A Rational Expression on Leakage Through Annular Clearance and its Useful Applications

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Fig. 1 shows an annular clearance formed by a shaft of diameter D and its bushing of (D+a) 1. D., then the amount of leakage through the clearance can be calculated by

$$Q=.273\frac{PDa^3}{L\upsilon}\cdots\cdots\cdots\cdots\cdots(1)$$

Where Q amount of leakage, GPM

P Pressure difference across the bushing, Ft.

D Diameter of the shaft, inches

L Lengthe of bushing, inches

a diametrical clearance between bushing and shaft, inches

υ Kinematic viscosity, Ft²/sec.

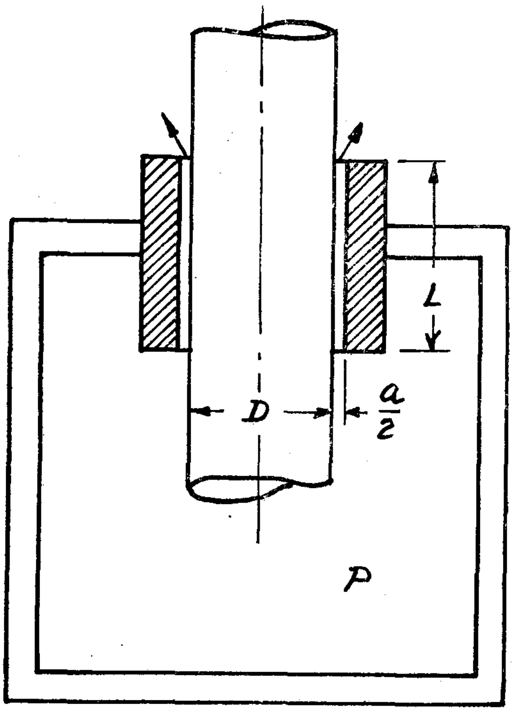

Fig. 1

The shaft can either be rotative or stationary. Since a stationary shaft does not present engineering problem in reducing or stopping the leakage, so our interest is naturely on the rotating shaft only.

Similar expressions appear in other literatures [1, 2]*, but equation (1) is the simplest and the most convenient one to use.

Both field observation and laboratory measurement check very closely with the calculated result. This adds to the confidence in using it.

One installation of high pressure vertical turbine pump involves following data:

$D=1''$

$a=.004''$

$L=3''$

$P=2800'$

$\upsilon=9.3\times10^{-6}$ Ft²/sec. (water viscosity at 80°F)

By calculation, Q=1.75 GPM

Actual leakage through seepage bypass line was estimated at 2 to 3 GPM

Our specially designed laboratory apparatus has shaft diameter of 1.375″, bushing clearance .006″, bushing length 3″ and at 300 psi or 693′ pressure, the leakage was measured .2 GPM for water temperature of 80 F. By calculation the result is same.

The derivation of equation (1) now follows:

use is made of two basic equations:

*Number in bracket refers to appended literatures

$$Q = VA \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$P = f\frac{L}{d}\frac{V^2}{2g} + .5\frac{V^2}{2g} + \frac{V^2}{2g} \quad (3)$$

Where f is frictional coefficient, dimmensionless.

A is the annular area, in Ft^2.

$$= \frac{a}{2}\pi D$$

Q is volume of leakage, in Ft^3/sec.

V is velocity of flow, in Ft/sec.

