Now, writing 2 equations for each rectangular grid, there will be a system of 2(N-1) equations as follows

$$\left.\begin{aligned}
&F_1\ (Q_1,\ E_1,\ Q_2,\ E_2\ )=0\\
&G_1\ (Q_1,\ E_1,\ Q_2,\ E_2\ )=0\\
&\cdots\cdots\cdots\cdots\\
&F_J\ (Q_J,\ E_J,\ Q_{J+1},\ E_{J+1})=0\\
&G_J\ (Q_J,\ E_J,\ Q_{J+1},\ E_{J+1})=0\\
&\cdots\cdots\cdots\cdots\\
&F_{N-1}(Q_{N-1},\ E_{N-1},\ Q_N,\ E_N\ )=0\\
&G_{N-1}(Q_{N-1},\ E_{N-1},\ Q_N,\ E_N\ )=0
\end{aligned}\right\}\cdots\cdots\cdots\cdots(12a)$$

Except the first and the last, each grid point is common to two adjacent rectangular grids. Therefore there are altogether 2N unknowns contained in the 2(N-1) equations. In order to solve for the 2N unknowns, there must be two additional equations describing the boundary conditions. The boundary conditions are normally a hydrograph (discharge or stage) at the upstream end and a hydrograph or a rating curve at the downstream end. They can be written in general form as

$$\left.\begin{aligned}
&F_0\ \ (Q_1,\ E_1\ )=0\\
&G_N\ (Q_N,\ E_N\ )=0
\end{aligned}\right\}\cdots\cdots\cdots\cdots(12b)$$

The system of Eqs. (12a) and (12b) comprises the simultaneous equations that are needed to determine the water surface elevations and discharges for all the N locations in the river reach under consideration. All other associated quantities such as areas, velocities, bed shear stresses, flow depths and surface widths can readily be computed from the discharges and water surface elevations.

3. *Solution Technique*

The set of simultaneous equations mentioned in the last Section can be solved by many different methods. In this study, the Newton's iteration method is employed. Basically the set of non-linear simultaneous equations is transformed into a set of linear ones in terms of the differentials of the original unknown variables as the new unkowns and then solved for these new unknowns. Suppose there is a set of estimated values for the original variables Q_J and E_J. Substituting these values into the simultaneous equations yields a set of residue R's. The residues will have non-zero values if the estimated Q_J and E_J are different from the true solution. Let the differences between the estimated values and the true solution be dQ_J and dE_J The relationship between the differences and the residues can be expressed as

$$\left.\begin{aligned}
&\frac{\partial F_0}{\partial Q_1}dQ_1+\frac{\partial F_0}{\partial E_1}dE_1=R_{1,0}\\
&\frac{\partial F_1}{\partial Q_1}dQ_1+\frac{\partial F_1}{\partial E_1}dE_1+\frac{\partial F_1}{\partial Q_2}dQ_2+\frac{\partial F_1}{\partial E_2}dE_2=R_{1,1}\\
&\cdots\cdots\cdots\cdots\\
&\frac{\partial F_J}{\partial Q_J}dQ_J+\frac{\partial F_J}{\partial E_J}dE_J+\frac{\partial F_J}{\partial Q_{J+1}}dQ_{J+1}+\frac{\partial F_J}{\partial E_{J+1}}dE_{J+1}=R_{1,J}\\
&\cdots\cdots\cdots\cdots\\
&\frac{\partial G_{N-1}}{\partial Q_{N-1}}dQ_{N-1}+\frac{\partial G_{N-1}}{\partial E_{N-1}}dE_{N-1}+\frac{\partial G_{N-1}}{\partial Q_N}dQ_N+\frac{\partial G_{N-1}}{\partial E_N}dE_N=R_{2,N}\\
&\frac{\partial G_N}{\partial Q_N}dQ_N+\frac{\partial G_N}{\partial E_N}dE_N=R_{2,N}
\end{aligned}\right\}\cdots\cdots(13)$$

