專論

整地工程填挖土方平衡之計算

Locating the cut Plane for Land Leveling

Joseph You-tsai Hung*

Abstract

A method of locating the cut plane in the plane method for land leveling has been found, The equations shown in the paper can be used to adjust the cut plane in such a way that it will provide the volume ratio (C/F or F/C) equal to the presumed C/F or F/C ratio.

The plane method is one of the most common methods used in land leveling design for a uniform down field and a uniform cross slope. The key point in applying it. is to locate a cut plane with appropriate slopes in both row and cross directions in order to ease irrigation and drainage practices. The first question is how to locate the cut plane in such a way that a minimum soil movement can be achieved.

Several methods have been cited to locate the cut plane in an attempt to provide a balance between cut and fill volumes. Givan (1940) used the least-square method for determining the slopes of least grading in a rectangular field. Chugg (1947) further tapplied Givan's method in a field with more than a rectangular block. Raju (1960) suggested the fixed-volume center method for btth a rectangular and more than a rectangular block field. Marr (1957) combined the least-square and average profiles method. The United States Department of Agriculture (1961) also cited another plane method in land laveling. Most recently, some computer techniques have been developed extensively by Shih and Kriz (1971) in comparing various types and methods of land forming designs.

Most methods mentioned are based on either the balance of the total depth of excavation and the total depth of fill or the balance of the total volume of excavation and the total volume of fill. However, it is obvious that due to the difference in types of soils, the balanced desig is not always true if the minimum earth moving is required. Engineers have to consider the possible shrinkage and swelling of th disturbed soils. Marr (1957) suggested a method for computations of cut and fill adjustments and used an approximate method for adjusting the shrinkage or swelling. The United States Department of Agriculture (1961) also mentioned a "cut/fill" ratio (by volume) approach. A trial-and-error method of obtaining the presumed cut/fill ratio has been extensively used by many workers.

The purpose of this paper is to provide a method which can be used to locate the exact location of the cut plane under a presumed C/F (or F/C) ratio (volume ratio) with known uniform row and cross slopes. This method can be applied to either a rectangular or more than a rectangular block.

* Associate Professor, Agricultural Engineering Department, California State Polytechnic University, Pomona,USA

Methods of Analysis

One of the conventional methods for estimating the quantities of cut and fill in a grid system for land leveling requires computation of the weighted average elevation of the field plot. The weighted average elevation used to estimate the center location of the cut plane is obtained by:

$$\text{Weighted Average Elevation} = \frac{\text{Sum of Products}}{\text{Sum of Multipliers}} \quad \cdots\cdots (1)$$

in which the sum of products is the summation of the individual grid point products which are obtained by the products of their ground elevations and the corresponding multipliers. The multiplier of each grid point is determined by the number of squares existing around the point.

Once the weighted elevation of the field plot has been found, use this as the mid-point elevation of the cut plane. The depth of cut (or fill) at each grid point can be obtained by comparing the ground elevations and the elevations of the cut plane at the corresponding points. The depth of cut (or fill) at each grid point multiplied by its corresponding multiplier will give a cut (or fill) product.

The earthwork balance between cuts and fills can be checked by comparing the sum of fill prpducts and the sum of cut products. If the per cent of excess is not desirable, a trial-and-error method must be applied to obtain a balanced design, or a preferable C/F (or F/C) volume ratio. This process is laborious unless a computer program is provided.

The following approach would furnish a rapid and exact solution to the problem without a trial-and-error method to locate the cut plane with a presumed C/F (or F/C) volume ratio.

The weighted average elevation would give the first estimate of the elevation on the cut plane in the center (or close to the center) of the field. After this elevation on the plane has been chosen at or near the center of the field, the elevations on the iplane at all grid points can be calculated with the known row and cross slopes. Thus, the sum of fill products (S_f) and the sum of cut products (S_c) can be found accord-ngly. To check the balance between cuts and fills, the following formulas can be used;

$$C/F = \frac{S_c}{S_f} \quad \cdots\cdots (2)$$

$$\text{or} \quad F/C = \frac{S_f}{S_c} \quad \cdots\cdots (3)$$

For a balanced design, assume $C/F = \frac{S_c}{S_f} = \frac{S_f}{S_c} = 1$. If the S_c/S_f or S_f/S_c is other than unity as desired, a certain per cent excess of one from the other can be assumed. For example, if an "a" per cent of cut volume in excess of fill volume is desired, the depth of the cut plane needs to be adjusted to obtain a presumed C/F (or F/C) volume ratio. This can be accomplished by the help of the following balanced design concepts.