D, L and a, same meaning as before, but all in Ft.

d is diameter of circular pipe, and $d = a$ as far as hydraulic radius

is concerned. (Fig. 2)

$$\text{Hydraulic radius, } m = \frac{\text{area}}{\text{wet perimeter}}$$

$$\text{For pipe, } m = \frac{\frac{\pi}{4}d^2}{\pi d} = \frac{d}{4}$$

For annular clearance,

$$m = \frac{\frac{1}{4}\pi(d_2^2 - d_1^2)}{\pi(d_2 + d_1)} = \frac{d_2 - d_1}{4} = \frac{a}{4}$$

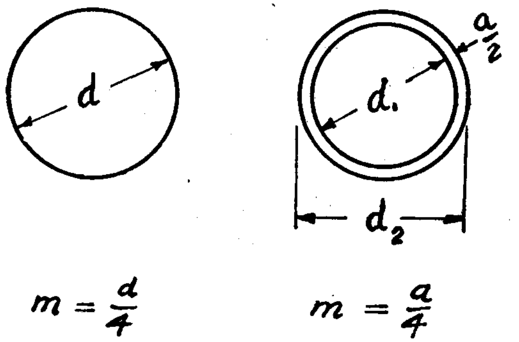

Fig. 2. Hydraulic Radius for Circular Pipe and Annular Clearance

P in eq. (3) represents the head in Ft. across the clearance. The first term represents the friction loss, the second the entrace loss and the last the velocity head. The last two terms are relatively small in compare with the first and hence are neglected. Eq.(3) then becomes,

$$P = f\frac{L}{a}\frac{V^2}{2g} \ldots\ldots\ldots\ldots (4)$$

For laminar flow, it can be determined theoretically

$$f = \frac{64}{R} \ldots\ldots\ldots\ldots (5)$$

$$f = \frac{96}{R} \ldots\ldots\ldots\ldots (6)$$

$$\text{R is Reynold's number } = \frac{aV}{\upsilon}$$

Substitute (6) into (4), we have

$$V = \frac{2g\ P\ a^2}{96\ L\ \upsilon}$$

$$\text{Then } Q = VA = \left(\frac{29\ P\ a^2}{96\ L\ \upsilon}\right)\left(\frac{a}{2}\pi D\right)$$

$$= \frac{\pi g\ PD\ a^3}{96\ L\ \upsilon} \ldots\ldots\ldots\ldots (7)$$

Give D, a, and L in inches, and Q in GPM, and combine all constants into one, equation (7) becomes (1).

No such neat expression can be derived for turbulent flow, because surface roughness enters the picture and complicates the whole thing. However for most of our leakage problems, laminar flow prevails. Therefore eq.(1) is valid for all practical purposes.

$$\text{For annular clearance, } R = \frac{aV}{\upsilon} = \frac{2Q}{\pi D\upsilon},$$

(Q in cu.ft./sec., D in ft.)

$$\text{or} = .0171\frac{Q}{D\upsilon}, \text{ (Q in GPM, D in inches)} \ldots\ldots\ldots\ldots (8)$$

Eq.(8) facilitates the calculation of Reynold's number.

In eq.(1), Q is propotional to a^3. The importance of small clearance in reducing leakage is very obvious. However, it is important but less obvious to relialize is the fact that it does not need really close clearance to bring down the leakage to a tolerable amount. For $a = .002''$, an easily obtainable clearance, a^3 becomes .000000008'' This fact makes several useful applications feasible:

A High pressure packing box (stuffing

box) or mechanical seal of all ratings incorporates a pressure relief line to bleed off the pressure. Fig. 3 & 4. Besides packing itself is nothing but a set a set of bushing whose clearance is adjustable.