In the above equations, all the partial derivatives are first evaluated using estimated values of Q_j and E_j. The objective now is to solve for the unknowns dQ_j and dE_j such that the left handside of each equation equals its respective residue. This is to say that if the Q_j and E_j are corrected by the amounts dQ_j and dE_j respectively, then the residues should become zero. However the solution here is only approximate because the coefficients of the set of simultaneous linear equations are evaluated with the estimated values of Q_j and E_j. As the new values of discharges and water surface elevations are substituted back into Eqs. (12a) and(12b), a new set of residues will result. This means then the solution to the set of linear equations has to be revised. This iterative process continues until the differences between two successive solutions reduce down to within a prescribed tolerance. After obtaining the solution for dQ_j and dE_j, the discharges and water surface elevations can be computed using

$$\left.\begin{aligned} Q_j &= Q_j^{\circ} - dQ_j \\ E_j &= E_j^{\circ} - dE_j \end{aligned}\right\} \qquad (14)$$

IV. THE THREE-TUBE MODEL

As stated earlier in this paper, there are shortcomings in the presently available techniques for flood routing in floodplain channels. In order to overcome these shortcomings, a computer model called "THREE-TUBE MODEL" [10] has been developed. Here the "three tubes" refer to the main channel, the left and the right floodplains. There are two versions of the "THREE-TUBE MODEL". In the first version, the method of characteristics has been used to solve for flow velocities and water surface elevations at various time levels. The exchanges of mass and of momentum arising from the interflow between the main channel and the floodplains are then computed based on the inequalities of water surface elevations between them at a cross-section. As stated earlier, the method of characteristics usually requires small time-steps and consequently a large computational time is needed for most of the practical problems. This requirement makes it uneconomical to use the method of characteristics for routing in floodplain channels. Therefore this method will not be discussed any further in this paper. For those who are interested in this particular scheme is referred to Reference 10.

In the second version of the "THREE-TUBE MODEL", the implicit method of finite difference scheme presented in the last Section is employed to solve for discharges and water suface elevations for all predetermined locations in a river reach at a time, In the process of computation, it is assumed, as the first approximation, that the water levels in all three tubes are the same at a section. The total discharge is then expressed in terms of the discharges in the three tubes, each of which in turn is a function of the flow depth, cross-sectional areas, and the Manning's coefficient of its respective tube. With these one can easily evaluate the coefficients in the simultaneous linear Equations (13) and can solve for dQ_j and dE_j, and consequently Q_j and E_j as stated in the last Section. It should be pointed out that the solution so obtained is only an approximation because the water surface is assumed to be at the same level for all three tubes. Adjustments have to be made now to account for the effect of uneven water surface across the section.

As the total flow increases with time on the rising side of a hydrograph, at certain stage the flow is going to be overbank running into the floodplains from the main channel. On the other hand, as the flood receeds some water flows back to the main channel from the floodplains. To maintain the interflow between the main channel and floodplains as flood rises or receeds, there must exist differences in water surface elevation between them. Now as the interflow goes on the continuity equation for the left floodplain alone can be written as

$$q_2 = -\frac{\partial Q_2}{\partial x} - \frac{\partial A_2}{\partial t} \quad \cdots\cdots (15a)$$

and similarly, for the right floodplain, it is

$$q_3 = -\frac{\partial Q_3}{\partial x} - \frac{\partial A_3}{\partial t} \quad \cdots\cdots (15b)$$

In the above equations, q_2 and q_3 are interflows per unit channel length for the left and right floodplains, respectively. The conventions are negative for flow from main channel to floodplain, and positive for the reverse. Q_2 and Q_3 are flow rates in the left and right floodplains, respectively. A_2 and A_3 are flow cross-sectional areas for the left and right, respectively. For the main channel the interflow is the sum of those for the left and the right floodplains. So, one can write

$$q_1 = -(q_2 + q_3) \quad \cdots\cdots (16)$$

Consider the uneven water surface as comprised of a number of small surges propagating in the transverse direction across. This propagation of small surges will produce a flow in its direction of propagation, which should be equal to the interflow obtained from Eqs. (15a) and (15b). For the left floodplain, one can write