1. Cut volume in excess of fill volume

$$S_c + Y[M_c + (1+a)\ M_f + M_o] = (1+a)\ S_f$$

$$\text{or} \quad Y = \frac{(1+a)\ S_f - S_c}{M_c + (1+a) M_f + M_o} \quad \cdots\cdots (4)$$

where Y = the depth to be adjusted for the cut plane

$Y = -$, to be raised

$Y = +$, to be lowered

a = desired excess in decimal form

Say 10%, use 0.1.

M_c = sum of cut multipliers

M_f = sum of fill multipliers

M_o = sum of non-cut and non-fill multipliers.

2. Fill volume in excess of cut volume

$$S_f + Y[M_f + (1+a)M_c + M_o] = (1+a)\ S_c$$

$$Y = \frac{(1+a)\ S_c - S_f}{M_f + (1+a)M_c + M_o} \qquad (5)$$

where Y = the depth to be adjusted for the cut plane

$Y = -$, to be lowered

$+$, to be raised

Since the sign conventions for Y in equation (4) is just opposite to that in equation (5), it is more convenient to modity equation (5) into

$$Y = \frac{S_f - (1+a)\ S_c}{M_f + (1+a)M_c + M_o} \qquad (6)$$

Accordingly, the sign conventions for the Y in equation (4) and (6) will be the same.

3. For cut and fill volume balanced design ($a = o$)

$$Y = \frac{S_f - S_c}{M_f + M_c + M_o} \qquad (7)$$

Applications of the New Approach

In Figure 1 the elevations shown at all grid points are the results of a grid survey, The slopes indicated in both row and cross directions are the slopes to be finished at the end of leveling. Table 1 shows the procedure of computations.

← Slope=0.2%

Slope=0.3% ↓

	A	B	C	D
1	13.0 11.242 11.2837 C=1.7163	14.0 11.442 11.4837 C=2.5163	15.0 11.642 11.6837 C=3.3163	14.0 11.842 11.8837 C=2.1163
2	12.0 10.942 10.9837 C=1.0163	11.01 11.142 11.1837 F=0.1837	13.0 11.342 11.3837 F=1.6163	12.0 11.542 11.5837 C=0.4163
3	10.0 10.642 10.6837 F=0.6837	10.0 10.842 10.8837 F=0.8837	11.0 11.042 11.0837 F=0.0837	13.0 11.242 11.2837 C=1.7163
4	9.0 10.342 10.3837 F=1.3837	10.0 10.542 10.5837 F=0.5838	9.0 10.742 10.7837 F=1.7837	12.0 10.942 10.9837 C=1.0163
5	8.0 10.042 10.0837 F=2.0837	9.0 10.242 10.2837 F=1.2837	8.0 10.442 10.4837 F=2.4837	11.0 10.642 10.4837 C=0.3163

A

13.0 original elev.
11.242 1st estimate
11.2837 final elev.
C=1.7163 final cut

Example;
Explanation of the values at grid point A_1

Fig. 1 Field Plot, 10% Excess Cut

Table 1. Land Leveling Calculations & Data Sheet (For 10% Excess Cut Example)

Sta	Elev	Mult	Prod	Fill	Fill prod	Cut	Cut prod	
A1	13.0	1	13.0			1.758 1.7163	1.758 1.7163	←1st. Estimate. ←2nd. or Final Value (C/F =1.1).
A2	12.0	2	24.0			1.058 1.0163	2.116 2.0326	
A3	10.0	2	20.0	0.642 0.6837	1.284 1.3674			
A4	9.0	2	18.0	1.342 1.3837	2.684 2.7674			
A5	8.0	1	8.0	2.042 2.0837	2.042 2.0837			
B1	14.0	2	28.0			2.558 2.5163	5.116 5.0326	
B2	11.0	4	44.0	0.142 0.1837	0.568 0.7348			
B3	10.0	4	40.0	0.842 0.8837	3.368 3.5348			
B4	10.0	4	40.0	0.542 0.5837	2.168 2.3348			
B5	9.0	2	18.0	1.242 1.2837	2.484 2.5674			
C1	15.0	2	30.0			3.358 3.3163	6.716 6.6326	
C2	13.0	4	52.0			1.658 1.6163	6.632 6.4652	
C3	11.0	4	44.0	0.042 0.0837	0.168 0.3348			
C4	9.0	4	36.0	1.742 1.7837	6.968 7.1348			
C5	8.0	2	16.0	2.442 2.4837	4.884 4.9674			
D1	14.0	1	14.0			2.158 2.1163	2.158 2.1163	
D2	12.0	2	24.0			0.458 0.4163	0.916 0.8326	
D3	13.0	2	26.0			1.758 1.7163	3.516 3.4326	
D4	12.0	2	24.0			1.058 1.0163	2.116 2.0326	
D5	11.0	1	11.0			0.358 0.3163	0.358 0.3163	
Total		48	530.0		26.618 27.8273		31.402 30.6097	←1st. Estimate. ←2nd. or Final Value (C/F =1.1).

