Using Linear Programming System for Land Forming Design

線性規劃系統在農田整地設計上之應用性

Researcher, Department of Agricultural Engineering,
McGill University, Quebec, Canada

李 醒 洲

Robin S.C. Lee

INTRODUCTION

Land Forming or Land Leveling for surface drainage and surface irrigation is modifying the surface relief of field to a planned grade to provide a more suitable land surface for successfully handle the irrigation and drainage water over that field without causing erosion.

Criteria that influence the land leveling are soil, slope, climate, crops, methods of irrigation, and the desires of the farmer. It is desirable to accomplish this modification at the cheapest cost within certain limits which relate to the aforementioned criteria.

Many methods of land forming design have been and are being used as has been indicated by Shih and Kriz (1971). Among which four basic methods that commonly used are: 1) The plane method 2) The profile method 3) The plan-inspection method 4) The contour-adjustment method. Each of these existing methods has its own unique way of determining the best design, but none of them uses mathematical optimization techniques for minimizing the earth moved in the entire field. Recently, a new method which is superior to these previously used has been introduced by Shih and Kriz (1971). It is using the linear programming method to determine the least cost pattern for moving the earth for a particular design. In addition to a plane surface design, Shih and Kriz also defined four other types of land forming design which involve broken slopes in one or both directions.

OBJECTIVES

This paper illustrates how linear programming can be applied to solve a land forming problem, and the usage of LPS/360 computer routine is also introduced. An optimal design can be obtained directly from the computer program output.

LINEAR PROGRAMMING MODEL

Linear programming is a mathematical optimization technique which optimizes some objective function subject to certain constraints. It is required that all the mathematical functions in the model are linear functions, and all the variables are restricted to be greater than or equal to zero. Thus, linear programming involves the planning of activities in order to obtain and "optimal" result, i.e. a result which reaches the specified goal best among all feasible alternatives.

The linear programming model has a general form as follows:

Maximize (or minimize)

$$Z = C_1X_1 + C_2X_2 + \cdots\cdots + C_nX_n$$

Subject to the restrictions

$$\begin{array}{l} A_{11}X_1 + A_{12}X_2 + \cdots\cdots + A_{1n}X_n \\ A_{21}X_1 + A_{22}X_2 + \cdots\cdots + A_{2n}X_n \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ A_{m1}X_1 + A_{m2}X_2 + \cdots\cdots + A_{mn}X_n \end{array} \left(\begin{array}{c} \geqq \\ = \\ \leqq \end{array}\right) \begin{array}{l} B_1 \\ B_2 \\ \cdots \\ B_m \end{array}$$

$$X_1,\ X_2,\ X_3 \cdots\cdots X_n \geqq 0$$

where C_i, A_{ij}, B_i are known constants; m and n are positive integers. The function, Z, being maximized (or minimized) is called the objective function. The restrictions are referred to as constraints or restraints. The variables, X_i, being solved for are called decision variables.

In the formulation of the linear programming model in land forming design, the objective function is to minimize the sum of cuts in the field subject to the constraints of the land slope and the depth ratio of cuts to fills. The design problem illustrated here is an irregular shaped field with five stations in both row and cross row directions as shown in Figure 1. Each station is placed 100 ft apart so it covers a subdivision of 100^2 ft^2. Those stations located on the boundary line of the field may have a subdivision which is larger than or less than the unit area of 100^2 ft^2, a weighting value is then determined for each respective station. The original elevation at each station is surveyed and is expressed on this figure. It is assumed that the station (1,1) be the station with highest elevation in the final design, and the uniform slope (plane surface) is desired on both directions (Figure 2). It is