$$q_2 = c_2 \cdot \Delta E_2 \quad \cdots\cdots (17)$$

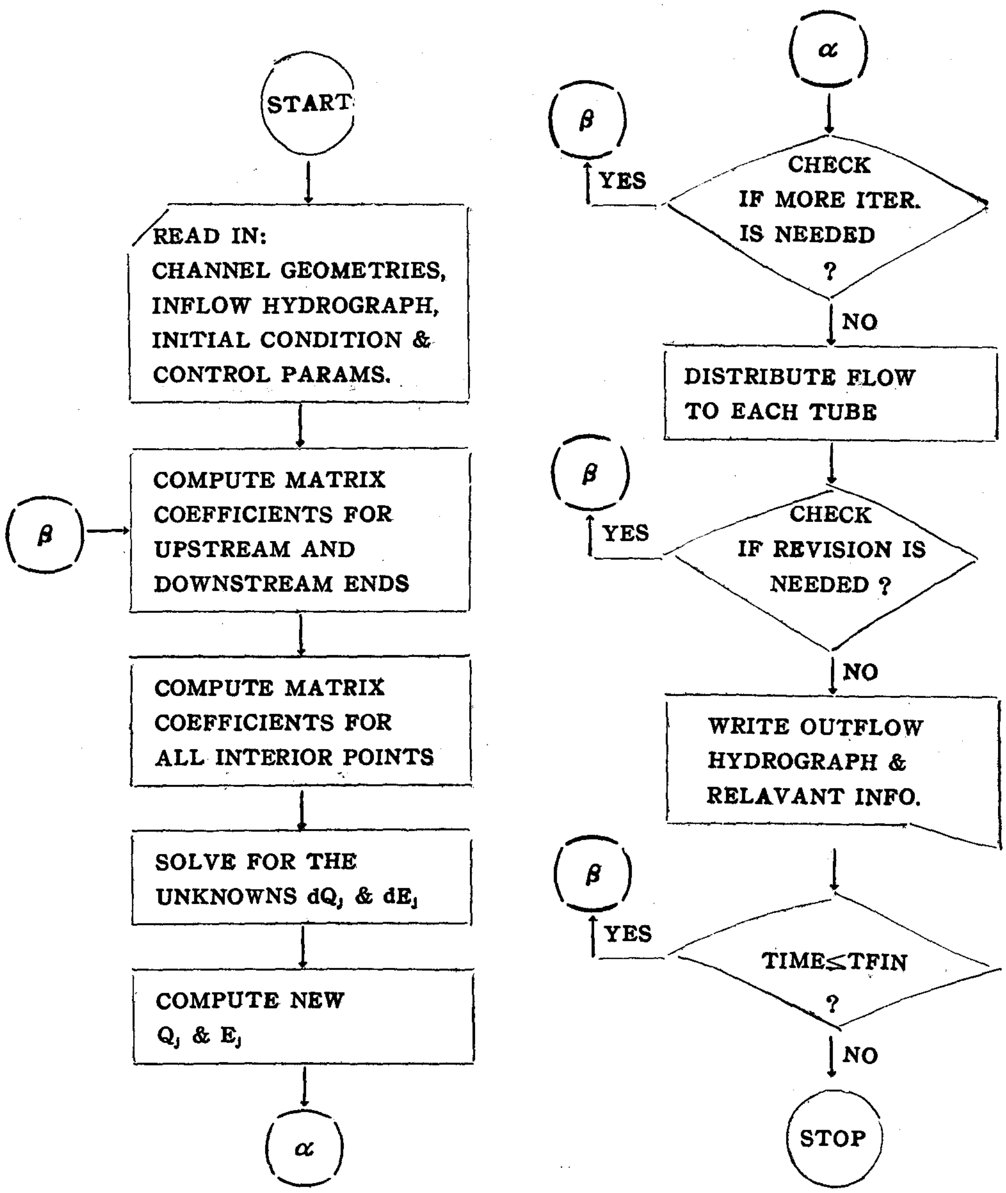
and the for the right floodplain

$$q_3 = c_3 \cdot \Delta E_3 \quad \cdots\cdots (18)$$

where c_2 and c_3 are wave celerities for the left and right floodplains, respectively. They can be expressed in terms of water depth in its respective floodplains. ΔE_2 and ΔE_3 are the differences in water surface levels required to produce q_2 and q_3, respectively.

