專　論

地下水層的數學模型

Analog Model Study of a Sand and Gravel Aquifer in Mossville Illinois

臺灣大學農工系客座副教授

胡　萬　旺

DR. WAN WANG HU

Visiting Associate Professor, National Taiwan University,
Bradley University, Illinois

中 文 摘 要

由於地層水流達賽微差公式與應用在電路網之寇克荷夫微差公式在數學上的類似，吾人可以依據土壤及滲透等各種資料設計一個相似的電路網。以此電路網在實驗室中所測到的資料數據加以模型計算可以預測到地層經過深井抽水後的水位等高線。此法簡易，省錢、省工，曾始用於美國伊利諾州並實地鑿井驗證，經證實其準確性極高。1967年密蘇里州聖路易城因鑿井過多，影響地層水位及基礎强度，經完成三度空間之全城地下水層模型，作爲以後鑿井之參考。

本文係作者在 Mossville, Illinois 所作之模型及電測結果，換算繪得因鑿井抽水而引成之地下水位圖。

鑒於臺北近年因城市繁榮，住家及工廠鑿井日多，影響地層水位及基礎強度至鉅，作者謹於此介紹此法與水利界諸位先生。

INTRODUCTION

In the past decade, both analog and digital compüters have been used by hydrologists in estimating possible ground water resources. The speed and accuracy of these computer techniques allow the hydrologists to compare with alternative methods at relatively low cost. Thus, a ground water problem may be solved in a better way.

This research is concentrated on an electronic analog approach to estimate the ground water distribution in a small area when a pump is imposed. An analog model was constructed and the test results are tabulated.

MOSSVILLE AREA

The study area, as shown in Fig. 1, is located in township 10N and range 8E of Peoria County, State of Illinois and designated the Mossville area as it incorporates the small town of Mossville, Illinois. The area contains surfacial sand and gravel underlain by Sankoty sand in the eastern section. The recharge of ground water in the area is quite high with values which may reach 500,000 gpd/mi^2,[1] The central portion of the area contains a thick strata of glacial drift which is also underlain by Sankoty sand. The topography of this area is rather rolling in the central region but flattens in eastwest direction. Surface elevations range from 500ft. to 800ft. aboove the mean sea level. The sand and gravel deposits vary in thickness up to 100 ft., while the total thickness above the bedrock ranges from 100 ft. to 300 ft., A 1933 survey showed the piezometric surface elevates from 400 ft. to 500 ft. above the mean sea level and according to the Illinois State Water Survey, it should not have appreciable change since then.

The important parameters affecting groud water are the coefficients of the perm eability, transmissibility and storage. Three test wells were driven by Illinois State Water Survey; the locations are shown in Fig. 1 and the soil logs are shown in Fig. 2. The test results showed that the coefficient of storage is quite variable, its magnitude was primarily dependent upon the pumping time and the distance from the pumping well. The varying nature of the storage coefficient proved to be a problem when a