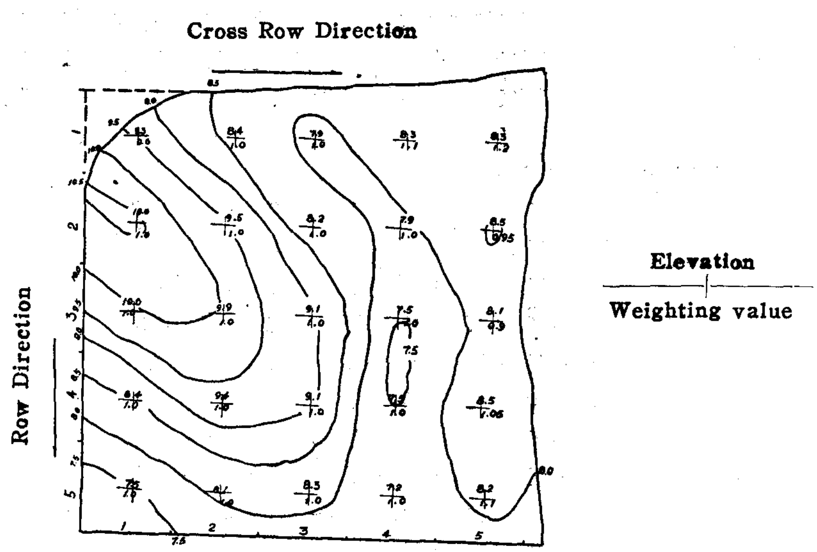


Fig. 1 Layout of the field

Scale: 1″=100′

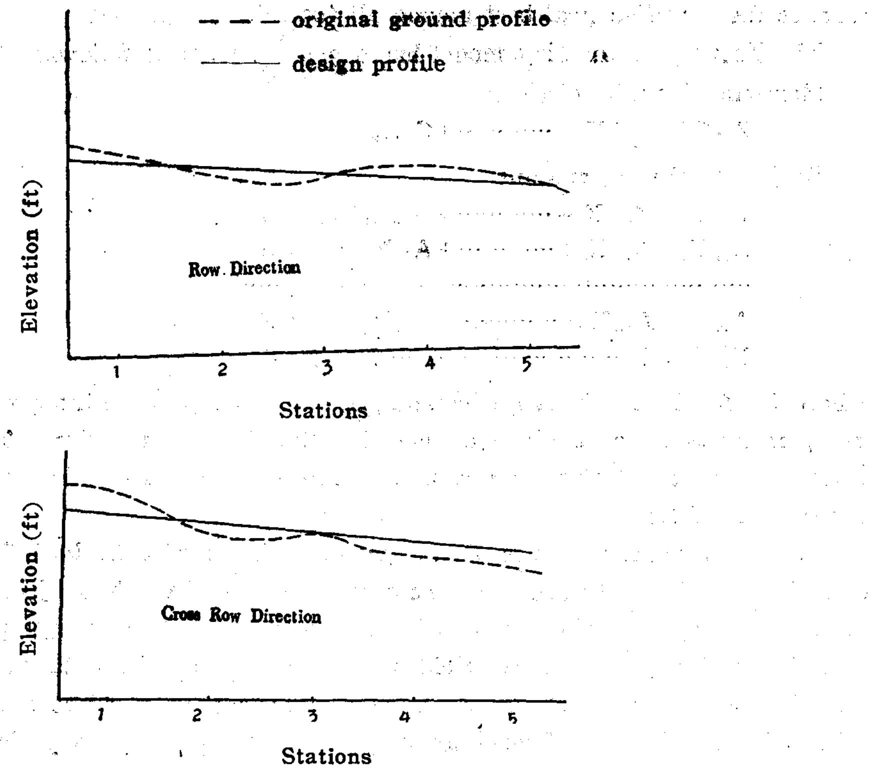


Fig. 2 Uniform slope on row and cross row directions.

also preassumed that the required cut-fill ratio be 1.40±0.06 (1.34~1.46) and the cross row slope SC=0~0.3% row slope SB=0~0.3%. Then the linear programming model for this problem can be formulated as below:

Minimize:
$$\begin{aligned} Z=\,&0.6X_{11}+1.0X_{12}+1.0X_{13}+1.1X_{14}+1.2X_{15}+1.0X_{21}\\ &+1.0X_{22}+1.0X_{23}+1.0X_{24}+0.95X_{25}+1.0X_{31}+1.0X_{32}\\ &+1.0X_{33}+1.0X_{34}+0.9X_{35}+1.0X_{41}+1.0X_{42}+1.0X_{43}\\ &+1.0X_{44}+1.05X_{45}+1.0X_{51}+1.0X_{52}+1.0X_{53}+1.0X_{54}\\ &+1.1X_{55}\end{aligned}$$