By substituting the q_2 and q_3 obtained from Eqs. (15a) and (15b) into Eqs. (17) and(18), one can solve for ΔE_2 and ΔE_3. With these differences in water levels, the water surface elevation in each tube can now be corrected so that the three tubes have different water levels. Since the quantities q_2 and q_3 are only approximate answers at this point because they are computed using Q_j and E_j obtained with the assumption of same water level at a section. Therefore, the resulting ΔE_2 and ΔE_3 are also only approximate corrections to be applied. So, the process of computing q_2, q_3, ΔE_2 and ΔE_3 should be repeated until the change in each of these quantities between two successive iterations falls within a prescribed tolerance.

A schematic flow chart showing the major computational steps for the THREE-TUBE MODEL for unsteady flow in rivers is given in the flow chart shown. A more detailed account of the MODEL and its program listing can be found in Reference 12.



FLOW CHART SHOWING MAJOR STEPS OF COMPUTATION

V. MODEL CALIBRATION AND VERIFICATION

In dealing with simulation model of unsteady flow in rivers, one must recognize the fact that it is impossible to reproduce a river reach or a system exactly. One has to proceed with various simplifications such as linear variation of geometries between two river sections, and with various assumptions such as resistance law and its associated resistance coefficient. This idealization of real system, simulating an infinitely complex situation by a finite number of computational points, can never truely represent the actual situation. Hence the calibration and verification (or validation) of the model are necessary. Usually when a simulating model is built the resistanee coefficient such as Manning's n is the first element that needs to be calibrated because the resistance coeffieient can not be measured directly. Another element needs to be calibrated is the spatial and temporal distributions of the lateral inflow to the river reach under study. Although the lateral flow into a river system can be measured, it is almost never done. Therefore the determination of its distributions must rely on calibration. The work of calibration often takes more time than anything else.

Since both the Manning's coefficient and lateral flow distributions affect the resulting simulated hydrograph, and since there is no clear-cut point as to where the influence of one of them terminates and the other begins, it is very difficult to decide when one should stop adjusting the Manning's n or when to begin adjusting the lateral flow distributions. Fortunately, the range of n values for a river with given condition is usually known. So, the calibration is based on the observed hydrogaph which should be reproduced by the model as closely as possible first by adjusting (within the given range) the n values. At this point the lateral flow distributions are assumed to be uniform along the river reach and to be of the same (percentage-wise) time-distribution as the inflow hydrograph. Upon obtaining the n value, the lateral flow distribution is then allowed to vary until a better fit between the observed and simulated hydrographs is obtained. A more reasonable way of distributing the lateral flow space-wise is to apportion it according to the contributing area along the river. The importance of the lateral flow distribution becomes trivial if the volume of lateral flow is only of a small percentage of the total volume of outflow hydrograph.

Verification is a step that must be taken to validate a simulating model for a particular river reach after it is calibrated. The procedure of verification is to simulate other flood events of the same river reach. Strictly speaking, only the values of roughness coefficient and the lateral flow distributions resulted from the calibration run can be used in the verification runs. If the simulated results obtained from these verification runs agree well with the actual records, the model is considered verified for the particular river reach concerned. Objectively, there should be a set of criterion against which the results of the verification runs can be measured. However, such a criterion has not been established yet for the present study. So, it is relied upon the judegement of the investigators, which can not be completely free from subjectivity.

The THREE-TUBE MODEL is calibrated and verified for the following three river reaches: South Yamhill River in Oregon, U. S. A., Cho-Shui River and Potzu River, both in Taiwan. The selection of these rivers is based on two major considerations. First, the exsistence of floodplain and occurence of overbank flow were reviewed. Second, the availability of data necessary to run the model and against which to compare the results was checked. The basic needs include inflow and outflow hydrographs, cross-sectional geometries and their locations. Presented in the followings are the results of the calibration and verification runs for these three river reaches.