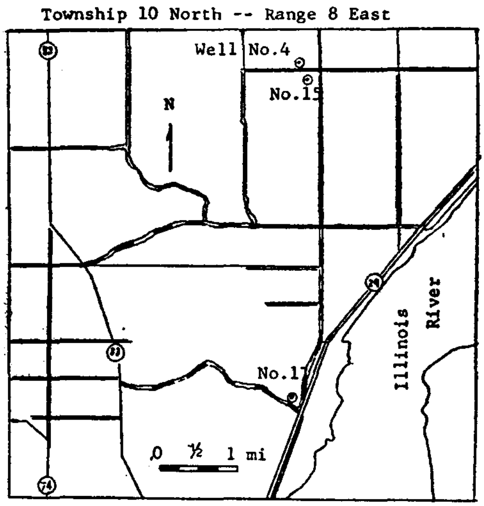


Figure 1. Mossville Area and Locations of Test Wells

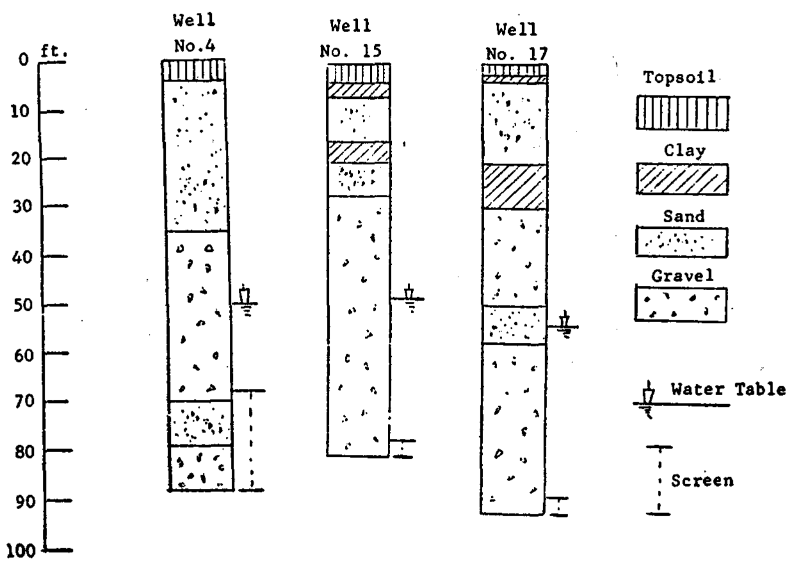


Figure 2. Soil Logs of Three Test Wells

digital computer solution was attempted. Answers inconsistent with the analog results were found and these inconsistencies produced a problem of determining an average value of the coefficient of storage. However, the analog solution has a special feature of being able to "average out" the variance. This is because of the interwoven nature of an analog model compared to a digital solution that considers only two locations at one time. Throughout this research, the coefficient of storage S is chosen as 0.2.

The Illinois State Water Survey has also provided the following hydrogeologic information maps for this area:

a) Coefficient of transmissibility T;
b) Coefficient of permeability P from test wells;
c) Thickness of deposits above the bedrock;
d) Thickness of sand and gravel strata;
e) Topographic map;
f) Glacial and geologic maps; and
g) Piezometric surface map.

The geologic study of this area indicates that the western boundary is a barrier type which is impervious and the other boundaries are recharge type which allow water to permeate into the storage region.

With the preceding hydrologic parameters known, it is possible to proceed with the actual construction of the analog model.

ANALOG MODEL

The analog model of this Mossville area was patterned after that developed by H. E. Skibitzke.[2] Its theory and application can be found in references 3, 4, 5 and many others.

The model consisted of a regular array of 470 resistors and 180 capacitors. It was constructed on a piece of one-eighth pegboard with an overall dimension of 2 ft. by 2 ft., It was scaled to correspond with the field dimensions of the area. Brass shoe eyelets were inserted into the pegboard holes to provide terminal nodes for the resistors and capacitors. Four resistors and one capacitor were connected to each interior terminal and the capacitor was secured to the ground wire. Two or three resistors, depending on the boundary conditions, and one capacitor were connected to each exterior terminal.

The model was developed on the premise that ground water flow in the area is essentially two dimensional. The finite-difference form of the partial differential equation governing the non-steady state of two dimensional flow is:

$$T\left(\sum_{2}^{5} h_i - 4h_1\right) = a^2 S \frac{\partial h}{\partial t} \qquad (1)$$

where h_1 is the head at node 1; h_i represents heads at neighbor nodes 2 to 5; a is the width of grid interval; T is the coefficient of transmissibility and S is the coefficient of storage.

The Kirchhoff's current law applied on the resistor-capacitor network yields the equation:

$$\frac{1}{R}\left(\sum_{2}^{5} V_i - 4V_1\right) = C\frac{\partial V}{\partial t} \quad \cdots\cdots (2)$$

where V_1 is the electric potential at nodel; V_i represents potentials at ands of conjuct resistors; R is the resistance and C the capacitance. Since the above equatons (1) and (2) are similar, a list of the electric-hydrologic analogies may be stated:

a) Water moves in an aquifer is similar as the current flows in an electric circuit.

b) Water is measured in gallons while electrical charge is in coulombs.

c) Rate of water flow is gallons per day and of electric is coulombs per second or amperes.

d) Hydraulic head loss h in feet is analogous to the electric potential drop V in volts.

e) Coefficient of transmissibility T in gpd/ft is analogous to the reciprocal of elecrical resistance 1/R in ohm^{-1}.

f) Product of the coefficient of storage S and the area of grid a^2 is analogous to the electrical capacitance C.

