側槽溢洪道控制點之奇點解

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Singular Point Solution of the Control Point in A Side Channel Spillway

By Chian Min Wu, [1] *AM. ASCE*

謹以本文歡迎即將回國任客座教授之本會會員胡萬旺博士，以酬彼在不連續流方面所做權威性之貢獻。

Introduction

To calculate the water surface profile in a side channel spillway, it is necessary to determine the control section from which the flow profile computation can be started. A method for determining the location of the critical depth control point was proposed by J. Hinds[2] using equivalent critical depth channel and was subsequently made generally appliciable.[3]

An alternative method is derived herein using the singular point method[3] which was proposed by Chow, however, was not extensively solved. The advantage of this solution as compared with the solution by Hinds is that the position of the control point can be determined with little additional computation. An example is also given for demonstration.

Dynamic Equation for Side Channel Spillway by Monentum Principle

The momentum equation is normally used for the derivation of the differential equation for the dynamic equation for the side channel spillway. Referring to the lateral spillway channel in Fig. 1, the momentum equation gives

$$\frac{w}{g}(Q+dQ)(V+dV)-\frac{w}{g}QV=P_1-P_2+\overline{W}\sin\theta-F_f \qquad \cdots\cdots(1)$$

where w is the unit weight of water, Q is the discharge, V is the velocity, dQ is the added discharge between sections 1 and 2, W is the weight of the body of water between the sections, F_f is the friction force along the channel wall, P is the total pressure, and g is the gravitational acceleration.

The pressure force, P_1 is

$$P_1 = w\bar{z}A$$

where $\bar{z}$ is the depth of the centroid of area, A, below the water surface.

Similarly, the total pressure on section 2 is

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1. Chief. Hyd-Lab. Water Resources Planning Commission and Assoc. Prof. Dept. of Civ. Engrg., National Taiwan University, Rep. of China.

2. Hinds, Julian, "Side Channel Spillways: Hydraulic Theory, Economic Factors, and Experimental Determination of Losses," Trans., ASCE, Vol. 89, 1926. pp 881-927.

3. V.T. Chow "Open-Chaanel Hydraulics," McGraw-Hill Book Co., Inc., 1959 pp 329-346.

4. Chian Min Wu "Discussion of" Hydraulics of Spatially Varied Pipe Flow"", (To be published).

5. Hu, Wan Wang, "Hydraulics of Spatially Varied Pipe Flow", *Journal of the Hydraulics Division,* ASCE, Vol. 93, No. HY6, Proc. Paper 5599, Nov. 1967, pp. 281-296.

$$P_2 = w\,(\bar{z} + dy)\,A + \frac{w}{2}\,dA\,dy$$

where dy is the difference between the depths of the two sections 1 and 2 as shown in Fig. 1. Neglecting the higher order terms, the resultant hydrostatic pressure acting on the body of water between sections 1 and 2 is