South Yamhill River: The South Yamhill is a relatively mild river in the State of Oregon, U. S. A. Geomorphologically, South Yamhill River is very stable. The river reach selected is of 4.03 miles long and has very distinct main channel and floodplain portions. The plan view of this reach is shown in Fig. 2. There is practically no

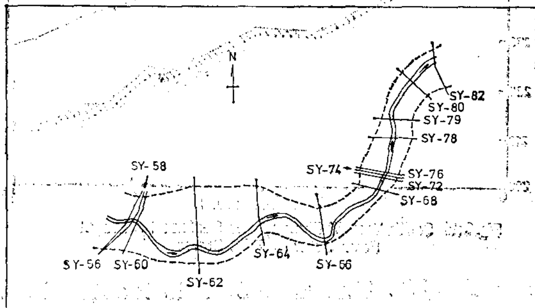


Fig.2 Plan of South Yamhill River
(SY 56−SY 82)

lateral flow coming into this reach. The Manning's n values have been well calibrated by the U. S. Soil Conservation Service (SCS) using the backwater computation method. So, it is felt that there is no need for another calibration by the THREE−TUBE MODEL. The n values, 0.05 for main channel, 0.118 for left floodplain and 0.125 for the right, obtained by the SCS are adopted directly for the verification run simulating the 1972 flood. The recorded inflow hydrograph and simulated outflow hydrograph are shown together in Fig. 3(a). There is no record of outflow hydrograph to check against,

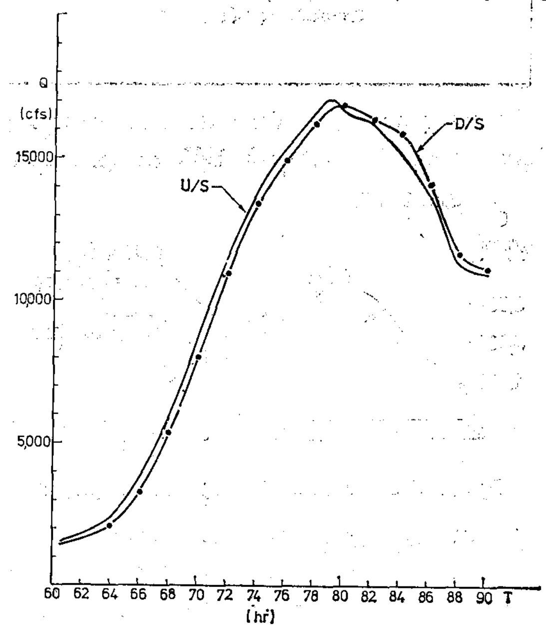


Fig.3(a) South Yamhill River 1972 Storm Hydrographs

but there is a record of high water levels during the flood event. The simulated water surface levels at the peak flow is plotted in Fig. 3(b) together with the recorded high water levels. They are in very good agreement.

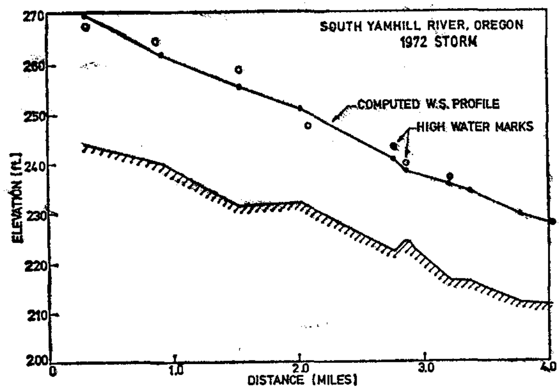


Fig.3(b) South Yamhill River Water Surface Profile at Peak Flow of 1972 Storm

Cho-Shui River (Meopu-Chichi): The Cho-Shui is located in central Taiwan and has a total length of 184.6 kilometers. The reach selected for calibration and verification is from Meopu to Chichi having a length of 12.8 kilometers. The plan view of this reach is shown in Fig. 4. Cho-Shui River is noted for its high rate of sediment transport

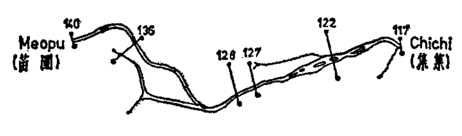


Fig. 4 Plan of Meopu-Chichi Reach Cho-shui River (濁水溪)

and is goemorphologically very unstable. Channel cross-sections can vary greatly from time to time. The flood event of October 9, 1973, as shown in Fig. 5 (a), was used for

