Hydraulic Jump In Horizontal Standard Horseshoe Channel

(馬蹄形渠道中之水躍現象)

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Synopsis

An investigation on the hydraulic jump in partially full horizontal standard horseshoe channel is presented. Analytical and experimental equations and graphs have been developed for the main flow parameters.

Introduction

The experimental and theoretical analysis of hydraulic jump in horizontal channel have been almost studied throughly. Recently, the phenomenon in all shapes of horizontal channels has been summarized.[1] However; the hydraulic jump in horizontal horseshoe channel, which is an important conveyance section, has been neglected.

In order to gain a more rational understanding of this phenomenom, a combination of theory and experiments were used and the analytical solutions for the sequent depths and energy loss of the jump are presented.

Definition Sketch

Based on the hydraulics of part full flow in horseshoe channel and the definition sketch (Fig. 1), the factors involved in the analysis are defined as follows:

Initial depth factor d_1/D, is defined as the ratio of the supercritical initial depth, d_1, to the diameter of horseshoe channel, D.

Sequent depth factor, d_2/d_1, is defined as the ratio of the sequent depth, d_2, t supercritical initial depth, d_1.

Froude number is defined as the ratio

$$F = V/\sqrt{gD_h} \qquad (1)$$

where D_h is the hydraulic depth and is defined as the cross-sectional area, A, of the water normal to the direction of flow in the channel divided by the width, b, of the free surface.

Flow parameter for horseshoe channel, F_p, is defined as the dimensionless ratio

$$F_p = Q^2/gAD^3 \qquad (2)$$

where Q is the discharge, g is the acceleration due to gravity.

Sequent Depth Factor

There is great utility in expressing the sequent depth factor and energy loss equations for any prismatic channel in terms of upstream Froude number. However, for design purpose, other dimensionless parametes may be obtained the similar result. For example, Q^2/gA_1D^3, which eliminates the numbers of factors involved in the originally defined Froude number, b_1Q^2/gA_1^3, can be used as a parameter for hydraulic jump in horseshoe channel.

(1) Richard Silvester. "Hydraulic jump In All Shapes of Horizontal Channels". Proc. ASCE Vol. 90, No. HY 1, January, 1964, pp 23-55.

The general form of the momentum equation is

$$(Ad_g)_2-(Ad_g)_1=\frac{Q}{g}(1/A_1-1/A_2) \qquad (3)$$

let $Ad_g=P$
yields

$$\frac{Q^2}{gA_1D^3}=\frac{(P_2-P_1)/D^3}{1-A_1/A_2} \qquad (4)$$

where $P/D^3=F_1(d/D)$ (5)

$A/D^2=F_2(d/D)$ (6)

and d_g is the distance of centre of area of cross section below water surface and P is the pressure on cross section of water prism in cubic units of water.

Functons 1 and 2 (equations 5 and 6) are geometric properties for different flow depths in horseshoe conduits. For standard horseshoe section, the properties have been tabulated(2) or can be obtained from the geometry of the figure. Thus, it is easily shown that equation 4, i.e, flow parameter, Q^2/gAD^3, is a function of initial depth factor, d_1/D, and sequent depth factor, d_2/d_1; or sequent depth factor, d_2/d_1, is a function of flow parameter, Q^2/gA_1D^3, and initial depth factor, d_1/D.
That is

$$d_2/d_1=F_3(Q^2/gA_1D^3,\ d_1/D) \qquad (7)$$

Then, equation 4 has been solved for various value of d_1/D and d_2/D. The solution is graphically given in figure 2, from which sequent depth factor d_2/d_1 can be obtained directly following evaluation of the dimensionless parameter Q^2/gA_1D^3.
Now

$$F^2=V^2/gD_h=bQ^2/gA^3=(Q^2/gAD^3)(b/D)(D^2/A)^2 \qquad (8)$$

and $b/D=F_4(d/D)$ (9)

Substitution for F_1 with various value of d_1/D gave a similar result to equation 4, and the solution is given in figure 3.