Subject to:
$$\left.\begin{aligned} -X_{11}+X_{12}+Y_{11}-Y_{12}-SC&=8.4-9.3\\ -X_{12}+X_{13}+Y_{12}-Y_{13}-SC&=7.9-8.4\\ -X_{13}+X_{14}+Y_{13}-Y_{14}-SC&=8.3-7.9\\ -X_{14}+X_{15}+Y_{14}-Y_{15}-SC&=8.3-8.3\\ -X_{21}+X_{22}+Y_{21}-Y_{22}-SC&=9.5-10.6\\ -X_{22}+X_{23}+Y_{22}-Y_{23}-SC&=8.2-9.5\\ -X_{23}+X_{24}+Y_{23}-Y_{24}-SC&=7.9-8.2\\ -X_{24}+X_{25}+Y_{24}-Y_{25}-SC&=8.5-7.9\\ -X_{31}+X_{32}+Y_{31}-Y_{32}-SC&=9.9-10.0\\ -X_{32}+X_{33}+Y_{32}-Y_{33}-SC&=9.1-9.9\\ -X_{33}+X_{34}+Y_{33}-Y_{34}-SC&=7.5-9.1\\ -X_{34}+X_{35}+Y_{34}-Y_{35}-SC&=8.1-7.5\end{aligned}\right\}\cdots\cdots\cdots\cdots(\mathrm{I})$$

$$
\left.\begin{array}{l}
-X_{41}+X_{42}+Y_{41}-Y_{42}-SC= 9.4- 8.4\\
-X_{42}+X_{43}+Y_{42}-Y_{43}-SC= 9.1- 9.4\\
-X_{43}+X_{44}+Y_{43}-Y_{44}-SC= 7.5- 9.1\\
-X_{44}+X_{45}+Y_{44}-Y_{45}-SC= 8.5- 7.5\\
-X_{52}+X_{53}+Y_{52}-Y_{53}-SC= 8.3- 8.1\\
-X_{53}+X_{54}+Y_{53}-Y_{54}-SC= 7.2- 8.3\\
-X_{54}+X_{55}+Y_{54}-Y_{55}-SC= 8.2- 7.2
\end{array}\right\}
$$

$$
\left.\begin{array}{l}
-X_{11}+X_{21}+Y_{11}-Y_{21}-SB=10.6- 9.3\\
-X_{21}+X_{31}+Y_{21}-Y_{31}-SB=10.0-10.6\\
-X_{31}+X_{41}+Y_{31}-Y_{41}-SB= 8.4-10.0\\
-X_{41}+X_{51}+Y_{41}-Y_{51}-SB= 7.5- 8.4\\
-X_{12}+X_{22}+Y_{12}-Y_{22}-SB= 9.5- 8.4\\
-X_{22}+X_{32}+Y_{22}-Y_{32}-SB= 9.9- 9.5\\
-X_{32}+X_{42}+Y_{32}-Y_{42}-SB= 9.4- 9.9\\
-X_{42}+X_{52}+Y_{42}-Y_{52}-SB= 8.1- 9.4\\
-X_{13}+X_{23}+Y_{13}-Y_{23}-SB= 8.2- 7.9\\
-X_{23}+X_{33}+Y_{23}-Y_{33}-SB= 9.1- 8.2\\
-X_{33}+X_{43}+Y_{33}-Y_{43}-SB= 9.1- 9.1\\
-X_{43}+X_{53}+Y_{43}-Y_{53}-SB= 8.3- 9.1\\
-X_{14}+X_{24}+Y_{14}-Y_{24}-SB= 7.9- 8.3\\
-X_{24}+X_{34}+Y_{24}-Y_{34}-SB= 7.5- 7.9\\
-X_{34}+X_{44}+Y_{34}-Y_{44}-SB= 7.5- 7.5\\
-X_{44}+X_{54}+Y_{44}-Y_{54}-SB= 7.2- 7.5\\
-X_{15}+X_{25}+Y_{15}-Y_{25}-SB= 8.5- 8.3\\
-X_{25}+X_{35}+Y_{25}-Y_{35}-SB= 8.1- 8.5\\
-X_{35}+X_{45}+Y_{35}-Y_{45}-SB= 8.5- 8.1\\
-X_{45}+X_{55}+Y_{45}-Y_{55}-SB= 8.2- 8.5
\end{array}\right\}\cdots\cdots\cdots\cdots(\mathrm{II})
$$