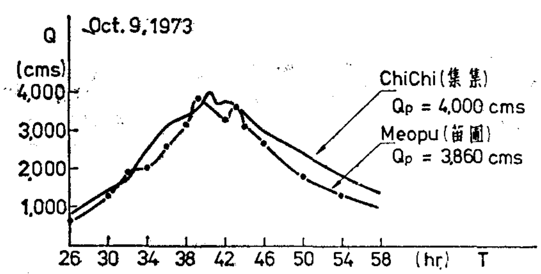


Fig.5(a) Cho-Shui River 1973 Storm Hydrographs

calibration. The cross-section survey of 1968 was adopted because this is the nearest to the flood event available. The Manning's n values for the Cho-Shui have been calibrated by Taiwan Provincial Water Conservancy Bureau (TPWCB) using also backwater computation method [3]. The n value of 0.02 and 0.041 for main channel and floodplains, respectively, are taken. The lateral flow for this particular flood event is only 19%. So, its spatial distribution is simply assumed to be uniform and time-distribution to be the same (percentage-wise) as the inflow hydrograph. There is no need for calibrations of Manning's n nor of lateral flow distributions. However, because of the ever changing channel cross-sections, it was thought that the rating curve at the downstream end of the reach may need some adjustments.

The first trial run without any change in the rating curve yieded no good results. It was then decided to subtract some amount from the depth of the rating curve [14]. After several experimental runs with different amounts of subtraction, it was found that by subtracting an amount of 1.5 meters from the depth coordinate of the rating curve the simulated hydrgoraph has a reasonably good agreement with recorded one. As can be seen from Fig.5(b), the simulated peak flow is 4,478 cms while the recorded

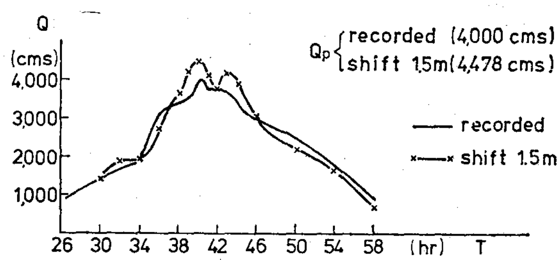


Fig.5(b) Results of Calibration of Cho-Shui River

is 4,000 cms. The simulated peak occurs 15 minutes earlier. Another flood event, of 1970 as shown in Fig. 6(a), was simulated with the same subtraction, but the results

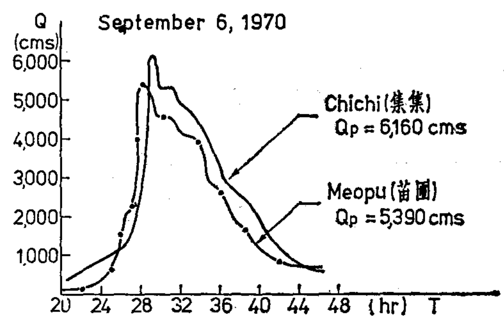


Fig. 6(a) Cho-Shui River 1970 Storm Hydrographs

were not good. Again, after several experimental runs, it was found with a subtraction of 1.1 meters reasonably good results can be obtained as shown in Fig. 6(b).

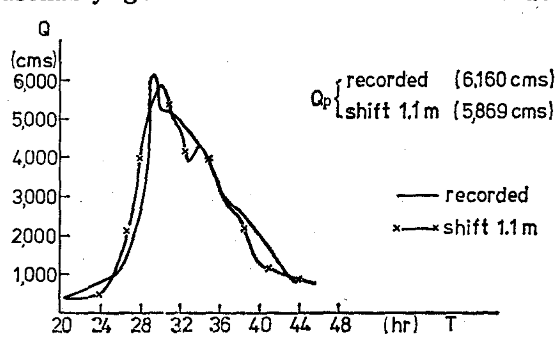


Fig.6(b) Results of Verification of Cho-Shui River

The possible explanation for shifting the depth coordinate of the rating curve is that during the intervening years between the cross-section survey and flood events the reference point of the rating curve might have changed without taking notes of it.