From equation (8) it is easily shown that the dimensionless parameter, Q^2/gA_1D, is a function of Froude number, $V_1/\sqrt{gA_1/b_1}$, and initial depth factor, d_1/D. However; comparison of Figures 2 and 3 shows that the flow parameter, Q^2/gA_1D^3, is much more significant than Froude number. Also the former can reduce the width factor in computation work and give a reasonable design chart.

To check the validity of equation 4, a horseshoe channel of 14.5 cm diameter was constructed. A series of experiments, covering average values of F_1 equal to 1.01 to 6.2, flow parameter, Q^2/gA_1D^3, of 0.01-0.53 and initial depth factors of 0.1-0.4 were performed. The value of flow parameters, Q^2/gA_1D^3, and Froude numbers for all the experiments were plotted against the theoretical values caluculated using equation 4 in Fig. 4 and 5. The correlation is seen to be good. Hence, it is recommended that Fig. 2 and 3 be used for design purpose.

Energy Loss

The general form of relative loss in horizontal jump can be expressed as E_L/E_1 and for all shapes of channel

(2)"Design of Small Dam" U.S.B.R.

$$E_L/E_1 = \frac{d_1+V_1^2/2g-d_2-{}_2^2V/2g}{d_1+V^2_1/2g} = \frac{d_1-d_2+(Q^2/2g)(1/A^2_1-1/A^2_2)}{d_1+Q^2/2gA^2_1} \qquad (10)$$

Now $F^2_1=b_1Q^2/gA^3_1$, $Q^2=F^2_1gA^3_1/b_1$

Therefore,

$$E_1/E_1 = \frac{2(d_1-d_2)+(F^2_1A_1/b_1)(1-A^2_1/A^2_2)}{2d_1+F^2_1A_1/b_1} = \frac{2\frac{d_1}{D}(1-\frac{d_1}{d_2})+(1-\frac{A^2_1}{A^2_2})\frac{F^2_1A_1}{b_1D}}{2(\frac{d_1}{D})+F^2_1\frac{A_1}{b_1D}} \qquad (11)$$

From equation (11), it is apparent that

$$E_L/E_1=F_5(\frac{d_1}{D},\frac{d_2}{d_1}) \qquad (12)$$

Again it is shown that

$$E_L/E_1=\frac{2(d_1/D)(1-d_2/d_1)+(Q^2/gA_1D^3)(D^2/A_1)(1-A^2_1/A^2_2)}{(2\frac{d_1}{D})+(Q^2/gA_1D^3)(D^2/A_1)} \qquad (13)$$

or $E_L/E_1=F_6(Q^2/gA_1D^3,\ d_1/D)$ (14)

Thus, the relative loss of energy in the hydraulic jump of horizontal standard horseshoe channel has been shown to be a function of only F_1 and d_1/D or Q^2/gA_1D^3 and d_1/D. To check the validity of equations 11 and 13, experimental values of E_L/E_1 were plotted against the theoretical values in Fig. 5.

General agreement with the Analytical curve is good even down to small Froude numbers. This could be expected because for smaller Froude numbers, the upstream high velocity jet enters the down stream over a smaller width and the resulting triangular "leading edges" of the jump at the downstream end permit a certain degree of reverse flow and recirculation.

This water is fed back into the jump and hence produces greater energy dissipation for the smaller Froude numbers ($F<2.5$). Thus, the characteristics of loss is well defined by the analytical curve than that is for rectangular and other channels. Hence, equation 11 and 13 have been solved for various values of d_1/D and the solution is given graphically in Figure 7 and 8.

Conclusion

1. The sequent depth ratio and the energy loss ratio have been simply derived in terms of the upstream Froude number and flow parameter Q^2/gAD^3 for the standard horseshoe shape.

General agreement has been obtained with experimental data, although further tests are desirable for Froude numbers in excess of 6.