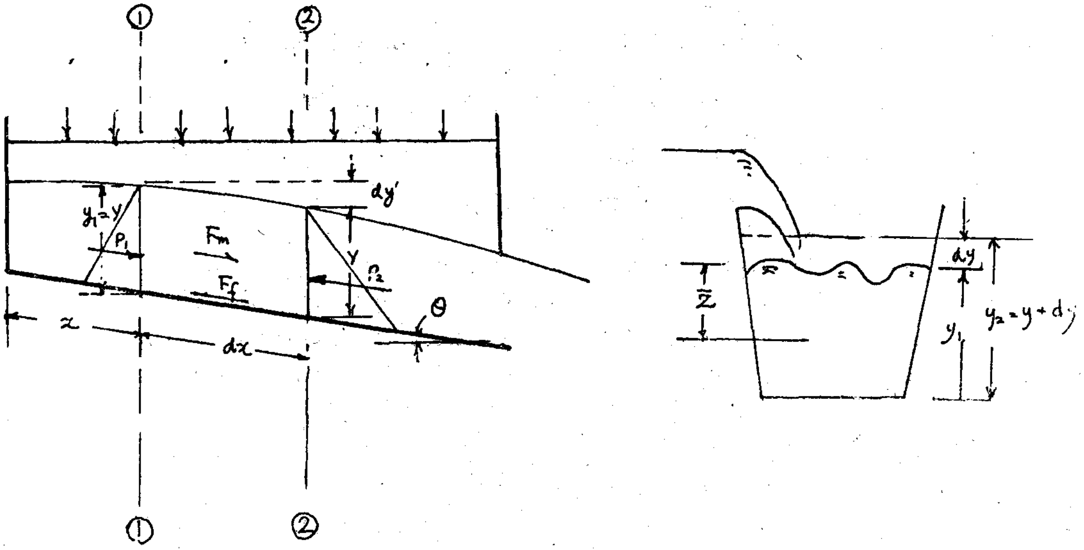

Fig. 1. Definition Sketch

$$P_1 - P_2 = -\,w\,A\,dy \quad \cdots\cdots(2)$$

The weight of the body of water between the sections can be in term of section area and its component in the direction of flow is

$$W \sin\theta = w\,S_o\,(A + \tfrac{1}{2}\,dA)\,dx = w\,S_o\,A\,dx \quad \cdots\cdots(3)$$

where slope S_o is equal to $\sin\theta$ and the term containing the product of differentials is dropped.

The frictional head between the two sections is equal to the friction slope, S_f, multiplied by the length, dx, or

$$h_f = S_f\,dx$$

If Manning formula is used

$$S_f = \frac{V^2\,n^2}{2.22\,R^{4/3}} = \frac{Q^2\,n^2}{2.22\,A^2\,R^{4/3}}$$

The frictional force along the channel wall is equivalent to the pressure due to friction head multiplied by the average area, or

$$F_f = w\,(A + \tfrac{1}{2}\,dA)\,S_f\,dx = w\,A\,S_f\,dx \quad \cdots\cdots(4)$$

After substituting all values of forces and simplifying, Eq. 1 reduced to

$$dy = -\,\frac{1}{g}\,(VdV + \frac{V}{A}\,dQ) + (S_o - S_f)\,dx \quad \cdots\cdots(5)$$

Since $V = Q/A$ and $V + dV = (Q+dQ)/(A+dA)$, and

$$dV = \frac{Q + dQ}{A + dA} - \frac{Q}{A} = \frac{A\,dQ - Q\,dA}{A\,(A+dA)}$$

$$VdQ = \frac{Q}{A}\,dQ = \frac{QAdQ + QdAdQ}{A\,(A + dA)}$$

Hence, the above equation becomes.

$$dy = -\frac{V}{g}\left(\frac{2AdQ - QdA + dAdQ}{A^2 + AdA}\right) + (S_o - S_f)\,dx \quad\text{(6)}$$

Neglecting dA in the denominator and dA dQ in the numerator, and simplifying

$$\frac{dy}{dx} = \frac{S_o - S_f - (2Q/gA^2)(dQ/dx)}{1 - Q^2/gA^2D} \quad\text{(7)}$$

If nonuniform velocity distribution in channel section is considered

$$\frac{dy}{dx} = \frac{S_o - S_f - 2\beta Q\, q_*/gA^2}{1 - \beta Q^2/gA^2D} \quad\text{(8)}$$

where $q_* = dQ/dx$ and $D = A/T$.

This is the dynamic equation for spatially varied flow with increasing discharge. Usually the energy coefficient is used instead of the momentum coefficient. Because in evaluating the friction slope, the Manning formula is used. Thus,

$$\frac{dy}{dx} = \frac{S_o - S_f - (2\alpha Q/gA^2)(dQ/dx)}{1 - (\alpha Q^2/gA^2D)} \quad\text{(9)}$$

However, it is considered this is merely a practical interpretation which has no theoretical basis.

Dynamic Equation for Side Channel Spillway by Energy Principle

Although the momentum principle is normally used for the derivation of the differential equation for the dynamic equation of the spatially varied flow with increasing discharge, it is convenient to deal with the general case of a nonuniform channel by means of the energy principle.