$$
\begin{array}{l}
0.6X_{11}+1.0X_{12}+1.0X_{13}+1.1X_{14}+1.2X_{15}+1.0X_{21}+1.0X_{22}\\
1.0X_{23}+1.0X_{24}+0.95X_{25}+1.0X_{31}+1.0X_{32}+1.0X_{33}+\\
1.0X_{34}+0.9X_{35}+1.0X_{41}+1.0X_{42}+1.0X_{43}+1.0X_{44}+\\
1.05X_{45}+1.0X_{51}+1.0X_{52}+1.0X_{53}+1.0X_{54}+1.1X_{55}-\\
1.34(0.6Y_{11}+1.0Y_{12}+1.0Y_{13}+1.1Y_{14}+1.2Y_{15}+1.0Y_{21}\\
+1.0Y_{22}+1.0Y_{23}+1.0Y_{24}+0.95Y_{25}+1.0Y_{31}+1.0Y_{32}+\\
1.0Y_{33}+1.0Y_{34}+0.9Y_{35}+1.0Y_{41}+1.0Y_{42}+1.0Y_{43}+\\
1.0Y_{44}+1.05Y_{45}+1.0Y_{51}+1.0Y_{52}+1.0Y_{53}+1.0Y_{54}+1.1Y_{55})\geqq 0\\
0.6X_{11}+1.0X_{12}+1.0X_{13}+1.1X_{14}+1.2X_{15}+1.0X_{21}+\\
1.0X_{22}+1.0X_{23}+1.0X_{24}+0.95X_{25}+1.0X_{31}+1.0X_{32}+\\
1.0X_{33}+1.0X_{34}+0.9X_{35}+1.0X_{41}+1.0X_{42}+1.0X_{43}+1.0X_{44}\\
+1.05X_{45}+1.0X_{51}+1.0X_{52}+1.0X_{53}+1.0X_{54}+1.1X_{55}-1.46\\
(0.6Y_{11}+1.0Y_{12}+1.0Y_{13}+1.1Y_{14}+1.2Y_{15}+1.0Y_{21}+1.0Y_{22}\\
+1.0Y_{23}+1.0Y_{24}+0.95Y_{25}+1.0Y_{31}+1.0Y_{32}+1.0Y_{33}+1.0Y_{34}\\
+0.9Y_{35}+1.0Y_{41}+1.0Y_{42}+1.0Y_{43}+1.0Y_{44}+1.05Y_{45}+\\
1.0Y_{51}+1.0Y_{52}+1.0Y_{53}+1.0Y_{54}+1.1Y_{55})\leqq 0
\end{array}\quad\cdots(\mathrm{III})
$$

$$\left.\begin{array}{l} SC \geqq +0.0 \\ SC \leqq 0.3 \\ SB \geqq +0.0 \\ SB \leqq 0.3 \end{array}\right\} \cdots\cdots (IV)$$

Where X_{ij}=depth of cut at station (i,j) in terms of ft.

Y_{ij}=depth of fill at station (i,j) in terms of ft.

SC=cross row slope in terms of ft/100ft.

SB=row slope in terms of ft/100ft.

Constraint group (I) and (II) assure that the slopes between each individual stations in cross row and row directions are equal to SC and SB respectively. Constraint group (III) makes sure that the cut-fill ratio is within upper and lower limits. Constraint group (IV) sets the restriction that the cross row slope, SC, and row slope, SB, are within desired tolerances.

The decision variables involved in this program are the cuts X's, fills Y's and the design slopes SC, SB. Since the objective function is to minimize the sum of cuts by depth per unit area, variables Y's, SC and SB are not contribute any to the objective. However, they are treated as artificial variables with zero coefficients in the objective function.