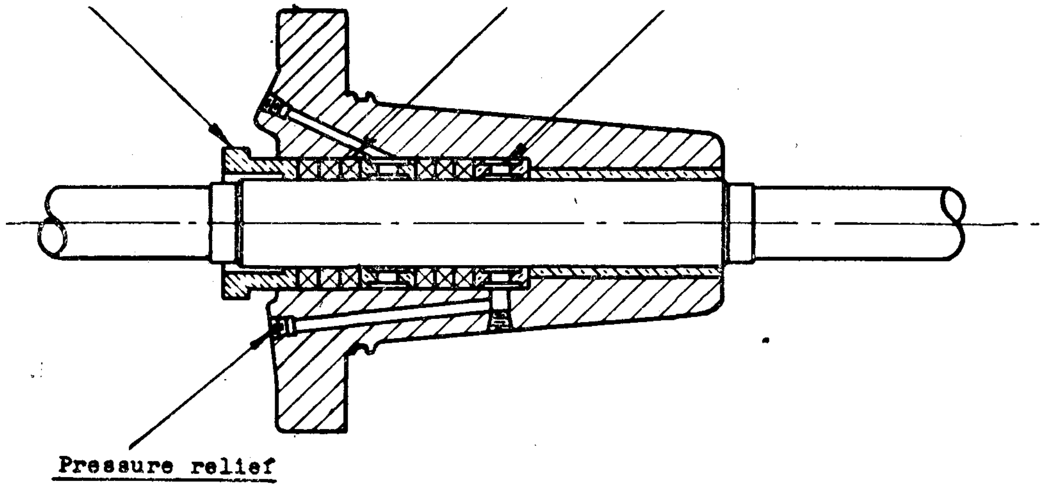

Fig. 3. Pressure relief in packing box

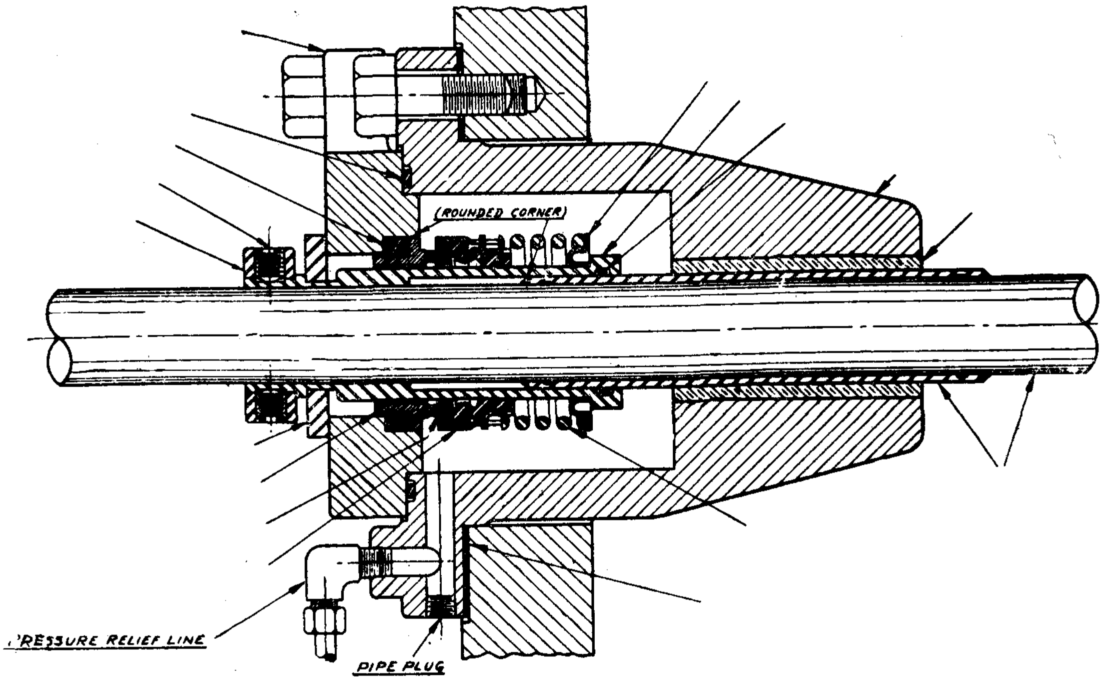

Fig. 4. Pressure relief in Mechanical Seal

For very high pressure and/or very high speed application, experience indicates that a plain bushing of proper clearance is the best way, sometimes the only way, to seal the fluid successfully.

B, Impeller balancing devices, such as balancing drum, balancing disc and balanced impeller, use eq.(1) to reduce the thrust. Fig. 5 is a balancing drum used in horizontal centrifugal pumps.