Four similarity equatons may be developed from the above mentioned analogies:

$$q = K_1 B \quad \cdots\cdots (3)$$

$$h = K_2 V \quad \cdots\cdots (4)$$

$$Q = K_3 I \quad \cdots\cdots (5)$$

$$t_d = K_4 t_s \quad \cdots\cdots (6)$$

in which q is gallons for water flow and B is coulombs for electric flow; h is in ft. and V in volts; Q is gal/day and I is amperes; t_d is the time in days and t_s in seconds. All K's are the scale factors which, according the analogous phenomenon, have the following relationships:

$$K_3 K_4 = K_1 \quad \cdots\cdots (7)$$

$$R = K_3/K_2 T \quad \cdots\cdots (8)$$

$$C = 7.48a^2 S(K_2/K_1) \quad \cdots\cdots (9)$$

In this model, a network grid spacing a is chosen to have the scale of 1 inch=2000 feet of prototype.

From tests run by the Illinois State Water Survey,[5] the various K values were determined as:

$K_1 = 1.82 \times 10^{11}$ gal/coulomb

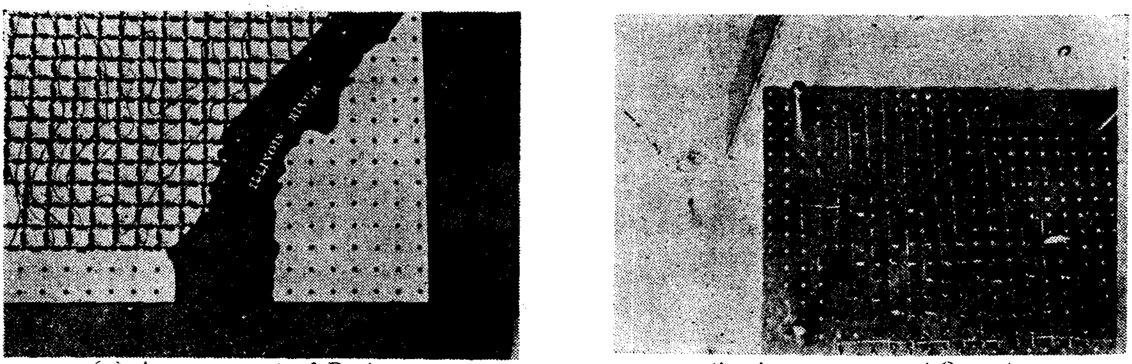
$K_2 = 1.0$ ft/volt

$K_3 = 1.0 \times 10^6$ gal/day/amp

$K_4 = 1.82 \times 10^5$ day/sec

Values of resistors were computed according the values of T. Resistors at the interior nodes ranged from 1,500,000 ohms near the barrier boundary where T is about 6000 gpd/ft to 180,000 ohms near the recharge boundaries where T is about 80,000 gpd/ft. Capacitors at the interior nodes were calculated to be 3300 micromicro-farads. The average S is 0.20.

A general layout of the model is shown on the two photographs in Fig. 3. Node No. 104 was chosen as the assumed pumping well because its central location. The pumping period td, was assumed to be 5 years and therefore, t_s was 10^{-2} seconds. Each of the nodes was similar to an observation well in field. As a test result, the drawdown at each node can be evaluated.



(a) Arrangement of Resistors (b) Arrangement of Capacitors

Figure 3. Analog Model Simulator

Excitation-Response Apparatus

The excitation-response apparatus forced electric energy in a proper time phase into the model and measured the energy levels within the model network. The apparatus consisted of a model 211-A square wave generator, an HP 130-B oscilloscope and the model network. The apparatus diagram is shown on Fig. 4.

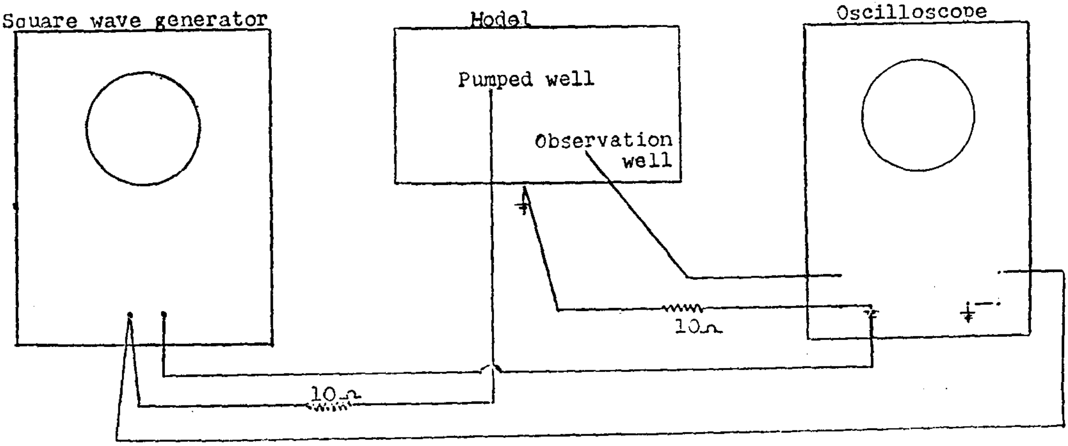


Figure 4. Excitation-Response apparatus.