COMPUTER PROGRAM DESCRIPTION

This problem is sloved on IBM 360/40 digital computer using packaged computer routines. A special data input format is required to input the problem data and to construct an input data deck for LPS/360. The LPS/360 provides the user with a simple yet efficient means of solving linear programming problems as is demonstrated here. Detailed information for preparing a problem data deck is provided by the IBM Program Description Manual (see References). Fig. 3 shows schematically all the necessary cards required for program control and system control in order to produce an output report for the solution of the problem. The actual printout from the computer is attached in Appendix.

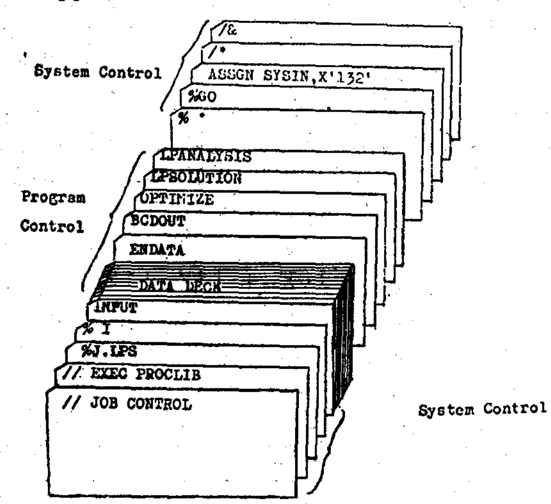


Fig. 3 Computer Deck Setup for LPS/360

In this particular program, the total memory requirements are 56567 bytes. For a field with 5X5 stations the problem contains 47 selected rows, 100 variables and 330 column elements, the computer run time requires 8.27 minutes.

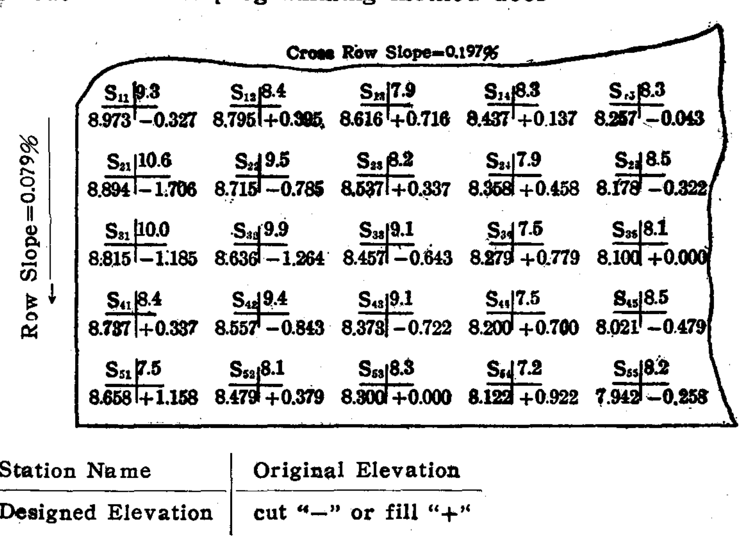
RESULTS AND DISCUSSION

Figure 4 presents the final results of this land forming design problem. The computer program calculates the value of cut or fill for each station. The designed elevation is obtained by adding or subtracting the depth of fill or cut from the original elevation. The cut-fill ratio falls on the lower bound of the given range which gives the value of 1.34. The value obtained for the objective function is 8.481, which is the total sum of cuts by depth. The total volume of cuts in terms of cubic yerd is calculated by multiplying the total depth of cuts by the unit area, 100^2 ft^2, then divided by 27 (1 cu.yd.=27 cu. ft.). The slopes in cross row and row directions are 0.179 % and 0.079% respectively. Both are within allowed tolerances. It is found that the weighting factor given for each subdivision only effects the volume of cuts as well as the cut-fill ratio, but not on the slope constraints.

In most cases every adjacent station is placed 100 ft apart so that the slope can be easily expressed in terms of percentage. It should be noted that if the grid space is not choosen as 100 ft, the slope can also be expressed in terms of percentage simply by dividing that distance.