Potzu River (*Neuchaochi Bridge-Potzu*): The Potzu is a relatively small stream located in South Central Taiwan and is quite stable geomorphologically. The reach selected, shown in Fig. 7, has a length of 26 kilometers. The flood event of June 11, 1974,

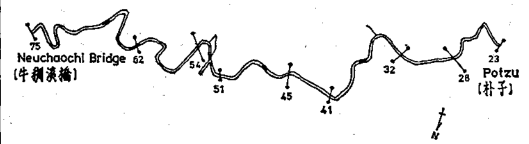


Fig.7 Plan of Neuchaochi Bridge—Potzu Reach, Potzu River (朴子溪)

as shown in Fig. 8(a), and the cross-section survey of 1974 were used for calibration. The volume of lateral inflow is 118% of that of the upstream inflow. So, it is

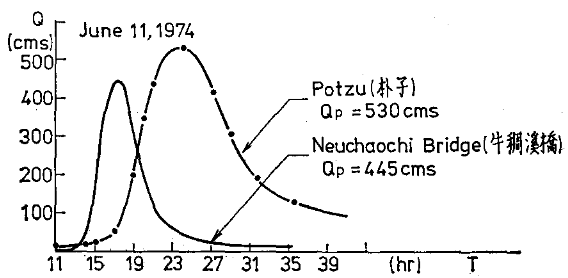


Fig. 8(a) Potzu River 1974 Storm Hydrographs

obvious that the lateral flow distribution will play quite an important role in determining the simulated results. The resulting simulated outflow hydrograph obtained after several trials of the calibration run [14] is shown in Fig. 8(b). The

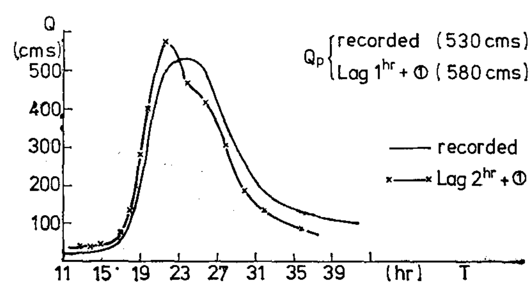


Fig. 8(b) Results of Calibration of Potzu River

results of this calibration run are: (1) Manning's n values are 0.041 for the main channel and 0.074 for the floodplains; (2) the spatial distribution of lateral flow is linear with the upstream end to downstream end ratio of 3:1; and (3) the temporal distribution is the same as that of the upstream end inflow hydrograph but with one hour delay. These results are applied to the flood of August 24, 1973, shown in Fig.

9(a). The resulting outflow hydrograph is shown in Fig. 9(b) along with the recorded. They compare fairly well.

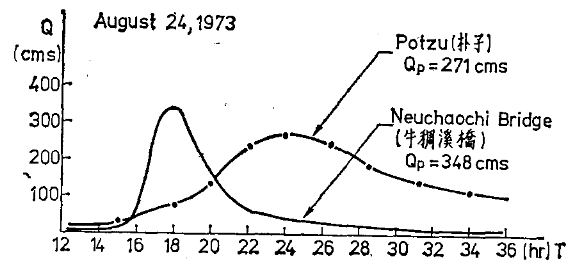


Fig. 9(a) Potzu River 1973 Storm Hydrographs

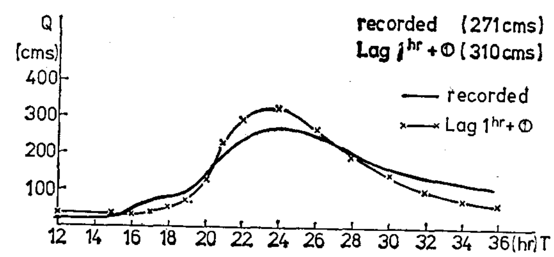


Fig. 9(b) Results of Verification of Potzu River

VI. CONCLUSIONS

1. Procedures for computation of unsteady flow in rivers have progressed in the past decade or so from the much simplified routing methods to sophisticated simulation models using finite difference methods. Of the many finite difference schemes, the implicit mehtod is considered to be the most adequate one for simulating unsteady flow in rivers.