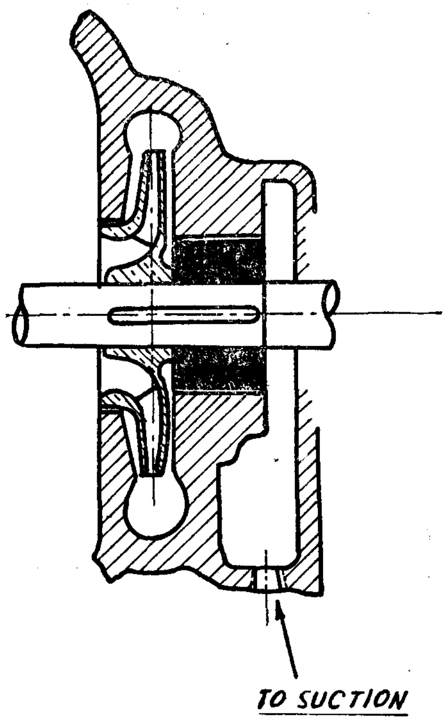

Fig. 5. Balancing Drum
The amount of thrust reduction equals the net drum area times pressure difference across the drum.

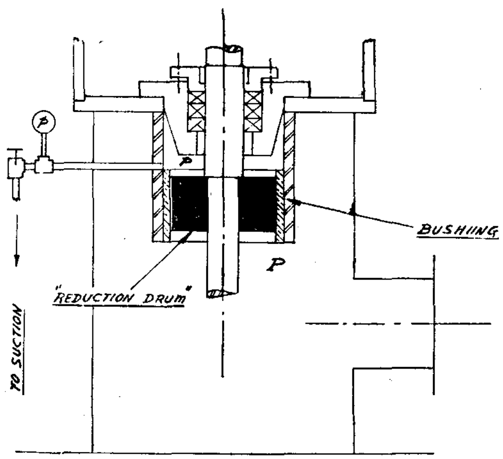

Fig. 6. Reduction Drum, Which reduces the thrust and pressure at same time.

C, Fig. 6 is a scheme conceived by the author, which can be used to reduce the down thrust on driver and to relieve the pressure on packing box at the same time. By installing a valve and a pressure gage, one can control the amount of reduction as desired. From time to time he can also, by measuring the volume of bypass, determine the proper time to replace the "reduction drum" or bushing.

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中 文 摘 要

求環隙漏量之公式及其應用

軸與軸承之間爲環形之間隙，稱爲環隙 (Annular Clearance) 如軸承之一方爲具有壓力之液體，則此液體將通過環隙而漏向他方，本文先敍述一個漏量公式之求得，繼從實驗所得與經驗所見，證實此公式在實用上之可靠性。此公式非作者所首創。類似之方程式，散見于文獻者有數處。唯本文所建議者爲最方便與合用。最後作者指出該公式之多種實用價值，使公式增添不少新意義。

漏量公式：$Q=.273\frac{PDa^3}{L\upsilon}$ (參看圖一)

Q	漏量，GPM	
P	軸承二方之壓差	Ft.
D	軸徑	Ins.
L	軸承長度	Ins.
a	直徑間隙	Ins.
υ	動力粘度	Ft^2./sec.

此公式之最值得注意一點，是漏量Q與間隙 a 之立方成正比。間隙影響漏量之大，極爲顯著。且間隙不必太小而漏量即已減至甚小。多種重要之應用，皆由此事實而來：

(一) 高壓塞漏器 (Packing Box) 與機械封 (Mechanical Seal) 類皆有減壓路之設，使液體從旁漏出少許而將壓力大大的降低。

(二) 抽水機操作時右軸向衝力 (Axial Thrust)

此衝力由馬達之鋼珠軸承承受之。但此力亦可由本公式之原理而減低或消除之，其引用之機件有所謂平衡鼓，平衡碟與平衡輪槳等。圖五示平衡鼓之構造。

（三） 在若干超高壓抽水機之設計與應用上，常見衝力之大非一般鋼珠軸承或他種軸承所能承受，壓力之高，亦非一般塞漏器或機械封所能抵擋。圖六示一種新想法，安置一減壓鼓于塞漏器之下，既可減壓又能減少衝力。一舉而二便也。