It is understood that the aim of a land forming design is to minimize the cost of reshaping the field for agriculture use. An agricultural engineer is seeking a design which gives the minimal total volume of earth required to be moved. Linear programming method offers a great advantage in this design in that it is capable of puting bounds on decision variables. This assures that the resulting solution is within desired limits.

It has been concluded that no other design procedure will resultin a field with a smaller sum of cuts as linear programming method does.



No. of station with cuts=12
No. of station with fills=11
No. of station with neither cut nor fill=2
Total cuts/Total fills=1.34
Total volume of cuts=$8.481\times 100^2/27=3141.111yd^3$.

Fig. 4 Final design of land forming problem

APPENDIX

Computer printout of the problem solution

Variable	Entries Type		Solution Activity	Upper Bound	Lower Bound	Current Cost	Reduced Cost
CUT	B*	0	8.481	—	—	-1.000	1.000
CR1	EQ	0	-0.900	-0.900	-0.900	0.0	0.0
CR2	EQ	0	-0.500	-0.500	-0.500	0.0	2.197
CR3	EQ	0	0.400	0.400	0.400	0.0	1.013
CR4	EQ	0	0.0	0.0	0.0	0.0	-0.822
CR6	EQ	0	-1.100	-1.100	-1.100	0.0	2.230
CR7	EQ	0	-1.300	-1.300	-1.300	0.0	0.0
CR8	EQ	0	-0.300	-0.300	-0.300	0.0	0.0
CR9	EQ	0	0.600	0.600	0.600	0.0	0.0
CR11	EQ	0	-0.100	-0.100	-0.100	0.0	-0.779
CR12	EQ	0	-0.800	-0.800	-0.800	0.0	-0.559
CR13	EQ	0	-1.600	-1.600	-1.600	0.0	0.0
CR14	EQ	0	0.600	0.600	0.600	0.0	0.0
CR16	EQ	0	1.000	1.000	1.000	0.0	0.0
CR17	EQ	0	-0.300	-0.300	-0.300	0.0	0.897
CR18	EQ	0	-1.600	-1.600	-1.600	0.0	0.0
CR19	EQ	0	1.000	1.000	1.000	0.0	0.0
CR21	EQ	0	0.600	0.600	0.600	0.0	-1.184
CR22	EQ	0	0.200	0.200	0.200	0.0	-1.776
CR23	EQ	0	-1.100	-1.100	-1.100	0.0	-0.017
CR24	EQ	0	1.000	1.000	1.000	0.0	-1.201
R1	EQ	0	1.300	1.300	1.300	0.0	0.335
R2	EQ	0	-0.600	-0.600	-0.600	0.0	-1.337
R3	EQ	0	-1.600	-1.600	-1.600	0.0	0.0
R4	EQ	0	-0.900	-0.900	-0.900	0.0	-0.592
R5	EQ	0	1.100	1.100	1.100	0.0	-2.789
R6	EQ	0	0.400	0.400	0.400	0.0	0.0
R7	EQ	0	-0.500	-0.500	-0.500	0.0	0.338
R8	EQ	0	-1.300	-1.300	-1.300	0.0	0.0
R9	EQ	0	0.300	0.300	0.300	0.0	0.592
R10	EQ	0	0.900	0.900	0.900	0.0	0.0
R11	EQ	0	0.0	0.0	0.0	0.0	0.0
R12	EQ	0	-0.800	-0.800	-0.800	0.0	1.455
R13	EQ	0	-0.400	-0.400	-0.400	0.0	1.184
R14	EQ	0	-0.400	-0.400	-0.400	0.0	0.592
R15	EQ	0	0.0	0.0	0.0	0.0	0.0
R16	EQ	0	-0.300	-0.300	-0.300	0.0	-0.592
R17	EQ	0	0.200	0.200	0.200	0.0	-0.152
R18	EQ	0	-0.400	-0.400	-0.400	0.0	0.379
R19	EQ	0	0.400	0.400	0.400	0.0	0.0
R20	EQ	0	-0.300	-0.300	-0.300	0.0	0.587
CFL	LL	0	0.0	—	0.0	0.0	-0.442
CFU	B*	0	-0.760	0.0	—	0.0	-0.000
SLRL	B*	0	0.079	—	0.0	0.0	-0.000
SLRU	B*	0	0.078	0.800	—	0.0	0.000