2. For Channel with floodplains, it is very difficult to construct a simulation model because there exist large differences in flow depth and in roughness between floodplains and the main channel. The THREE-TUBE MODEL has been developed trying to overcome this difficulty by treating main channel and floodplains separately and allowing interflow to take place between them. In this way the mass and momentum exchages arising from the interflow can be taken into consideration.

3. The MODEL has been calibrated and verified in three river reaches: South Yamhill River, Cho-Shui River (Meopu-Chichi), and Potzu River (Neuchaochi Bridge-Potzu). In the case of the South Yamhill, the roughess coefficient has been well calibrated by backwater method. So only a verification run was carried out. The resulting water surface profile agrees well with the recorded. In the case of Cho-Shui, after adjusting the downstream rating curve, reasonably good results were obtained. Fairly good agreement between simulated and recorded results can also be seen in the case of Potzu river.

4. From the above, it appears that the MODEL can be used in Taiwan's streams

to simulate flood flows. However, it is recommended that more calibration and verification works be carried out before it is adopted for wide use.

ACKNOWLEDGMENTS

The results reported herin is a protion of a research project carried out under the U.S.-Rep. of China Cooperative Science Program, supported jointly by the National Science Foundation (U.S.) and National Science Council (Rep. of China). The senior author is indebted to Howard University, Washington, D. C. for granting him sabbatical leave while stationing in Taiwan to carry out the research. The authors are very grateful to National Taiwan University for providing facilities and orther generous assistances. Thanks are due to Taiwan Provincial Water Conservancy Bureau, and to Mr. Owen P. Lee of SCS-USDA for providing valuable data needed to run the computer program. Last but not least, Mr. Yuan-Jia Lei is to be thanked for his assistance in running the computer program.

REFERENCES

1. Baltzer R. A., and C. Lai, "Compuetr Simulation of Unsteady Flow in Waterways," J. of the Hyd. Div., Proc. of ASCE, Vol. 94, No. HY4, July 1968.

2. Chow, V.T., *Open Channel Hydraulics*, McGraw-Hill Book Co., Inc., N.Y. 1959.

3. Cunge, J. A., "On the Subject of a Flood Propagation Method," J. of Hyd. Res., IAHR, 7, No. 2, 1969.

4. Hayami, S., *On Propagation of Flood Waves*, Bulletin No. 1, Disaster Prevention Research Institute, Kyoto University, Dec. 1951.

5. Hegly, V. M., "Flow in Eastern Canals of Compound Cross-section," Trans., ASCE, Nov. 1937

6. Lighthill, M. J. and G. B. Whitham, "On Kinematic Waves - I. Flood Movement in Long Rivers," Proc. Royal Soc. of London, Series A, 229, 1955.

7. McCarthy, G. T., *The Unit Hydrograph and Flood Routing*, The U. S. Engineer School, Fort Belvoir, Va. 1938.

8. Mahmood, K. and V. Yevjevich, Ed., *Unsteady Flow in Open Channels*, Vol. I, Water Resources Publications, Fort Collins, Colorado, 1975.

9. Price, R. K., *Flood Routing Methods for British Rivers*, Report No. INT 111, Hydraulic Research Station, Wallingford, Berkshire, England, 1973.

10. Yen, C. L., G. H. Comer and H. N. Holtan, *USDAHL-70 Flood Routing Method*, PPhI Report No. 7, ARS-USDA, August 1974.

11. Yen, C. L., and D. E. Overton, "Shape Effects on Resistance in Floodplain Channels," J. of the Hyd. Div., Proc. of ASCE, Vol 99, No. HY1, Jan. 1973.

12. Yen, C. L. and T. W. Wang, *Flood Routing in Floodplain Channels - Field Testing*, Final Project Report to NSC (Rep. of China) and NSF (U.S.), Sept. 1976 (in preparation),

13. Taiwan Provincial Water Conservancy Bureau, *Review on Safety of Dikes along Cho-Shui River and Plans for Heightening and Strengthening*, Publications No. 82, 1972. (in Chinese).

14. Lei, Yuan-Jia, *Testing and Application of THREE-TUBE FLOOD ROUTING MODEL in Taiwan's Streams*, M. S. Thesis, Dept. of Civil Engrg., National Taiwan University, Taipei, June 1976 (in Chinese).