The weighted average elevation of the field is found by equation (1) as:

$$\text{Wtd. Ave. Elev.} = \frac{530}{48} = 11.042 \text{ (ft.)}$$

Choose elevation 11.042 at either point B_3 or C_3 (near the center of the field). Say, C_3.

The elevations encircled are the first estimated elevations on the cut plane at all grid points. The second estimated elevations (or final values) are also indicated in Figure 1. The depths and products of fill and cut are found in columns 5, 7, 6 and 8 in Table 1 (computations sheet) respectively. The total of fill and cut products are 26.818 (S_f) and 31.402 (S_c).

Case 1. Assume that 10% of cut volume in excess to the fill volume is desirable.

The volume ratio (C/F) was found

$$\frac{C}{F}=\frac{\text{Cut products}}{\text{Fill products}}=\frac{31.402}{26.618}=1.18$$

This shows an 18% of cut volume more than fill volume. In order to obtain 10%, formula (4) is applied,

$$Y=\frac{(1+0.1)\ 26.618-31.402}{19+(1+0.1)29+0}$$
$$=-0.0417\ (\text{ft})$$

This indicates that in order to obtain 10% of cut volume in excess of fill volume the estimated cut plane has to be raised 0.0417 foot. After having raised the cut plane 0.0417 foot, the new cut and fill products are found as 30.6097 and 27.8273 which give the C/F volume ratio of 1.1.

Case 2. For a=0, C/F=1. (Also using Table 1 as source data.)
Applying either equation (4), (6) or (7) gives
Y=−0.0997 and the sum of cut products and the sum of fill products are 294.96 and 29.509 respectively. These figures exhibit that the quantities of cut and fill are the same.

To obtain cut and fill volumes, both the 4-point and the weighted products methods were used. The formula for computing earthwork quantities (mimeograph, California State Polytechnic University, Pomona) is;

$$\text{Volume (yd}^3)=\frac{S_c\ (\text{or } S_f)\ A}{108}$$

where A=area of the grid square in ft^2.

The results obtained from both methods are shown in Table 2.

Table 2. Results of earthwork computations by the 4-point and the weighted products methods.

	Presumed C/F=1.0		Resulted	Presumed C/F=1.1		Resulted
	cut (yd^3)	Fill (yd^3)	C/F	cut (yd^3)	Fill (yd^3)	C/F
4-pt Method	2350	2355	1.0	2475	2223	1.11
Wtd. Products Method	2730	2730	1.0	2832	2580	1.10

Discussions

The example shown is chosen from one of many examples. The resulting C/F ratio through the adjustment of the depth of cut plane by the derived equation has an excellent agreement with the predetermined C/F ratio in either 10% or 0% case. The results of C/F ratios strongly support the validity of the derived equations.

The earthwork volumes computed by the weighted products method are on the higher side compared with those by the 4-point method. However, these results exhibit only a matter of choosing the method in computing earthwork quantities after the location of the cut plane has been determined. The U. S. Department of Agriculture

(1961) also shows that the earthwork quantities computed through the 4-point method are less than those through the horizontal plane and summation methods. Regardless of the choice of methods of determining earthwork quantities, the equations 4, 6 and 7 are valid and provide great sreat savings in time versus the conventional trial-and-error method.

References

1. California State Polytechnic University, Pomona, California Land Forming. Mimeograph.
2. Chugg, G. E. (1947). Calculation for Land Grading. Agricultural Engineering 28 (10): 461-463.
3. Givan, J. L. (1940). Land Grading Calculations. Agricultural Engineering 21(1): 11-12.
4. Marr, J. C. (1957). Grading Land for Surface Irrigation. California Agricultral Experimental Station Extension Service.
5. Raju, V. S. (1960). Land Grading for Irrigation. Transaction of the ASAE 3 (1): 38-41.
6. Shih, Sun-Fu and George J. Kriz (1971). Comparisons of Types and Methods of Lan dForming Designs. Transaction of the ASAE, 14(5): 990-994.
7. U. S. Department of Agriculture, (Soil Conservation Service), (1961). Section 15, Irrigation: Chapter 12, Land Leveling.