2. Because of the greater percentage energy dissipation of horseshoe channel, more use could be expected of this section in stilling basin.

3. With the theoretical analysis now available for the ratios of both sequent depth and energy loss the efficiency of other energy dissipators can easily be evaluated.

4. The length of the jump can be expressed in the form

$$L/d_2=F(F,\ d_1/D)=F(Q^2/gAD^3,d_1/D)$$

This must be obtained from experimental data. As the experiments were performed in a 14.5 cm diamenter channel, having a limiting testing head of 2 m, it is resoned that the scale effect might be affected, hence, is not presented.

摘　　要

馬蹄型渠道爲輸水慣用之斷面，惟其水躍現象迄今未爲水理界人士所注意。本文特就其消能及水躍特性加以分析，除利用慣用之福祿數（Froude number）爲分析之參數外，另引用參數Q^2/gAD^3以簡化計算過程；分析所得成果均以試驗證實其可靠性，並經繪製成圖表，以供設計之需。

Fig 1. Definition sketch

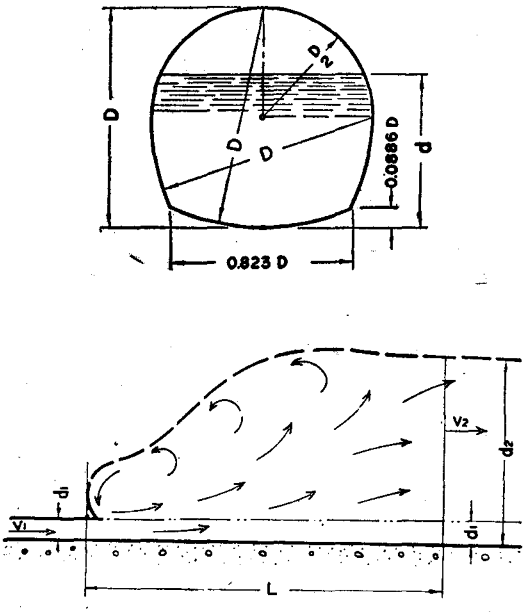

Fig 2. Q^2/gA_1D^3 versus d_1/D and d_2/d_1

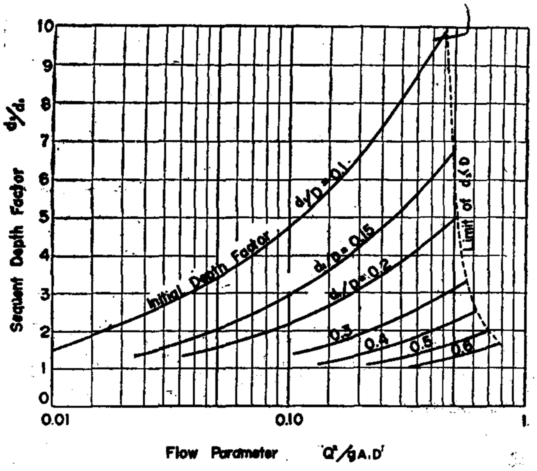

Fig 3. F vevsus d_1/D and d_2/d_1

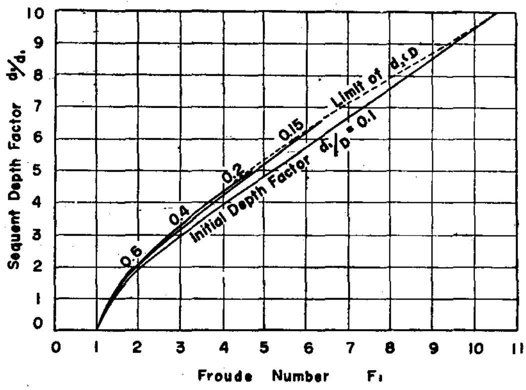

Fig 4. verification of Experimental and Theoretical Froude Numbers

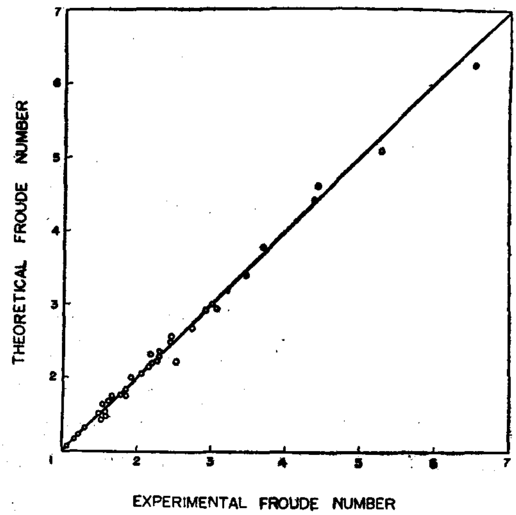

Fig 5. Verification of Flow Parameter.

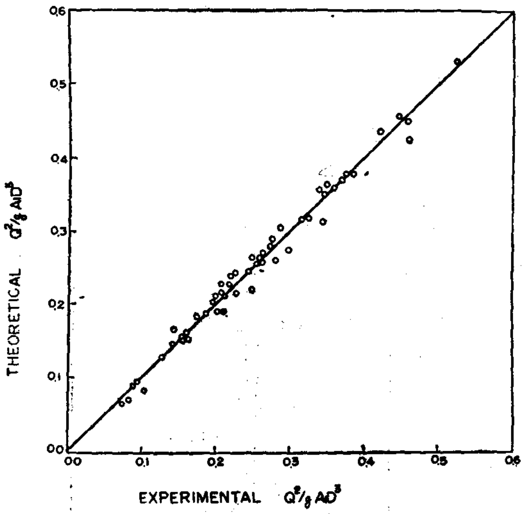

Fig 7. E_L/E_1 versus Q^2/gA_1D^3 and d_1/D

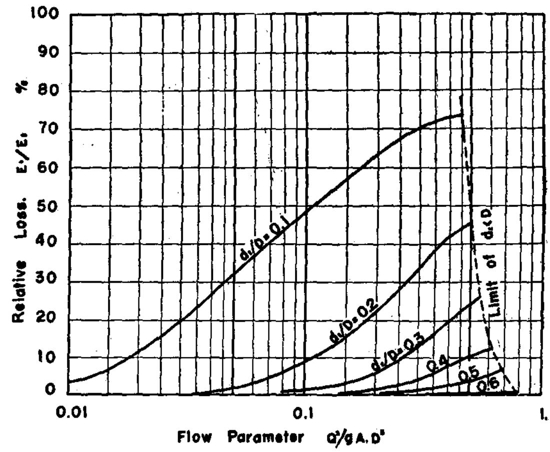

Fig 6. Verificati on of E_L/E_1

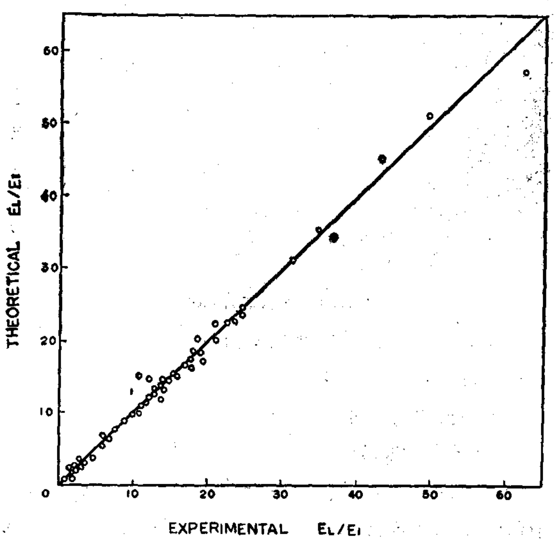

Fig 8. E_L/E_1 versus F and d_1/D