The square wave generator applied a current to the model in a square wave. The positive pole of the generator was connected to the pumping well node No. 104 of the model and also to the oscilloscope. The oscillosope was used as a triggering device for the system. The negative pole of the generator and the model were both connected to the ground on the oscilloscope. The well (or node) under observation was connected to the input of the oscilloscope by means of an alligator clamp which is very easy to operate.

To eliminate the effects of circuit loading when making voltage measurements, a small 10-ohm resistor was placed in series with the pumping well. The discharge rate was calculated by using the following equation:

$$Q=\frac{V_R K_3}{1.44\times 10^3 R_i} \text{ gal/min} \quad \cdots\cdots (10)$$

where V_R is the voltage drop across the resistor R_i, in volts; and R_i is the calibrated resistance in ohms.

TABLE 1

Values of Model Resistors, Capacitors and Test Drawdowns

Node No.	$R\times 10^3$ ohm	$C\times 10^{-6}$ farad	Drawdown ft	Node No.	$R\times 10^3$ ohm	$C\times 10^{-6}$ farad	Drawdown ft
1	330	390	1.2	34	270	0	8.0
2	220	1500	1.8	37	1200	0	1.5
3	180	1500	3.0	38	1000	0	1.8
4	180	1500	4.6	39	330	3300	2.3
5	150	1500	5.2	40	270	3300	3.0
6	150	1500	9.0	41	180	3300	5.0
7	150	1500	6.4	42	180	3300	6.0
8	180	1500	6.2	43	150	3300	7.2
9	180	1500	6.8	44	150	3300	8.0
10	220	1500	7.2	45	150	3300	8.3
11	220	1500	7.8	46	180	3300	8.5
12	220	1500	8.0	47	220	3300	8.5
13	270	1500	8.0	48	270	3300	8.6
14	270	1500	7.9	49	270	3300	8.8
18	820	0	1.3	50	270	3800	8.8
19	560	8000	1.5	51	330	3800	8.8
20	270	3300	1.8	53	1500	3000	1.7
21	220	3300	2.2	54	1200	1000	2.2
22	180	3300	3.8	55	680	3300	3.5
23	120	3300	5.0	56	330	3300	5.0
24	150	3300	6.0	57	220	3300	6.8
25	150	3300	6.5	58	180	3300	9.2
26	150	3800	6.8	59	180	3300	10.0
27	180	3300	7.0	60	150	3300	11.0
28	180	3300	7.5	61	150	3300	10.0
29	220	3300	8.0	62	180	3300	10.5
30	220	3300	8.4	63	220	3300	10.0
31	270	3300	8.5	64	270	3300	10.0
32	270	3300	3.5	65	270	3300	9.0
33	270	3300	8.4	66	270	3300	9.6

TABLE 1— (continued)