The energy principle is only applicable if due allowance is made for the work done in accelerating the lateral inflow up to the longitudinal velocity of flow in the channel.

With reference to Fig. 1, in a length dx the lateral inflow is (dQ/dx)dx. All the energy of this water will be destroyed by impact and turbulence, except that appropriate to the elevation y of the water surface.

The energy required to accelerate the lateral inflow, (dQ/dx) dx, in the longitudinal direction of the channel from 0 to V+dV is

$$\frac{1}{2}\,\alpha\,(\text{Mass})(V + dV)^2 \doteqdot \frac{\frac{1}{2g}\left(w\,\frac{dQ}{dx}\bullet dx\bullet dt\right)\alpha V^2}{w\,A\,dx} \quad\text{(10)}$$

To maintain the continuity of the process in space and time

$$\tfrac{1}{2}\,(V + dV)\,dt = dx$$

or approximately

$$(\tfrac{1}{2}V)\,dt = dx \quad\text{(11)}$$

Inserting this value in Eq. 10 gives the energy required per unit weight of water as

$$\frac{\alpha}{g}\,\frac{V}{A}\,\frac{dQ}{dx}\bullet dx$$

or

$$\alpha\,(Q/gA^2)\,(dQ/dx)\,dx$$

This may be expressed as a slope term as

$$\alpha\,\frac{Q}{gA^2}\,\frac{dQ}{dx}$$

If H = the total head, the energy equation may be written:

$$H = y + z + \frac{\alpha Q^2}{2gA^2} + \text{(energy required to accelerate later inflow)}$$

Differentiating and inserting Eq. 11 for the last term,

$$dH/dx = dy/dx + dz/dx + \frac{\alpha}{2g}\left(\frac{2Q}{A^2}\frac{dQ}{dx} - \frac{2Q^2}{A^3}\frac{dA}{dx}\right) + \alpha\frac{Q}{gA^2}\frac{dQ}{dx} \quad \cdots\cdots\cdots (12)$$

By definition

$$S_f = -dH/dx$$

$$S_o = -dz/dx$$

and let T=water surface width,

$$dA/dx = (dA/dy)(dy/dx) = T(dy/dx)$$

or $dA/dx = (A/D)(dy/dx)$

Then Eq. 12 gives

$$\frac{dy}{dx} = -\frac{2\alpha Q}{gA^2}\frac{dQ}{dx} + \frac{\alpha Q^2}{gA^2D}\frac{dy}{dx} + S_o - S_f$$

or

$$\frac{dy}{dx} = \frac{S_o - S_f - (2\alpha Q/gA^2)(dQ/dx)}{1 - \alpha Q^2/gA^2D}$$

The resulting equation is identical with Eq. (9) as derived by using the momentum principle.

Transitional Profile for Side Channel Spillway

At the critical section, the value of gD approaches V^2 since its Froude number must equal to unity. The numerator on the right side of Eq. 9 must approach zero to give dy/dx a finite value.

Accordingly,

$$S_o - S_f - (2\alpha Q/gA^2)(dQ/dx=)\ 0$$

or

$$S_o - S_f - (2\alpha Q/gA^2)(1/Q)(dQ/dx) = 0 \quad \cdots\cdots\cdots (13)$$

However, continuity equation yields

$$Q = (dQ/dx)\,x$$

Hence, $(1/Q)(dQ/dx) = 1/x$ ……………… (14)

And Eq. (13) becomes

$$S_o - S_f - (2\alpha Q^2/gA^2)(1/x) = 0$$

or

$$S_o - S_f - 2D/x = 0$$

or

$$x = 2D/(S_o - S_f) \quad \cdots\cdots\cdots (15)$$

Eq. 15 is the equation of transitional profile as obtained by the singular point method. The left side of Eq. (15) as can be seen is only determined by the value of x and the right side only by the paticular depth selected at a given cross section. Determination of the value of the right side of Eq. (15) for a number of selected depths at a given cross section will enable the elevation of the transitional profile to be interpolated.