Variable Eniries Type			Solution Activity	Upper Bound	Lower Bound	Current Cost	Reduced Cost
SLCRL	B*	0	0.179	—	0.0	0.0	0.000
SLCRU	B*	0	0.178	0.300	—	0.0	-0.000
CU11	B*	5	0.327	—	0.0	0.600	0.0
CU12	LL	6	0.0	—	0.0	1.000	-1.151
CU13	LL	6	0.0	—	0.0	1.000	-1 151
CU14	LL	6	0.0	—	0.0	1.100	-1.264
CU15	B*	5	0.043	—	0.0	1.200	0.0
CU21	B*	6	1.706	—	0.0	1.000	0.0
CU22	B*	7	0.785	—	0.0	1.000	0.0
CU23	LL	7	0.0	—	0.0	1.000	-1.151
CU24	LL	7	0.0	—	0.0	1.000	-1.151
CU25	B*	6	0.322	—	0.0	0.950	0.0
CU31	B*	6	1.185	—	0.0	1.000	0.0
CU32	B*	7	1.264	—	0.0	1.000	0.0
CU33	B*	7	0.643	—	0.0	1.000	0.0
CU34	LL	7	0.0	—	0.0	1.000	-1.151
CU35	LL	6	0.0	—	0.0	0.900	-0.882
CU41	LL	6	0.0	—	0.0	1.000	-1.151
CU42	B*	7	0.843	—	0.0	1.000	0.0
CU43	B*	7	0.722	—	0.0	1.000	0.0
CU44	LL	7	0.0	—	0.0	1.000	-1.151
CU45	B*	6	0.479	—	0.0	1.050	0.0
CU51	LL	5	0.0	—	0.0	1.000	-1.151
CU52	LL	6	0.0	—	0.0	1.000	-1.151
CU53	LL	6	0.0	—	0.0	1.000	-0.253
CU54	LL	6	0.0	—	0.0	1.000	-1.151
CU55	B*	5	0.258	—	0.0	1.100	0.0
FI11	LL	4	0.0	—	0.0	0.0	-0.689
FI12	B*	5	0.395	—	0.0	0.0	0.0
FI13	B*	5	0.716	—	0.0	0.0	0.0
FI14	B*	5	0.137	—	0.0	0.0	0.0
FI15	LL	4	0.0	—	0.0	0.0	-1.382
FI21	LL	5	0.0	—	0.0	0.0	-1.151
FI22	LL	6	0.0	—	0.0	0.0	-1.151
FI23	B*	6	0.337	—	0.0	0.0	0.0
FI24	B*	6	0.458	—	0.0	0.0	0.0
FI25	LL	5	0.0	—	0.0	0.0	-10.92
FI31	LL	5	0.0	—	0.0	0.0	-1.151
FI32	LL	6	0.0	—	0.0	0.0	-1.151
FI33	LL	6	0.0	—	0.0	0.0	-1.151
FI34	B*	6	0.779	—	0.0	0.0	0.0
FI35	LL	5	0.0	—	0.0	0.0	-0.152
FI41	B*	5	0.337	—	0.0	0.0	0.0
FI42	LL	6	0.0	—	0.0	0.0	-1.151
FI43	LL	6	0.0	—	0.0	0.0	-1.151
FI44	B*	6	0.700	—	0.0	0.0	0.0
FI45	LL	5	0.0	—	0.0	0.0	-1.205
FI51	B*	4	1.158	—	0.0	0.0	0.0
FI52	B*	5	0.379	—	0.0	0.0	0.0
FI53	LL	5	0.0	—	0.0	0.0	-0.898
FI54	B*	5	0.922	—	0.0	0.0	0.0
FI55	LL	4	0.0	—	0.0	0.0	-1.264
SB	B*	22	0.079	—	0.0	0.0	0.0
SC	B*	22	0.179	—	0.0	0.0	0.0

—下轉43頁—