Node No.	$R \times 10^3$ ohm	$C \times 10^{-6}$ farad	Drawdown ft	Node No.	$R \times 10^3$ ohm	$C \times 10^{-6}$ farad	Drawdown ft
67	330	1000	8.8	114	680	3300	1.4
69	1200	1600	1.0	115	470	3300	10.0
70	1200	3300	2.7	116	330	3300	17.0
71	820	3300	5.2	117	270	3300	22.5
72	390	3300	8.5	118	220	3300	32.5
73	330	3300	12.5	119	180	3300	40.0
74	220	3300	16.0	120	180	3300	32.0
75	180	8300	18.0	121	220	3300	23.0
76	180	3300	19.5	122	270	3300	18.0
77	180	3300	14.2	123	270	3300	16.0
78	180	3300	13.2	127	1500	2200	0.5
79	220	3300	11.0	128	1000	2500	1.8
80	270	3300	11.0	129	560	3000	4.5
81	270	3300	11.5	130	470	3300	10.0
82	330	1800	9.2	131	470	3300	17.0
84	1200	2700	1.6	132	220	3300	21.0
85	1000	3300	4.6	133	220	3300	25.0
86	470	3300	8.8	134	220	3300	28.0
87	330	3300	13.5	135	220	3300	25.0
88	330	3300	19.0	136	220	3300	22.0
89	220	3300	25.0	137	270	3300	18.0
90	180	3300	34.0	140	820	3300	6.5
91	180	3300	25.0	141	330	3300	11.0
92	180	3300	18.0	142	220	3300	14.5
93	220	3300	15.0	143	220	3300	17.5
94	220	3300	13.0	144	180	3300	20.0
95	270	3300	12.5	145	220	3300	21.0
96	330	3300	11.5	146	220	3300	20.0
98	1200	1600	2.0	147	270	3300	18.0
99	820	3300	4.6	148	270	3300	16.0
100	470	3300	10.0	151	1000	3300	5.1
101	330	3300	17.0	152	470	3300	9.5
102	330	3300	23.0	153	330	3300	13.0
103	220	3300	38.0	154	270	3300	15.0
105	180	3300	38.0	155	220	3300	17.0
104	—	—	—	156	220	3300	17.0
106	220	3300	24.5	157	270	3300	17.0
107	220	3300	16.0	158	330	3300	15.0
108	270	3300	15.0	159	330	3300	14.0
109	270	3300	13.0	162	1100	3000	3.4
112	1500	1600	0.8	163	1000	3300	7.6
113	1200	3300	1.1	164	330	3300	10.5

TABLE 1—(continued)

Node No.	R×10³ ohm	C×10⁻⁶ farad	Drawdown ft	Node No.	R×10³ ohm	C×10⁻⁶ farad	Drawdown ft
165	270	3300	13.0	177	270	3300	12.0
166	270	3300	14.0	178	330	3300	11.0
167	270	3300	14.5	179	390	3300	10.0
168	270	3300	13.0	182	1500	1600	2.0
169	330	3300	12.3	183	1200	1600	4.0
170	330	1500	12.3	184	680	1600	7.5
172	1200	3300	2.0	185	330	1600	9.0
173	1100	3300	5.6	186	270	1600	9.6
174	470	3300	10.0	187	330	1600	10.0
175	330	3300	10.8	188	390	1600	10.0
176	270	3300	11.2				

Summary of Resuluts

The test results, together with the values of resistors and capacitors of th model network, are tabulated in Table I. A drawdown contour map is then plotted in Fig. 5.

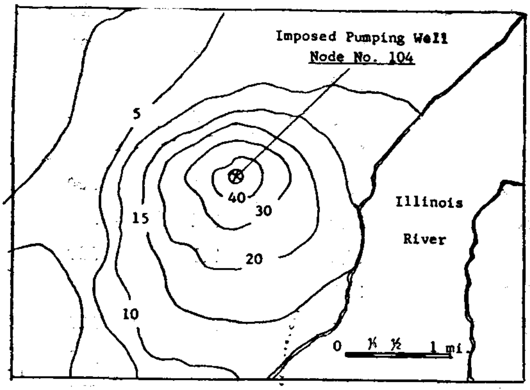


Figure 5. Drawdown Contour Map

Conclusion

The drawdown contours indicate that the drawdown decreases as the distance from the pumping well increases, which is in accordance with the theory, The assumed long period pumping of five years at a high rate would cause a considerable amount of drawdown; over 40 ft. was detected in the vicinity of the pumping well. The model study and its result prove to be an adequate representation of the ground water distribution of the studied area. A further detailed study using image well theory would probably better the results.

The method of using analog model to estimate the ground water condition has been used widely; this research is another example of application to a small area.

Notation

a = width of grid interval in feet;
B = quantity of electrical charge in coulombs;
C = capacittance in farads;
h = hydraulic heads at well in feet;
I = current in amperes;
K_1 = scale factor in fal/coulomb;
K_2 = scale factor in ft/volt;
K_3 = scale factor in fal/day/coulomb;
K_4 = scale factor in day/sec;
q = quantity of water in gallons;
Q = pumping rate in gal/day or gal/min;
R = resistance inohms;
S = coefficient of storage;
T = coefficient of transmissibility in gpd/ft;
t = time;
V = electric potential at node; and
V_R = voltage drop across resistors.

References

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