Example of the Method

The transitional profile method is used herein to determine the control point in a lateral spillway channel of uniform cross section. To allow a comparison, same example as solved by Hinds using the equivalent "Critical Depth Channel" is used[8]

The cnannel is a trapezoidal section of 400 ft long with a lateral inflow of 40 cfs/ft of length. The cross section has a bottom width of 10 ft., the side slopes are 2 vertical to 1 horizontal and the longitudinal slope of the channel is 0.1505, starting at an upstream bottom elevation of 73,70 ft. Manning's n is taken as 0.015 and $\alpha=1$.

The calculation for the position of the transitional profile is given in Table 1. Distance x is measured from the upper end of the spillway channel.

For a range of depths from 4 ft to 28 ft, the hydraulic properties and the values of Eq. (15) have been determined. To compute S_f the conveyance factor

$$K = 1.49\ AR^{2/3} / 0.015$$

is used to simplify the computation.

Table 1. Computation of Transitional Profile

(1) d	(2) A	(3) T	(4) $D=A/T$	(5) $V_c=(gD)^{1/2}$	(6) $Q_c=V_cA$	(7) R	(8) $R^{2/3}$	(9) K	(10) $x=\frac{2D}{S_o-S_f}$
4	48	14	3.42	10.49	504	2.52	1.85	8,800	46
8	112	18	6.22	14.15	1,585	4.01	2.53	28,100	85
12	192	22	8.72	16.75	3,216	5.22	3.02	57,400	119
16	288	26	11.08	18.88	5,440	6.29	3.40	97,200	150
20	400	30	13.34	20.71	8,284	7.31	3.77	149,000	181
24	528	34	15.52	22.34	11,800	8.29	4.10	212,000	211
28	672	38	17.68	23.84	16,020	9.26	4.41	293,000	239

Corresponding values in Cols. 1 and 10 enable the transitional profile to be plotted on a longtidunal elevation of the channel, Fig. 2.

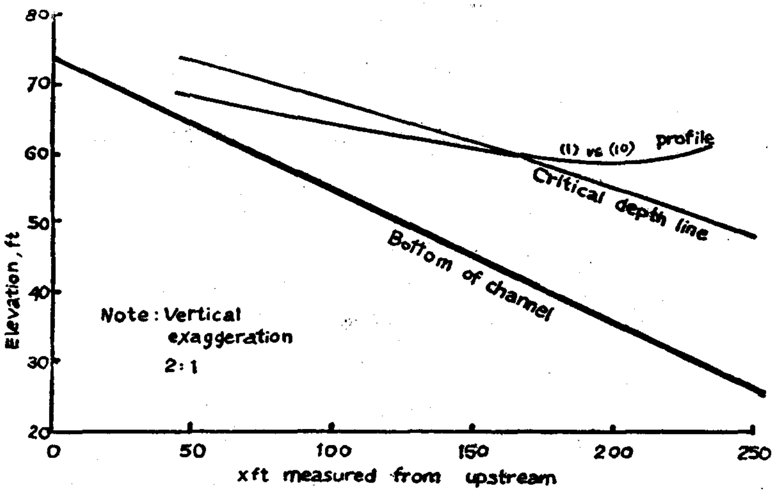

Fig.2. Singular Point Solution of Critical Point

To locate the control point, it is necessary to determine the point in the channel for which the actual discharge Q is equal to the identical values of Q_c on the transitional profile. This can be done by plotting Cols. 10 vs. 6 and their correponding actual discharge, i.e. 40x in this example. The intersection of the actual vs critical discharge lines gives the point at which $Q=Q_c$ i.e. the control point. From Fig. 3, the control point is located at $x=164$ ft.

A similar method can be adopted by using Fig. 2. The "critical depth line" which gives the critical depth for the actual discharge at any point can be plotted on the same plotting. The intersection of the critical depth line with the transition profile gives the point at which $Q=Q_c$. From Fig. 2, the control point is located at $x=164$ ft and at a depth of 17.7 ft, showing full agreement with Hinds' method[2)3)] and result obtained by using Fig 3.