A Rational Expression on Leakage Through Annular Clearance and its Useful Application

補充更正 (閱七卷四期第31頁)

(A correction on the "Leakage Formula" No.4, Vol.7)

本刊七卷四期刊載拙作〞求環隙漏量之公式及其應用〞其公式之演譯上，有重大之疏忽，且非一言二語可以補救，爰請簡加說明如次：

按文中所指公式之幾種應用，實作者所指派之幾個研究項目，各項目性質各殊，但共同需要一個求漏量的公式。參考〔1〕中有此公式，應用後却不滿意，乃自下二基本方程式中自演之：

$$Q = VA \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (2)$$

$$P = f\frac{L}{d}\frac{V^2}{2g} \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (3a)$$

所得之公式與實驗結果相符，並證明參考〔1〕中之公式確有錯誤。本人所製呈服務公司之工程報告中，其公式之演譯，即係由此而來者。

後來本刊擬在美國會員中籌集一點經費，本人即將上述工程報告抽取一部份，寫成該文，送交〞機械設計〞投稿，希圖取得一筆稿費，而資助本刊。〔註一〕以事關私事，不便交公司打字員打字，乃自爲之，一面打字，一面又增損，將方程式 (3a) 易爲 (3)：

$$P = f\frac{L}{d}\frac{V^2}{2g} + 0.5\frac{V^2}{2g} + \frac{V^2}{2g} \cdots\cdots\cdots\cdots\cdots\cdots (3)$$

(見參考〔3〕)

又以二與三兩項微不足計爲由而除去之，而錯誤即由此生，因二、三兩項非但不小，且遠比第一項爲大，如此則本公式不能成立。即合實用，亦不能以合理公式而稱之。

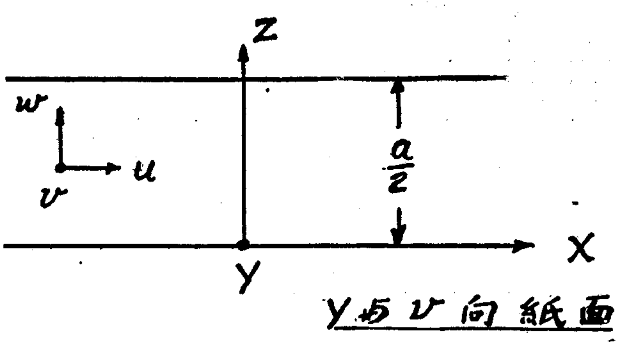
參考〔1〕與〔3〕之作者，俱爲當代美國之抽水機權威，上述之困難，彼等亦有一份在內，如能從不合理中而得合理之解決，則畫蛇添足之粗疏，並非完全無意義。

同輩留美學人吳耀祖兄，在附近加省理工大學任教，精於流體力學之理論，即以此事就商。彼另闢蹊徑，以液體流過微小之環隙實與流過二平行面者相等。因之援用基本之連續方程式 (Continuity Eq.) 與動量方程式 (Momentum Eqs.)〔註二〕，卒證得公式本身之無誤。同時亦斷定方程式(3)中之二、三兩項實不應存在，有機會時擬再與原作者討論之。本會國內外會員中，以流體之理論見長者，頗不乏人，還祈指正爲幸。

又文中方程式(5)"For circular pipe"與方程式(6)"For annular channel"漏落，亦併爲補正。

註一、"MACHINE DESING"以文太專門化，不合其一般讀者之胃口，未取用，後本刊徵得敝人同意而發表之。

註二、Continuity Equation (圖A)



$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \cdots\cdots\cdots\cdots\cdots\cdots (a)$$

Momentum Equations

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) \cdots\cdots\cdots (b)$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) \cdots\cdots\cdots (c)$$

$$u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) \cdots\cdots\cdots (d)$$