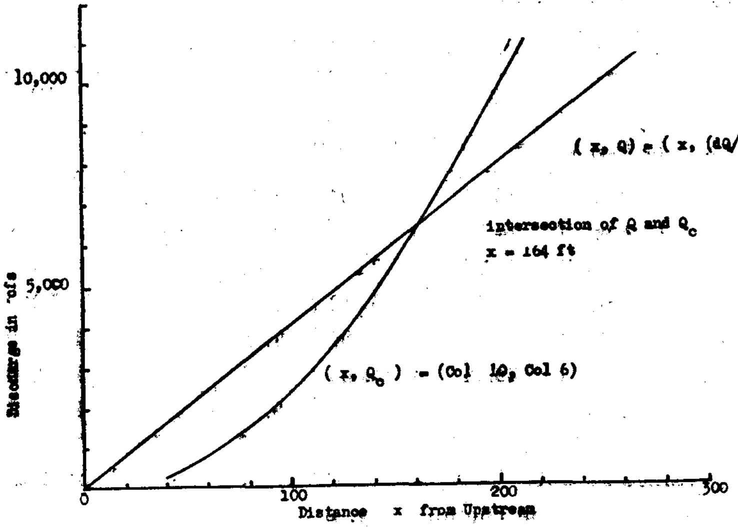

Fig. 3. Solution of Critical Control Point

Conclusions

The dynamic equation for a side channel spillway can be derived by using both momentum and energy principles. The resulting equation is

$$\frac{dy}{dx} = \frac{S_o - S_f - x(2\,Q/gA^2)\,(dQ/dx)}{1 - x(\,Q^2\,/\,gA^2D\,)}$$

To solve the location of the control point, the singlar point method, i.e.. the numerator on the right side of Eq. 9 must approach 0, can be used. As a result, a transitional profile can be obtained as

$$x = 2D\,/\,(S_o - S_f) \quad \cdots\cdots (14)$$

Simultaneous plotting of x vs. Q_c and x vs. Q leads to a solution of the control point in a side channel spillway.

The proposed adaptation of the transional profile method can be used for the determination of a critical depth control point in a side channel spillway and is simpler than Hinds' method[2), 3)]

Acknowledgments

The work described herein is a byproduct of a discussion[4] of "Hydranlics of Spatially Varied Pipe Flow" by Dr. Wang Hu[5]. Dr. Hu, Asst. Prof., Dept. of Civ. Engrg., Bradley Univ., Peoria, Ill. U.S.A., is recognized for offering the writer the benifit of his experience in studying the problem.

摘　　要

側槽溢洪道之水理遠在1926年J. Hinds 則曾用動量方程式誘導，並創「等值臨界槽」控制點決定法[2]，周文德氏曾廣泛介紹於水利界[3]，最近斯氏亦以能量方程式證實其可靠性，筆者最近與胡萬旺博士討論管流中之不連續流問題[4]，倡奇點分析法[5]，特將其內容詳細分析如下，以供參考，並求指正。

側槽溢洪道之水理，在水力學上屬於不連續定量流 (Spatially Steady Flow, Discontineous Flow)；爲典型之不連續增量變速漸變流(Spatially Gradually Varied Flow with Increasing Discharge)，其運動方程式可從動量及能量方程式獲得如下：

$$\frac{dy}{dx} = \frac{S_o - S_f - (2\alpha Q/gA^2)\ (dQ/dx)}{1 - \alpha Q^2/gA^2D}$$

如利用奇點法求臨界水深產生時，其軌跡可設分子分母均爲0而獲得，其解爲

$$x = 2D / (S_o - S_f)^{5)}$$

如與連續方程式 $Q = \frac{dQ}{dx}$，x 聯立解之，即可獲得控制點之位置，手續較 Hinds 氏解法簡便，請參閱例題。

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